METEOROLOGICAL OFFICE

FORECASTING TECHNIQUES BRANCH MEMORANDUM No. 6

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Abstract

Twenty five 300 mb objective numerical forecasts, produced at weekly intervals by a regression equation technique and based on objective numerical forecasts for 1000, 500, 200 and 100 mb, were compared with subjective forecasts prepared at London (Heathrow) Airport. The comparisons were made in terms of equivalent headwind along seven standard tracks from London, in a manner related to the use which is made of wind forecasts by airline companies. Verification data were derived from Heathrow actual charts. The results are summarised in para 8, Figs 6-9 and Tables 3,4. In Annex A some characteristics of equivalent headwinds are discussed. Annex B deals with the variances due to chart construction and extraction of equivalent headwinds.

1. Form and content of forecasts

a. During the period covered by these tests all subjective forecasts issued by Heathrow were in chart form. Forecasts were for 26 hours from the latest completed actual chart but those for European flights were not issued until after receipt of the intermediate six-hourly wind observations and amendments based on these observations could be incorporated. All objective forecasts were for 24 hours and in chart form.

b. In strong winds, it is rare for the actual wind direction to differ from the geostrophic direction by more than ten degrees (1). The practical effect of a directional error of ten degrees on a fast aircraft flying at 400 kt. is to change the equivalent headwind about 10 kt. in 300 mb average wind speeds of 50 kt but it may produce errors in equivalent headwind of up to 28 kt in a jet core speed of 150 kts when this is orientated at about 110° to the track of the aircraft (Annex A). This effect is neglected in forecasts, where wind direction is specified by the direction of the contours. However, departures of wind speed from geostrophic values can be considerable and cyclostrophic effects in strongly curved flow may reach 15% of the geostrophic wind speed. These and other departures from geostrophic wind speed are incorporated with the geostrophic component in the Heathrow subjective isotach patterns. No special care is taken to ensure that the contour spacing on these charts reflects the forecast geostrophic wind and contours are intended to indicate wind directions only. Winds were extracted from the objective forecasts by applying a geostrophic wind scale to the contour spacing. No corrections were made for curvature of the air trajectory.

2. Data Extraction

a. An equivalent headwind scale was employed which combined the headwind and beamwind components into a single equivalent headwind for a flight at a true air speed of 400 kt. The scale was applied to seven standard tracks from London, three over the Atlantic and four over Europe (Fig.1). These tracks do not correspond precisely to routes actually flown but they sample the forecast area adequately. The three Atlantic routes represent the eastern portions of the Lindy Line*, Great Circle and Rhumb Line routes to New York. The western limits are imposed by the western edge of the numerical forecast area which, at the time of these tests, extended to about 45°W only. Routes close to the three standard tracks are regularly used because of the varying destinations in North America - for example, the Great Circle route to Montreal lies close to the Lindy Line route to New York.

b. For the test to be realistic it should simulate the methods of least-time track selection practised by the air-line companies, in which the choice of track depends both on the forecast winds and the ground distance along each route. When track selection is involved forecasts must be assessed in two ways: firstly as to whether the least-time track has been forecast correctly - failure to do this causes an increase in flight time and operating costs - secondly, as to whether the winds on the

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* A Lindy Line (also called a Polar Curve) is a reflection of a Rhumb Line in a Great Circle.

indicated least-time track have been correctly forecast - failure here affects the timing of the operation. A completely realistic analysis is difficult because air line companies are not restricted to the three tracks used in these tests. Also the available objective forecasts extended to 45°W only and it is very doubtful whether conditions on the eastern halves typify the whole routes. The results (Figs 8,9, Table 3) must therefore be treated with reserve.

c. It was necessary to devise a means of allowing for the different distances involved on the three routes. The shortest practical route from London to New York which is permissible under current operating requirements, is a Great Circle via Shannon and Gander (3046 n. miles). The practical tracks which lie closest to the Lindy Line and Rhumb Line used in these tests are a Lindy Line via Belfast (3155 n. miles = Great Circle + 109 n. miles) and a Rhumb Line via Cape Cod (3117 n. miles = Great Circle + 71 n. miles). The extra distances involved, on the Lindy Line and Rhumb Line tracks were converted into equivalent headwinds by dividing by the total times of flight:-

$$\text{Lindy Line} - \frac{109 \times 400}{3155} \simeq 14 \text{ kt}$$

$$\text{Rhumb Line} - \frac{71 \times 400}{3117} \simeq 9 \text{ kt}$$

The equivalent headwinds measured from the charts were increased accordingly for the purpose of the track selection tests. The adjusted equivalent headwinds for all three tracks on both forecast and actual charts were inspected and the smallest forecast value compared with the smallest actual value. Additionally, to assess the timing errors, the equivalent headwind on each forecast least-time track was compared with the actual equivalent headwind for the same track.

3. The standard for comparison

A recent Canadian investigation (2) employed observations to assess forecast errors but this is not practicable on routes such as the Rhumb Line to New York and winds measured from drawn actual charts were used instead. The limited comparison at Table 1 of equivalent headwinds based on actual observations and on drawn charts indicates no advantage in using observations even when these are ideally placed on the route and if the corrections proposed at Annex B are applied then the charts may provide better assessments of actual conditions. The Heathrow actual chart was therefore adopted as the standard. Isotachs are not drawn on actual charts as routine and these had to be constructed before equivalent headwinds were measured.

4. Use of the headwind scale

The scale was applied to each of the seven standard tracks. Mean winds were estimated for distances of approximately 300 n. mile along each route. Each section involved a critical estimate of the mean wind direction and speed over this distance. In effect, the procedure required the extraction of a mean vector wind, which can be difficult to obtain when the patterns of contours and isotachs are complex. If a sharp trough or ridge occurred in any sector then the scale was applied more than once, and a suitably weighted mean equivalent headwind obtained. This procedure ensured that beamwinds from both sides of the track contributed towards the equivalent headwind. There were five zones of approximately 300 n. mile on each of the three Atlantic routes and two each on the four European routes.

