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METEOROLOGICAL OFFICE

INVESTIGATIONS DIVISION TECHNICAL NOTE NO 3

PROBABILITIES OF AIRCRAFT ENCOUNTERS WITH HEAVY RAIN

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1 Introduction

The possibility of erosion of aircraft structures by rain becomes increasingly important as the operating speed of aircraft increases. Designers and airworthiness authorities require estimates of the chances that an aeroplane will meet rain that might cause structural damage or might lead to the wearing-away of materials.

This note presents some estimates of the probability that an aircraft will meet heavy rain. The estimates are based on available rainfall data and are limited to flight in the vicinity of a few stations but the method can be used for any place or route for which data are to hand. Since the problem of rain erosion is of great importance to supersonic aircraft the estimates have been based on the probable operating conditions for Concorde.

2 Method

Suppose that the probability of a particular rate of rainfall at a point on the ground is P_p whilst the corresponding probability of the same rainfall rate somewhere inside a unit area is P_a. Then if the typical diameter of a rain cell of the intensity being considered is R, where R is considerably less than unity, we have

$$P_a \simeq \frac{P_p}{\pi R^2}$$

Now assuming that the rainfall probability is the same aloft as at the ground and that no avoiding action is taken then the chance of an aircraft encounter with a rain cell during the crossing of an area of unit radius is

$$\frac{4 P_a R}{\pi} \quad \text{of} \quad \frac{4P_p}{\pi R}$$

If the aircraft speed is V then the time taken to cross the area is 2/V and so the probability of an encounter with rain of the specified intensity in unit time is

$$\frac{4P_p}{\pi R} \cdot \frac{V}{2}$$

or

$$\frac{2P_p V}{\pi R} \quad \text{-------------} \quad 1$$

3 Estimates of P_p

Much rainfall information is available for many stations though the data are often limited to daily or monthly rainfall totals whereas the problem here relates to instantaneous rainfall rates. However, for an increasing number of places the use of recording rain gauges has permitted hourly rainfall totals to be obtained and these totals can be used to determine the required probabilities of instantaneous rainfall intensities.

Briggs and Harker (1) have obtained typical distributions of two-minute rainfall rates about the clock-hour totals and have hence derived conversion factors which enable occurrence of instantaneous rates of rainfall to be assessed on the basis of available clock-hour data. Estimates of P_p thus obtained are presented in Table 1 for a limited number of stations and rainfall rates.

Table 1

Probability of occurrence of instantaneous rainfall at or exceeding specified intensities

	Rainfall intensity (mm/hr)		
Place	25	50	100
Heathrow	1.26×10^{-4}	1.94×10^{-5}	1.14×10^{-6}
Singapore	1.83×10^{-3}	6.85×10^{-4}	1.48×10^{-4}
Freetown	3.31×10^{-3}	1.26×10^{-3}	3.19×10^{-4}

4 Rain cell diameters

The pattern of rainfall during the passage of a heavy shower can vary widely but in general each period of light or moderate rain will include a shorter period of heavier rain. On the average the shower profile will have a reasonably smooth intensity - time distribution and the higher the intensity considered the shorter will by the typical duration.

Durations of rainfall can be determined by inspection of the charts of recording rain gauges and may then be combined with estimated speeds of movement of the rain-bearing clouds to give values for the rain cell diameters. Table 2 presents estimates of typical rain cell diameters obtained for the places listed in Table 1.

Table 2

Average cell diameter (km) for rainfall at or exceeding specified intensities

	Rainfall Intensity (mm/hr)		
Place	25	50	100
Heathrow	3	2	1.5
Singapore	3.5	2.5	2
Freetown	4	3	2

As must be expected the typical diameter decreases as the intensity of the rainfall increases. The table also reflects the influence of the average temperatures of the places concerned - the amount of water vapour which is ultimately available for release as rain is temperature dependent and so the average cell diameter increases as the temperature rises. The figures of Table 2 are also in good accord with the diameters suggested by radar studies of the cores of heavy showers.

Rain cell diameters will vary with altitude but it will be assumed here that the diameter remains reasonably constant throughout the depth of a heavy shower.

5 Variation of the probability of rainfall occurrence with height

The rainfall rates of interest in the rain erosion problem are mainly those exceeding, say, 10 mm/hr and such rates are normally limited to showery conditions though orographic intensification can cause such intensities inside wide spread frontal-type rain. Radar studies, see, for example, Hamilton (2), give some indication of the distribution of precipitation in the vertical. Although widespread rain usually shows a steady decrease of precipitation content with height increase it seems that the large precipitation densities occurring in the core of severe showers have a fairly uniform distribution throughout the bulk of the shower cloud.

Our problem here is to determine how the probability of rainfall occurrence varies with height on the average, not just for one particular shower cloud. The probability at a given height will be compounded of the probability of a shower cloud top reaching to that height together with the probability of a given rainfall rate inside a shower cloud which extends to a given height.