5. Data presentation

Forecast errors were expressed as differences between the forecast equivalent headwinds and the verifying equivalent headwinds extracted from the Heathrow actual charts. Accumulated frequency curves were plotted on normal (arithmetic) probability graph paper for each of the seven tracks separately (Figs 2,3,4,5). The indicated departures from gaussian distributions are not surprising in such small data samples, especially in view of the dependence of the error distribution on the orientation of each track with respect to the wind flow (Annex A). Data for the three Atlantic routes were combined into subjective and objective Atlantic populations (Fig 6) and data for the four European routes were combined into subjective and objective European populations (Fig 7). Table 2 shows the

/mean

mean errors and standard deviations for the seven tracks separately and also for the combined 'Atlantic' and combined 'European' routes. The justification for combining the routes in this way is provided in Table 2 by the 'Students' t-tests of the null hypotheses that the differences between the data for the individual routes and the data for the respective combined route could arise by chance if the various data samples belonged to the same population. All the probability levels exceed 10% and are especially high for the Atlantic routes. The adjusted standard deviations for the combined Atlantic and combined European routes (shown in brackets in Table 2) were obtained by correcting for the standard deviations due to drawing of contours and isotachs and also due to measurement of equivalent headwinds (see Annex B).

6. Results

a. On the Atlantic routes (Fig 6) the objective forecasts show a markedly smaller mean error and standard deviation than the subjective forecasts. There are fewer large negative errors (underestimate of headwind) in the subjective forecasts than in the objective but this only occurs because there is a marked bias towards positive errors in the entire subjective distribution. This bias may represent a tendency on the part of forecasters to overestimate headwinds systematically on Atlantic routes. This can easily occur when isotachs are used to define wind speed because it is no longer necessary for contour spacing to conform to geostrophic wind speed (para 1). This bias is not shown in other tests (Harley[7], McGain[8]) which treated forecasts of contour pattern without isotachs. The standard deviations for all three of these tests of subjective forecasts are comparable if adjustments are made for route length. A tendency towards equivalent tailwind bias in eastbound Atlantic flights (corresponding to headwinds for westbound flights) is reported in the Canadian investigation (2). The Canadian forecasts were based on the U.S. Weather Bureau barotropic numerical forecasts and the authors suggest that the bias may be due to any of the following:-

(1) flattening of troughs and ridges

(2) a tendency to extrapolate jet streams too far and produce excessively high maxima

(3) actual winds may be ageostrophic.

Whatever the reason, there should be no tendency for Canadian forecasters to bias deliberately towards tailwinds.

b. For the European routes (Fig 7) both the subjective and objective forecasts yield mean equivalent headwind errors close to zero and also large standard deviations. The bias towards positive errors, evident in the subjective Atlantic forecasts, is not apparent in the subjective European forecasts. This lack of bias is possibly because the equivalent headwinds on European routes are more a function of the contour pattern and its position with respect to each route than of the isotach pattern (see Annex A). Consequently any tendency to bias the forecast towards positive equivalent headwinds on one route by adjusting the position of a trough or ridge would tend to produce negative bias on adjacent routes.

c. Fig 8 shows graphs of the accumulated frequencies of errors in forecasting the least-time track to New York and Fig 9 shows the distribution of timing errors on the indicated least-time track (para 2b). All four curves display smaller extreme positive errors than might be expected in a normal distribution and the objective forecasts also show smaller extreme negative errors. The standard deviations of both the least-time track error and the timing error (summarised in Table 3) are larger for the subjective forecasts than for the objective forecasts. There appears to be a small positive bias in the subjective least-time track errors and a small negative bias in the objective timing errors. Inspection of the 25 cases showed that, although the objective forecasts indicated the correct least-time track on 18 occasions, against 14 by the subjective, three of the four cases 'missed' by the subjective forecasts produced errors of 5 kt or less. Fig 10 shows the frequency distribution of the differences, taken from the verifying actual charts,

/between

between equivalent headwinds on the Great Circle and least-time routes. By definition of the least-time route the differences can only be positive and Fig 10 shows the expected bounded distribution. The 5-year means shown in Table 4 indicate that the mean advantages offered by the Lindy Line and Rhumb Line are effectively extinguished by the extra ground distances which are equivalent to 14 kt and 9 kt respectively. Therefore the Great Circle route should be the least-time route on roughly one occasion in three and advantages should only accrue on the other two routes on about two occasions in three. In the small sample of 25 synoptic situations the Great Circle route was the least-time track on seven occasions . Because the distribution in Fig 10 is bounded, the mean (16.4 kt) and standard deviation (17.1) were calculated from the standard formulae. If the seven cases where the Great Circle is the least-time track are excluded then the mean is 22·8 kt and the standard deviation 16·0. Although these means may decrease if the entire route from London to New York were considered: nevertheless they indicate that material advantages can be gained from track selection if the least-time track is forecast correctly. The above means and standard deviations are appended to Table 3 for comparison with the values for the subjective and objective forecasts.

7. Representativeness of the data samples

Fig 11 shows accumulated frequencies of actual equivalent headwinds, taken from the Heathrow verifying charts, for the three Atlantic routes combined. The distribution is clearly very close to normal, although slightly leptokurtic in the top and bottom five percentiles. Means and standard deviations of the Heathrow verifying equivalent headwinds for all seven routes were compared with values based on five years of geostrophic winds from contour charts (3). The long-period values compared with the Lindy Line to New York are based on a route via Keflavik to Narssassuaq and those compared with the Rhumb Line to New York are based on part of the Great Circle route to Bermuda (see Table 4). The short-period means for the three Atlantic routes are all 7 or 8 kt less than the long-period means - differences which, if they belong to the same population, could arise by chance in about three to five occasions in ten. The short-period standard deviations for all three routes are noticeably smaller than those for the long-period samples. The short-period means for the European routes differ from the long-period means by amounts ranging from +14 to -11 kts and, except for the London - Rome route, the differences would arise by chance on less than one occasion in ten, assuming they came from the same populations as the long-period samples. The standard deviations for three of the four short-period means are very similar to those of the long-period samples. Hence it would seem that, for the Atlantic routes, although the short-period samples appear to be reasonably typical of mean values, they represent a relatively quiet period when disturbances from average conditions were less than normal. The short-period European samples are somewhat less typical of the long-period means but the standard deviations suggest a variability typical of the long-period samples.

8. Summary

a. A sample of 25 objective numerical forecasts prepared at weekly intervals by a regression equation technique was compared by means of equivalent headwinds, with a corresponding sample of 25 subjective forecasts. The comparison showed that, for the small sample compared,

(1) The objective forecasts for European routes are probably as reliable as the subjective forecasts

(2) The objective forecasts for Atlantic routes are significantly better than the subjective forecasts

(3) Atlantic least-time tracks are probably detected more reliably by means of the objective forecasts and the timing errors on the selected tracks are less than those in the subjective forecasts.

(4) The data samples are small and it is not certain whether the above results are typical of a longer series.

(5) The least-time track selection tests are particularly open to criticism because the objective forecasts covered only half the Atlantic.

The following additional conclusions were drawn from the Annexes.

/(6)

(6) Annex A

For a wide range of angles between the actual wind and the track, errors in forecasting the wind direction in degrees have a greater effect on the equivalent headwind than errors of similar magnitude in forecasting the wind speed in knots.

(7) Annex B

Errors occur during the extraction of equivalent headwinds from forecast charts and it is possible that, with the advent of objective numerical methods, forecasts might be more accurately presented in the form of mean winds for each grid square. This method of data presentation would gain if a smaller grid length is eventually adopted and would have obvious appeal if airline operators employ computers for track selection and flight planning.

Acknowledgements

Thanks are due to A Binding, R. Dalgleish and P. Sowden who constructed isotachs on the verifying actual charts; also to C.L. Hawson who read the text and offered a number of helpful suggestions, especially regarding the treatment of the statistics.