Some indication of the probability of a given precipitation rate inside a shower cloud has been obtained by Donaldson (3) who measured radar reflectivities inside some 233 thunderstorms over New England. Radar reflectivity is essentially a measure of the water-substance contained in the larger rain drops, snow and hail but in interpreting the radar observations it is necessary to consider how the normal fall-off of temperature with height affects the rain/snow ratio. In many rainfall situations most of the large raindrops can be expected to have become frozen at heights above that corresponding to temperature of $-20^{\circ}C$, ie above about 7 km for temperate climates and about 8 km for tropical climates. However the heavy rain of main interest occurs in heavy showers or thunderstorms where strong updraughts rapidly distribute the water drops and so give good reason to ignore the freezing of the drops in the time available before cloud top is approached. Thus it is thought that Donaldson's profiles of radar reflectivity in thunderstorms give a good picture of the height distribution of the large rainfall rates now being considered.

Figure 1 presents the median profiles obtained by Donaldson. The figure suggests that the median rainfall rate is nearly constant to about 6 km and then decreases rapidly. Variation of median rainfall rate with height is not quite what we need - we wish to assess the variation in frequency of occurrence of a specific rainfall rate as height varies though of course Figure 1 does suggest that there is little variation up to about 6 km. Again Donaldson gives us some guidance by presenting frequency distributions for specified reflectivity values at levels of 5000, 20000, 30000 and 40000 feet. These frequency distributions yield, approximately, the rainfall rate distributions summarized by Table 3.

Table 3

Percentage of New England thunderstorms which have precipitation rates equal to or exceeding specified values

	Precipitation Rate (mm/hr)		
Height (km)	25	50	100
$1\frac{1}{2}$	55	30	20
6	40	20	10
9	10	3	1
12	5	1	<1

Table 3 refers to total precipitation whereas we are trying to determine variations in rainfall only. It is seen that the fall-off with height in the frequency of occurrence of a given total precipitation rate is most marked for high values of that rate. Now the largest precipitation rates are associated with the largest up draughts and so with the lowest likihood of drops becoming frozen by a given height. Hence freezing will affect relatively more raindrops at rates of 25 mm/hr than at 50 mm/hr, or again at 100 mm/hr, and so will tend to even out the differences between the columns of Table 3 when the probability of rainfall only is being considered. Again this implies that column 3 of the table is the most likely to indicate how the frequency of occurrence of a given rate of rainfall falls off with height. Thus, at least for New England storms which extend to over 12 km, it seems that the probability of occurrence of a given rate of rainfall is nearly constant to about 6 km and then decreases by an order of magnitude for each 3 km increase in height.

As indicated above, the overall probability of occurrence of a given rainfall rate depends also on the height distribution of shower cloud tops. Radar studies indicate these distributions and, for example, Moore (unpublished) has obtained the following percentage frequencies for radar echoes around Singapore.

Table 4

Frequency (%) for height of highest radar echo near Singapore

Height (km)	Frequency (%)	Height (km)	Frequency (%)
≥ 9	85.1	≥ 16	16.6
≥ 10	83.0	≥ 17	9.7
≥ 11	77.7	≥ 18	4.0
≥ 12	68.2	≥ 19	1.4
≥ 13	53.2	≥ 20	0.5
≥ 14	40.2	≥ 21	0.2
≥ 15	27.6		

Table 4 suggests that the frequency of cloud tops at a given height begins to decrease at some height near mid-troposphere and that near the tropopause (around 16 km at Singapore) the rate of fall-off in frequency of occurrence of cloud tops is about an order of magnitude for each 3 km increase in height.

Combining the indications of the two preceding paragraphs it seems that the probability of occurrence of a given rainfall intensity is near constant to about mid-troposphere then decreases by somewhat more than one order of magnitude in 3 km the rate of decrease approaching one order of magnitude - 3/2 km around the tropopause. So the probability of occurrence of a given rainfall intensity, P_H, at a given height H can be expressed as

$$P_H = P_p \quad \text{for } H \leq T/2$$

$$P_H = P_p 10^{-k(H-T/2)} \quad \text{for } H > T/2 \qquad \text{------------ 2}$$

where T = tropopause average height

and k has a value somewhere between $\frac{1}{2}$ and $\frac{2}{3}$.

In using the relation 2 above it must be noted that since water drops tend to freeze spontaneously even without nuclei when the temperature approaches -40°C then it is likely that all rain drops will be frozen at heights somewhere about 13 km so rainfall probabilities above 13 km should approximate to zero. However at heights above 13 km or so mushy hail may produce effects equivalent to those of rain and so the use of relation 2 may still be informative.

6 Estimates of rainfall encounters for Concorde

Relations 1 and 2 can be used together with the appropriate values of P_p, R and T to determine probabilities of aircraft encounters with rainfall of a specified intensity. It is necessary to know the height-speed profile of the aircraft and for Concorde the following values are indicated by probable operational routines:-

Height (km)	Speed (m/s)	Height (km)	Speed (m/s)
3	200	12	375
6	250	15	500
9	300	18	600

Using these values and relevant rainfall data for Heathrow, Singapore and Freetown the intensity of rain likely to be met once in 10^5 flight hours has been determined. Figure 2 presents the results using two values, ie $\frac{1}{2}$ and $\frac{2}{3}$, for the factor k in relation 2. It is thought that the dashed lines corresponding to $k=\frac{1}{2}$ are more likely to correspond to actual experience especially at the highest levels though the solid lines corresponding to $k=\frac{1}{2}$ may provide safer planning guidance.