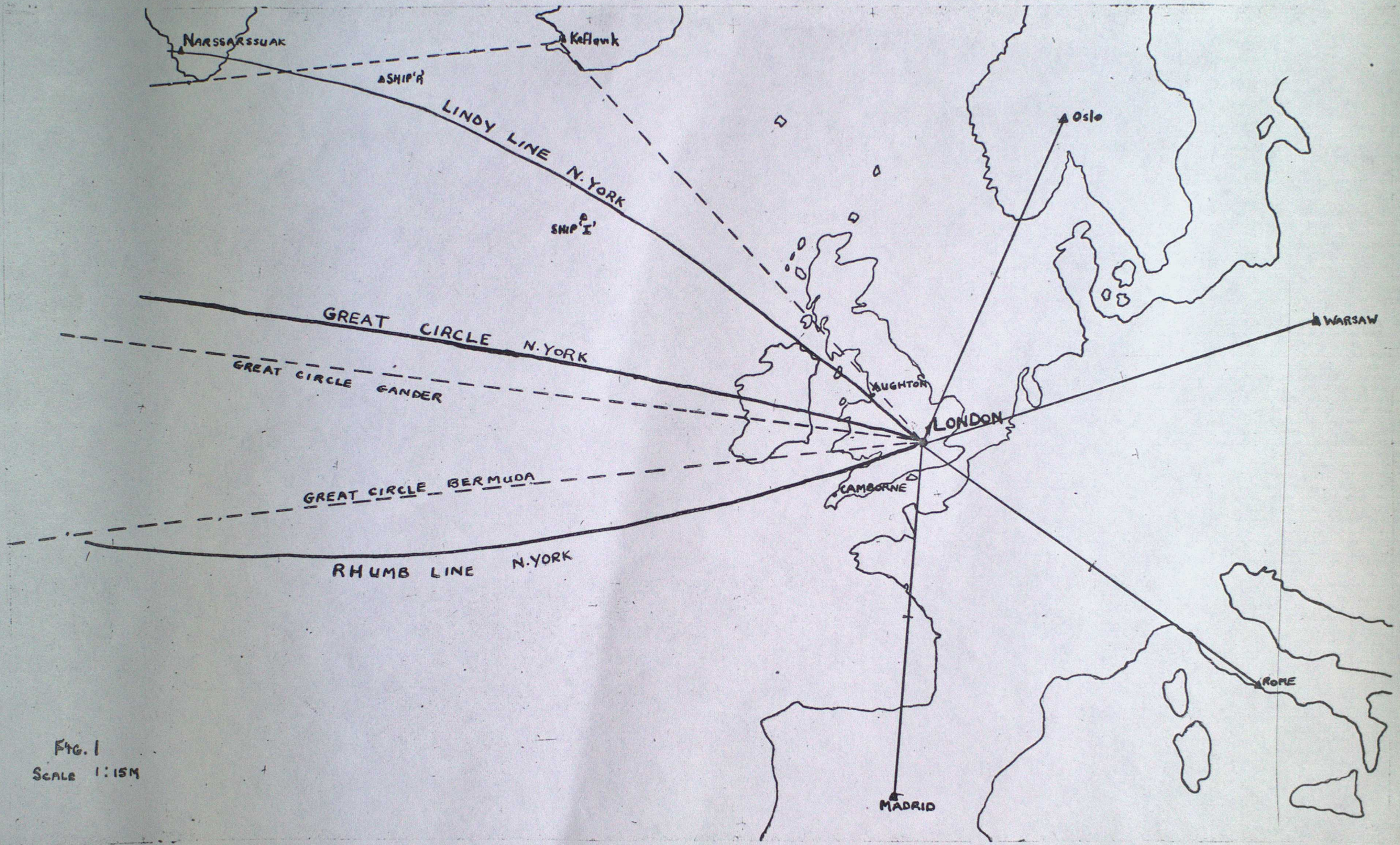

FIG. 1
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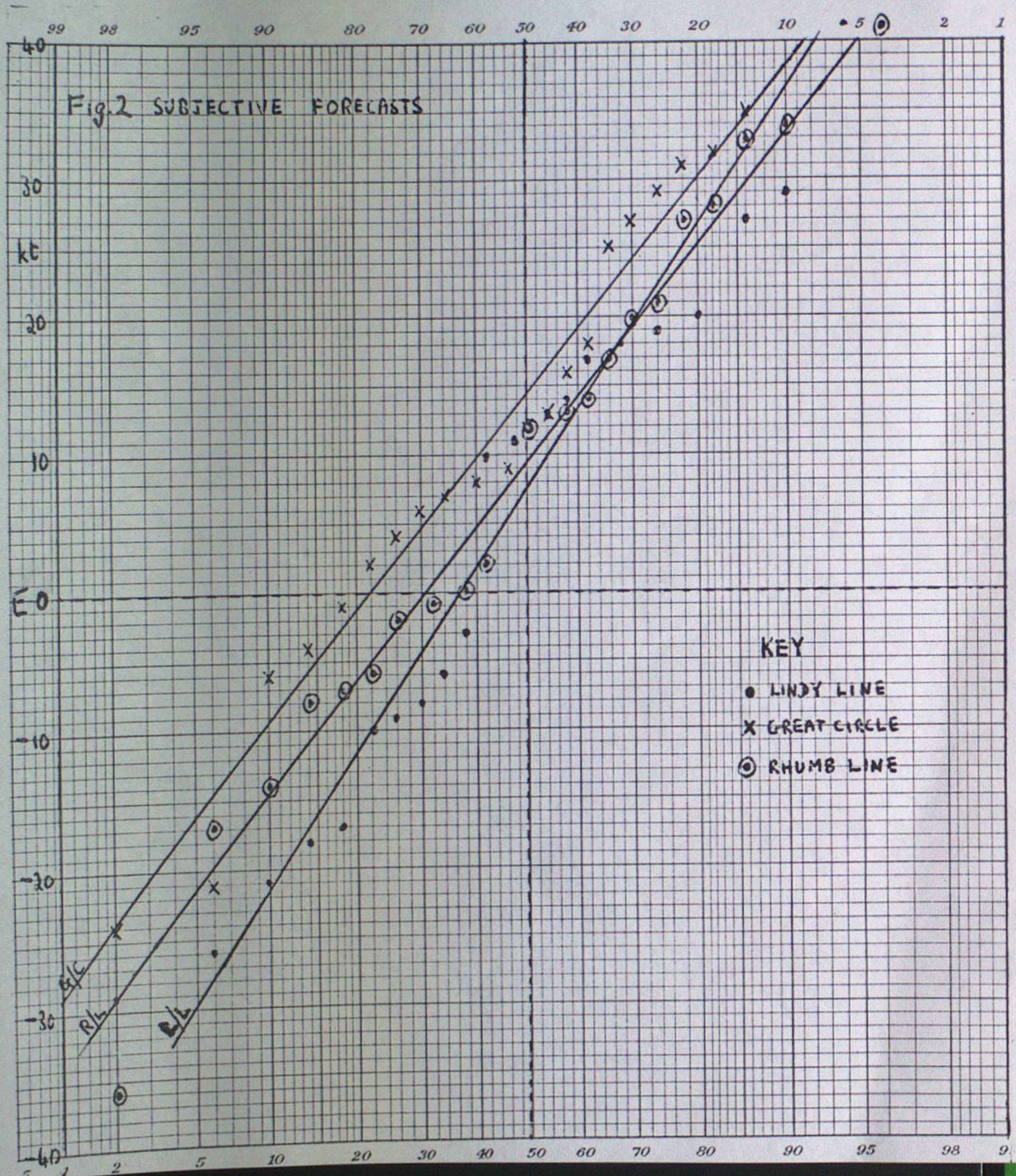
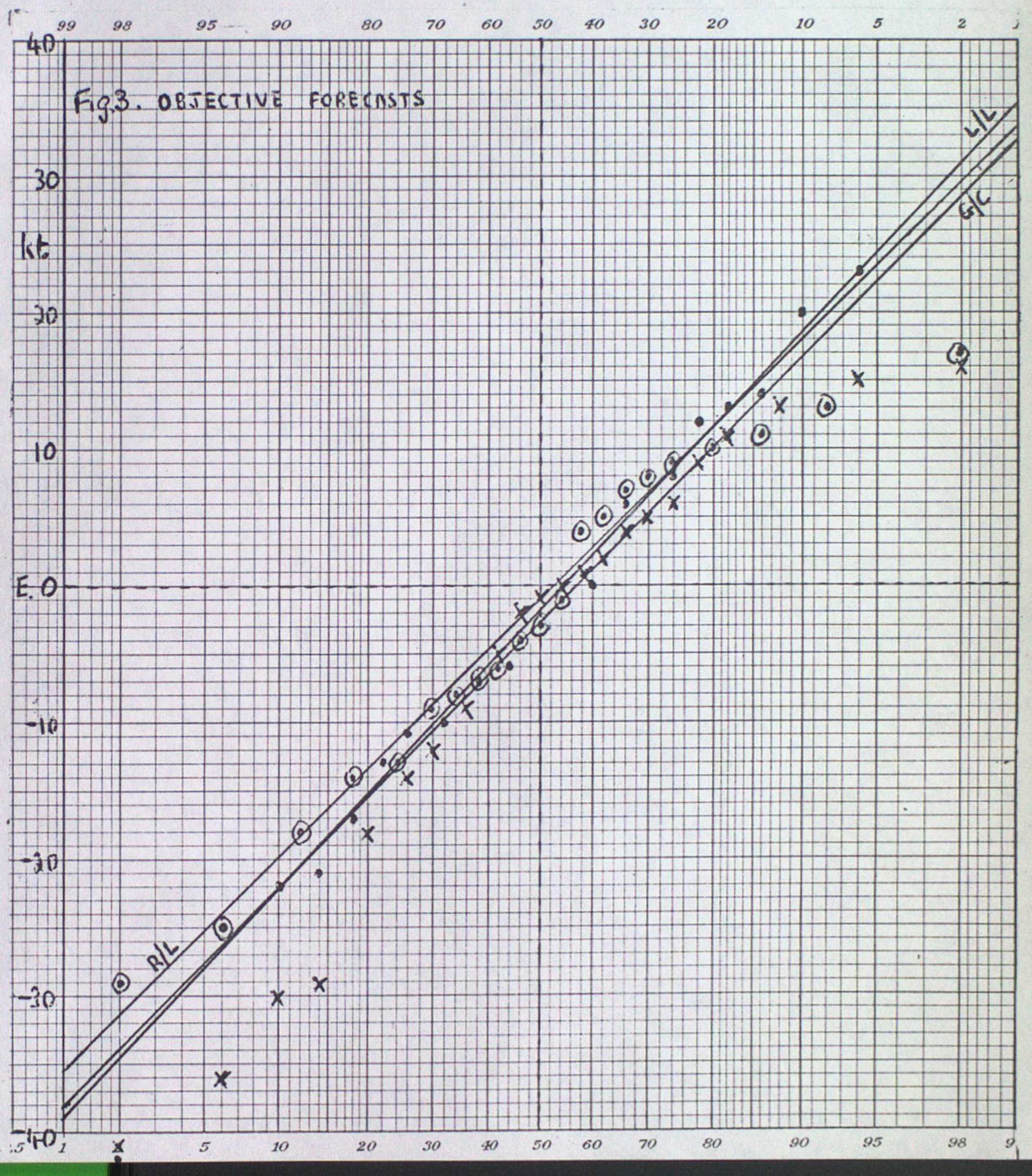
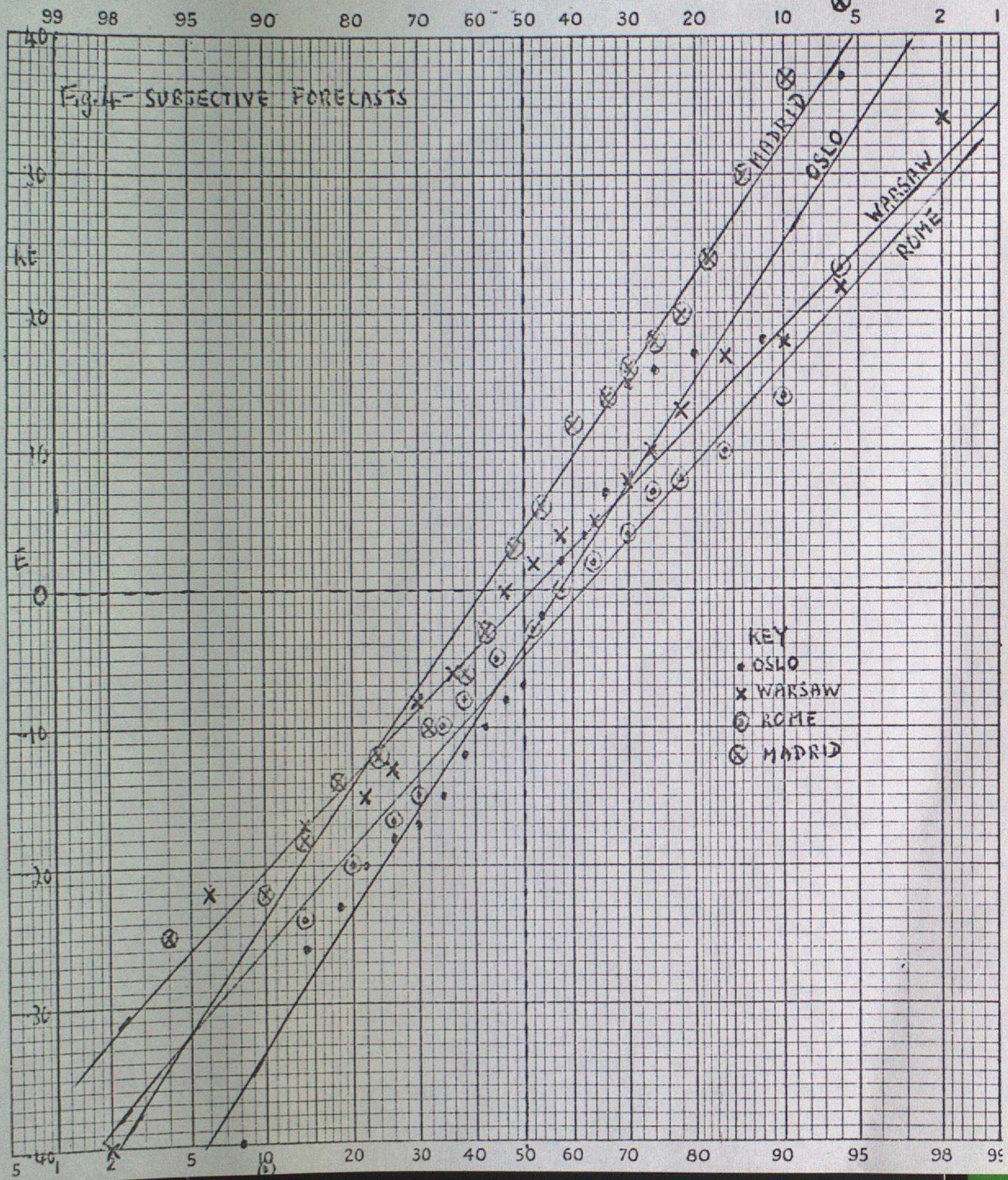
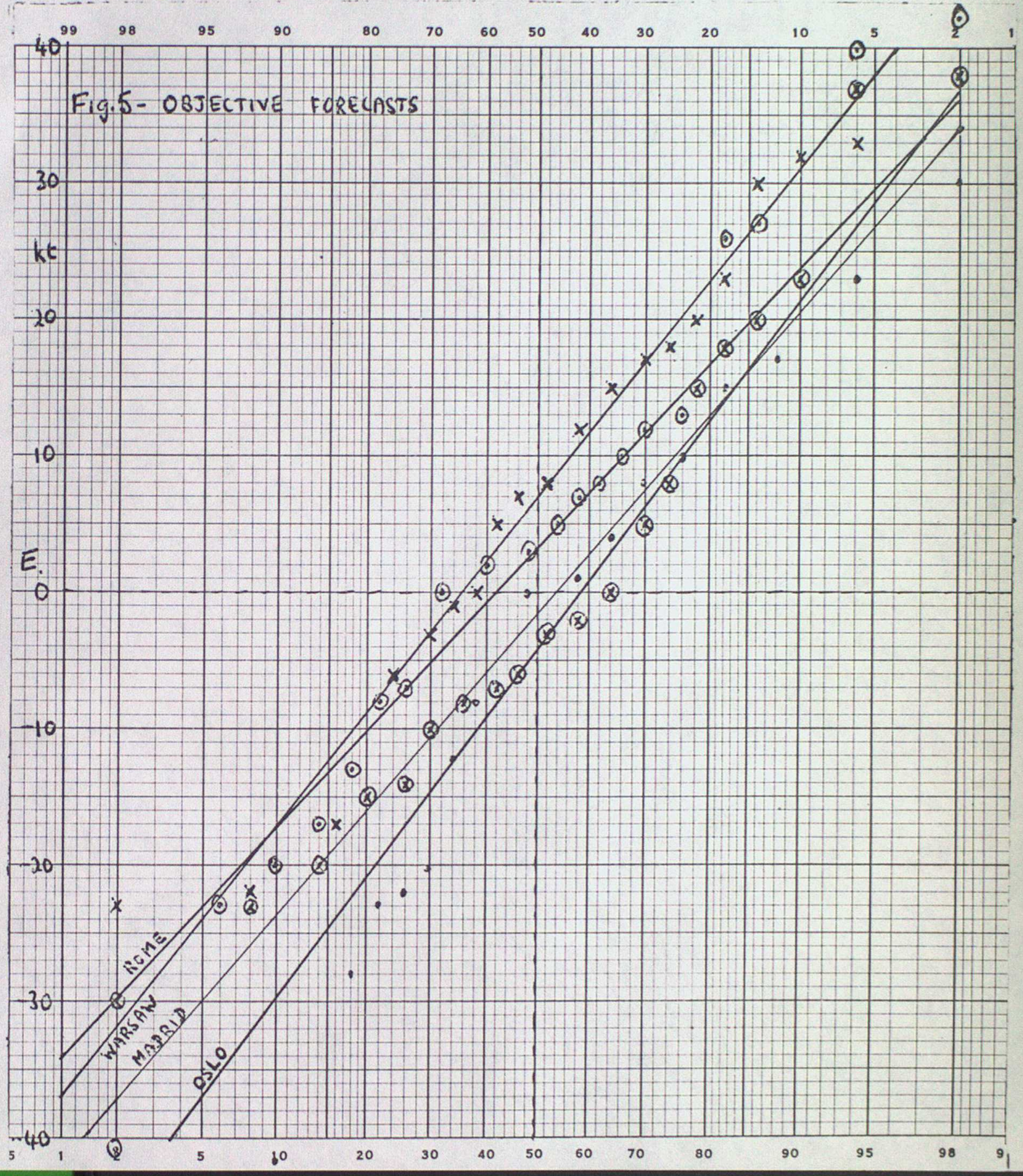
Figs2,3 Accumulated frequencies of equivalent headwind errors for three Atlantic routes separately.