7 Summary

Estimates of probabilities of aircraft encounters with heavy rain have been obtained for three localities. The estimates are necessarily based on somewhat arbitrary assumptions, especially as regards the variation in rainfall probabilities with variation of height. However the assumptions are reasonably supported by observational evidence and the method used has the merit that estimates can be made fairly readily for any area where the available rainfall data are adequate.

References

(1) Briggs J and Harker J A — Estimates of the duration of short-period rainfall rates based on clock-hour values. Met Mag 98 1969 p246

(2) Hamilton P M — Vertical profiles of total precipitation in shower situations QJR Met Soc 92 1966 p346

(3) Donaldson R J D — Radar reflectivity profiles in thunderstorms. J Met 18 June 1961 p292

February 1971
AF/M 1447/71
MET O 9

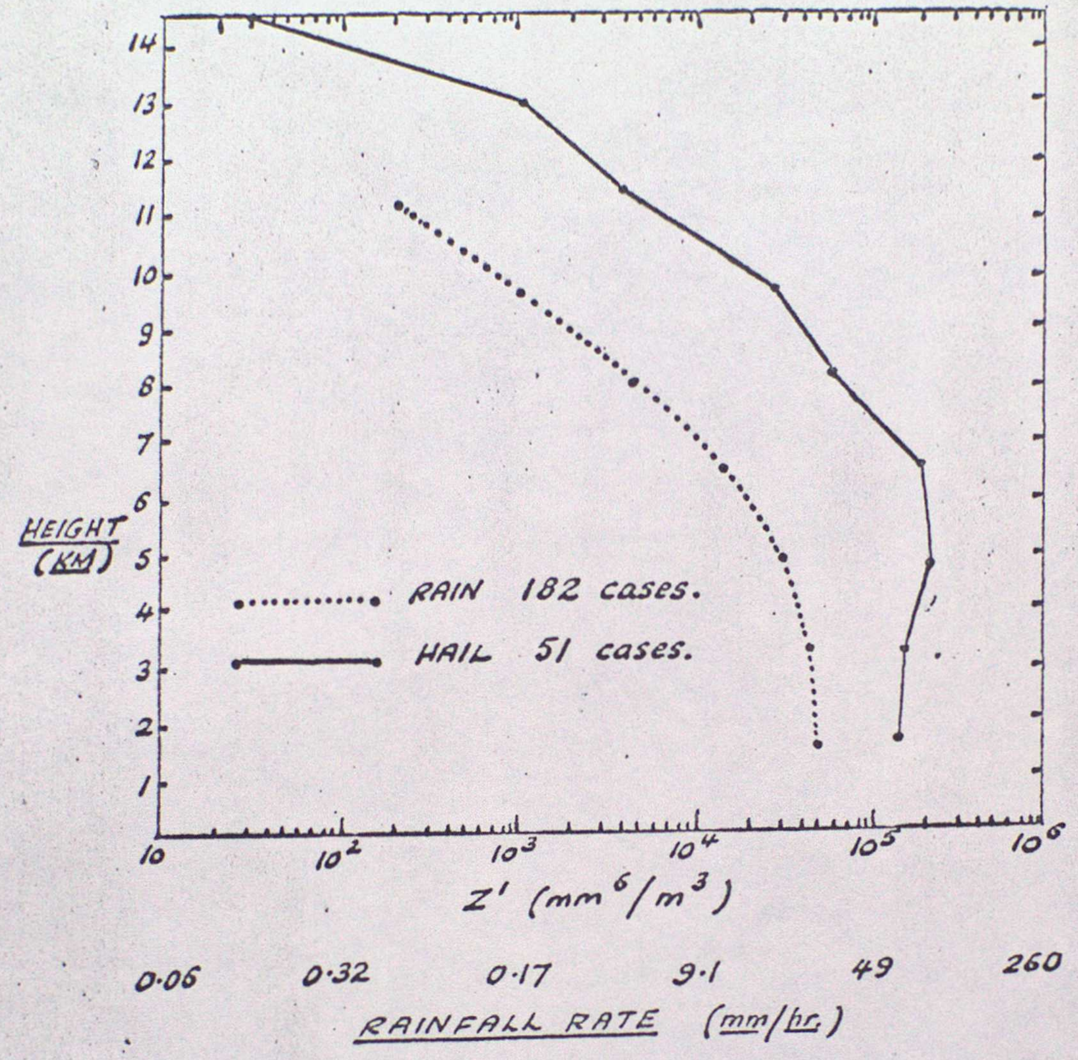

FIGURE 1. MEDIAN VALUES OF RADAR REFLECTIVITY AND OF EQUIVALENT RAINFALL RATE FOR NEW ENGLAND THUNDERSTORMS (After DONALDSON (3))

FIGURE 2

Intensity of rain met once in 10^5 flight hours (all at constant height) against height for three localities a) HEATHROW b) SINGAPORE c) FREETOWN.