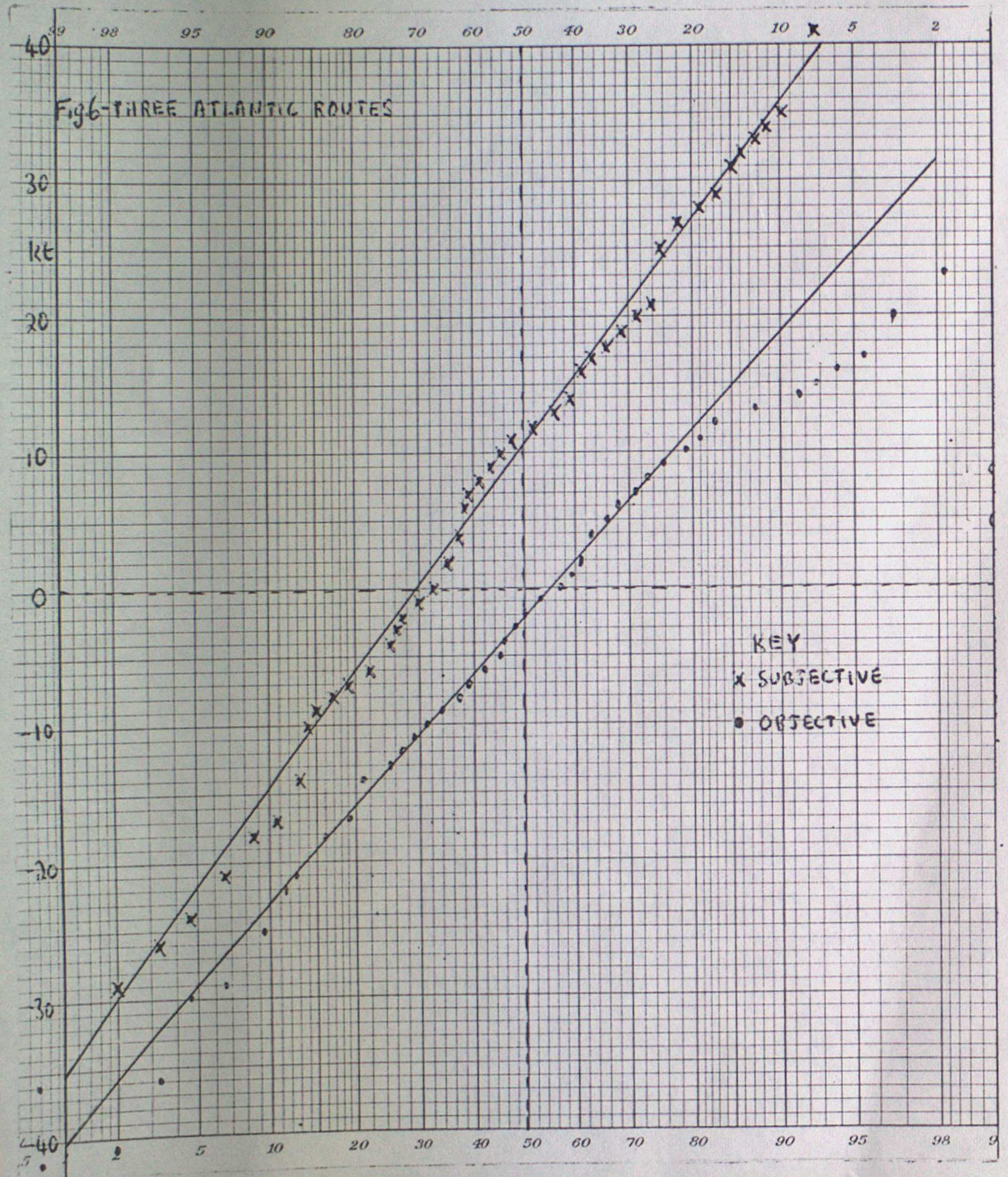


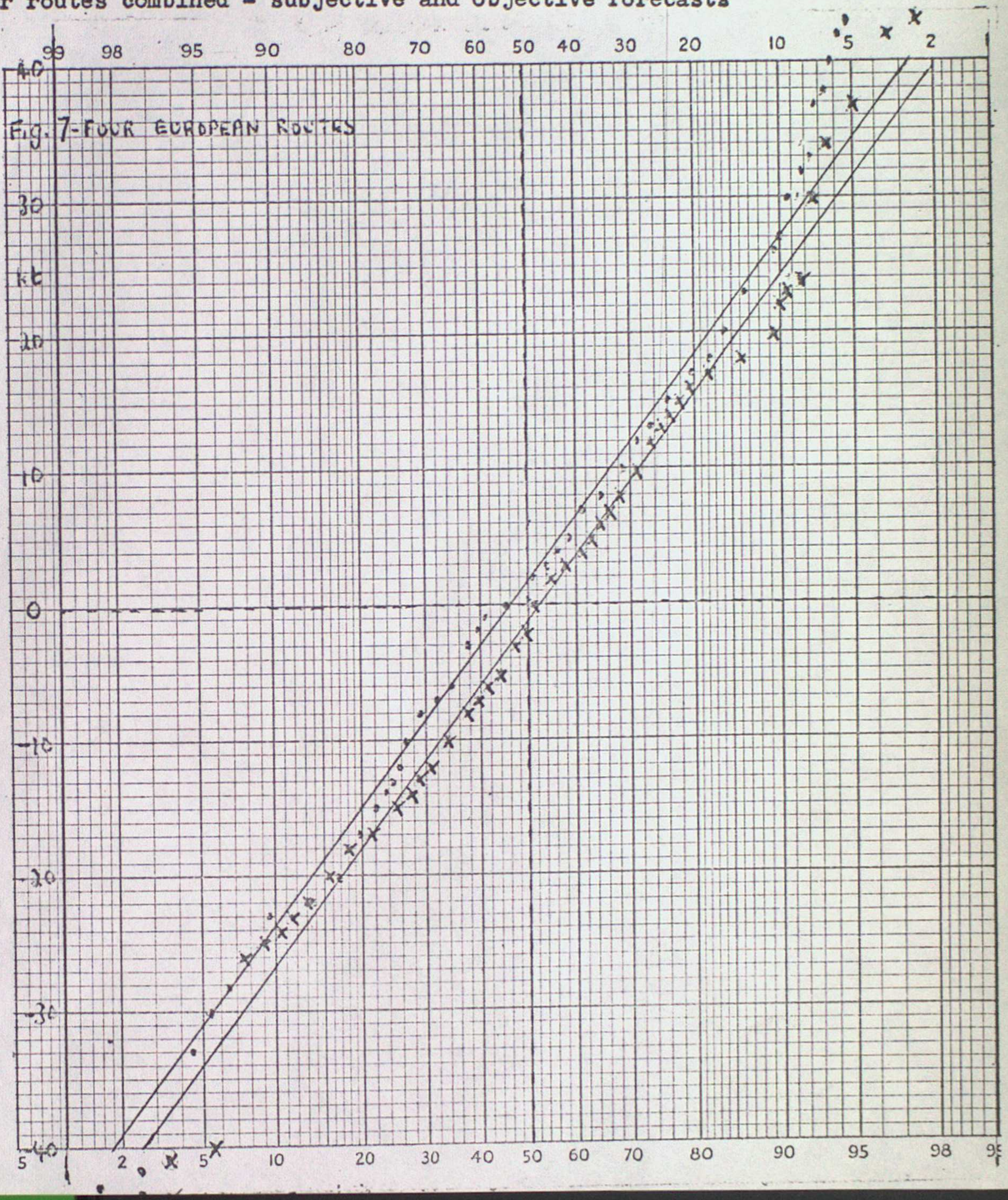
Figs 4,5 Accumulated frequencies of equivalent headwind errors for four European routes separately



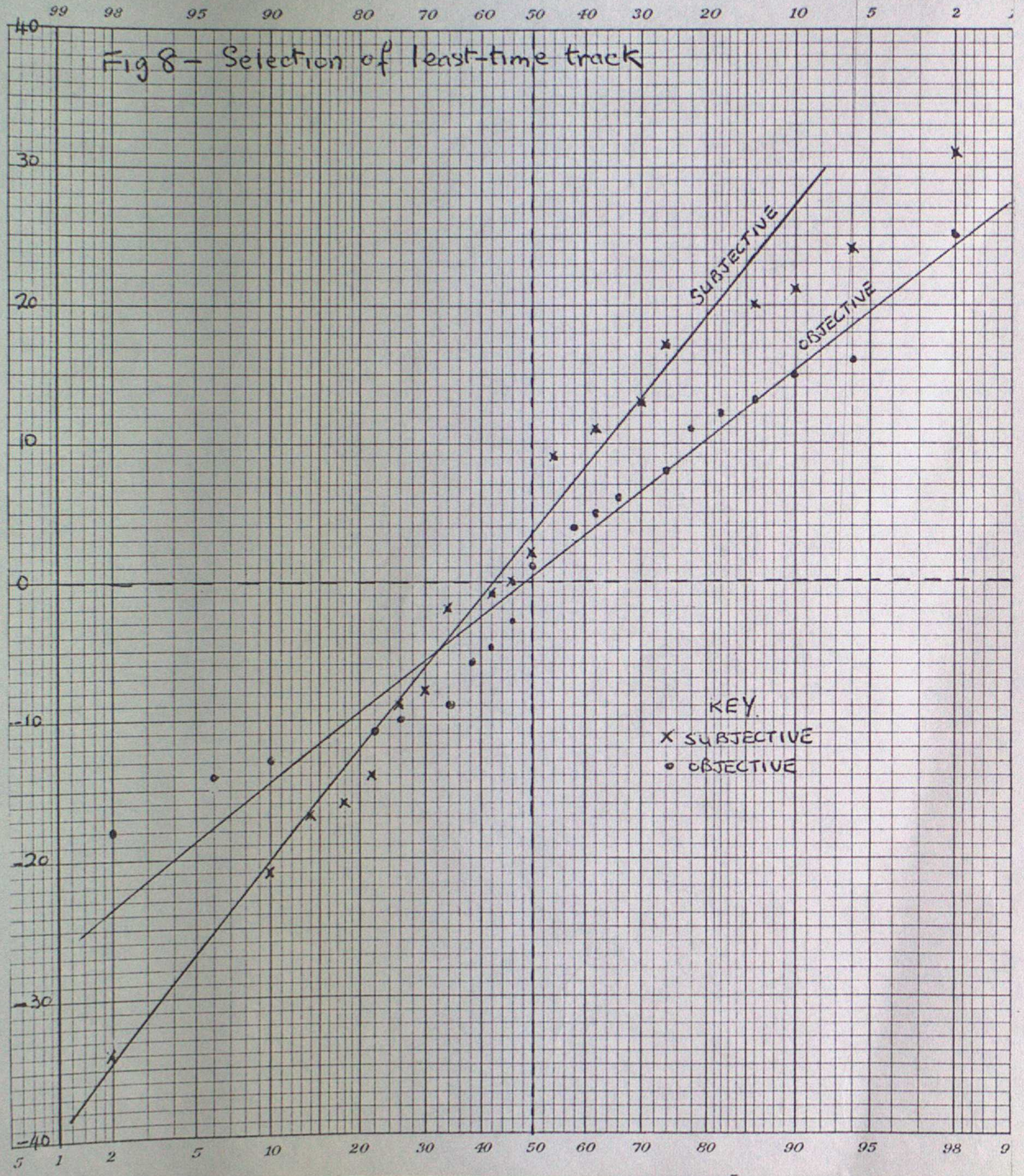


Figs 6,7 Accumulated frequencies of equivalent headwind errors for routes combined - subjective and objective forecasts





Figs 8 and 9 - Accumulated frequencies of errors, in terms of equivalent headwind London - New York subjective and objective track selection



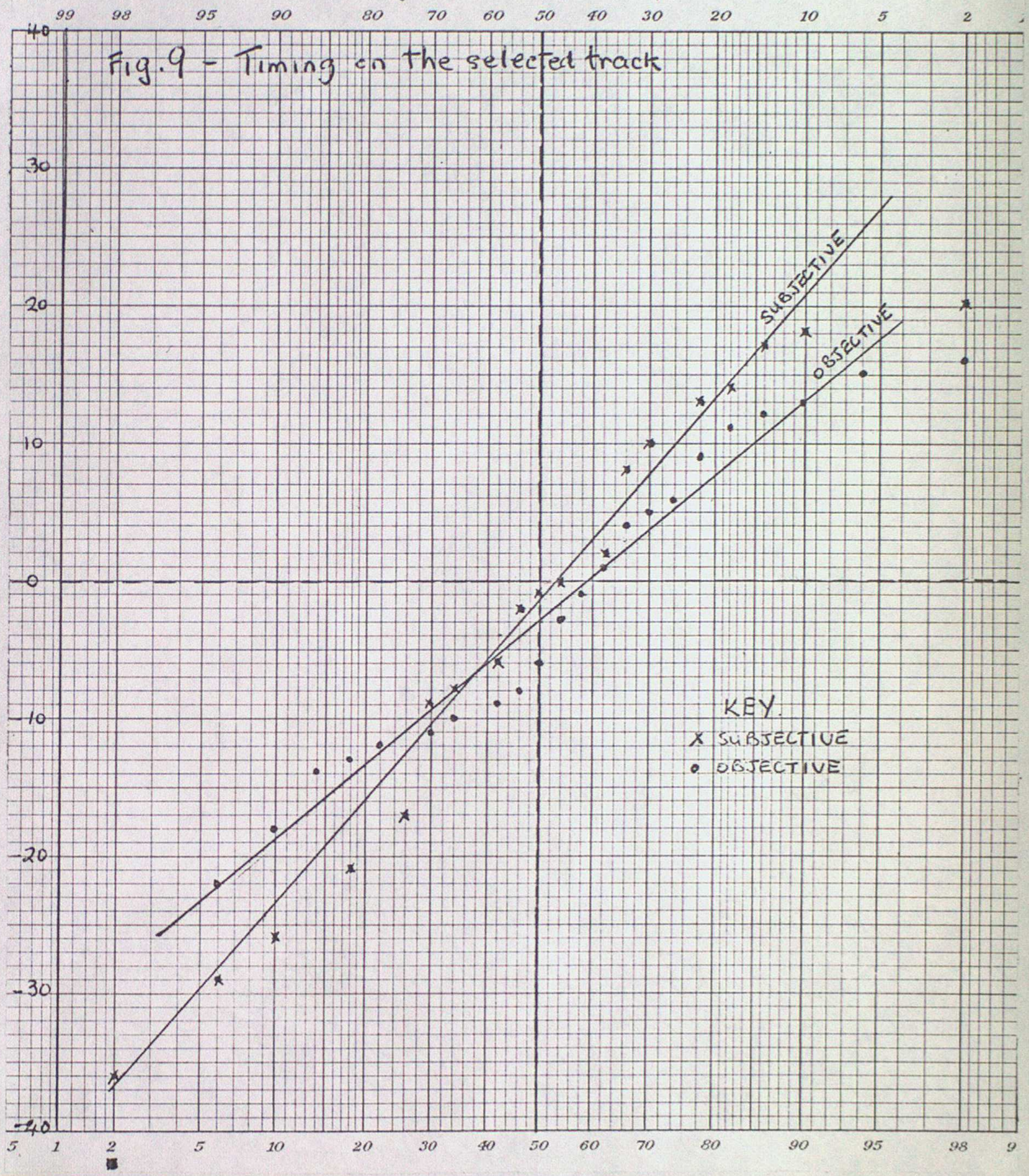


Fig 10 - Accumulated frequencies of advantages in least-time track selection, in terms of equivalent headwind, from Heathrow actual charts.

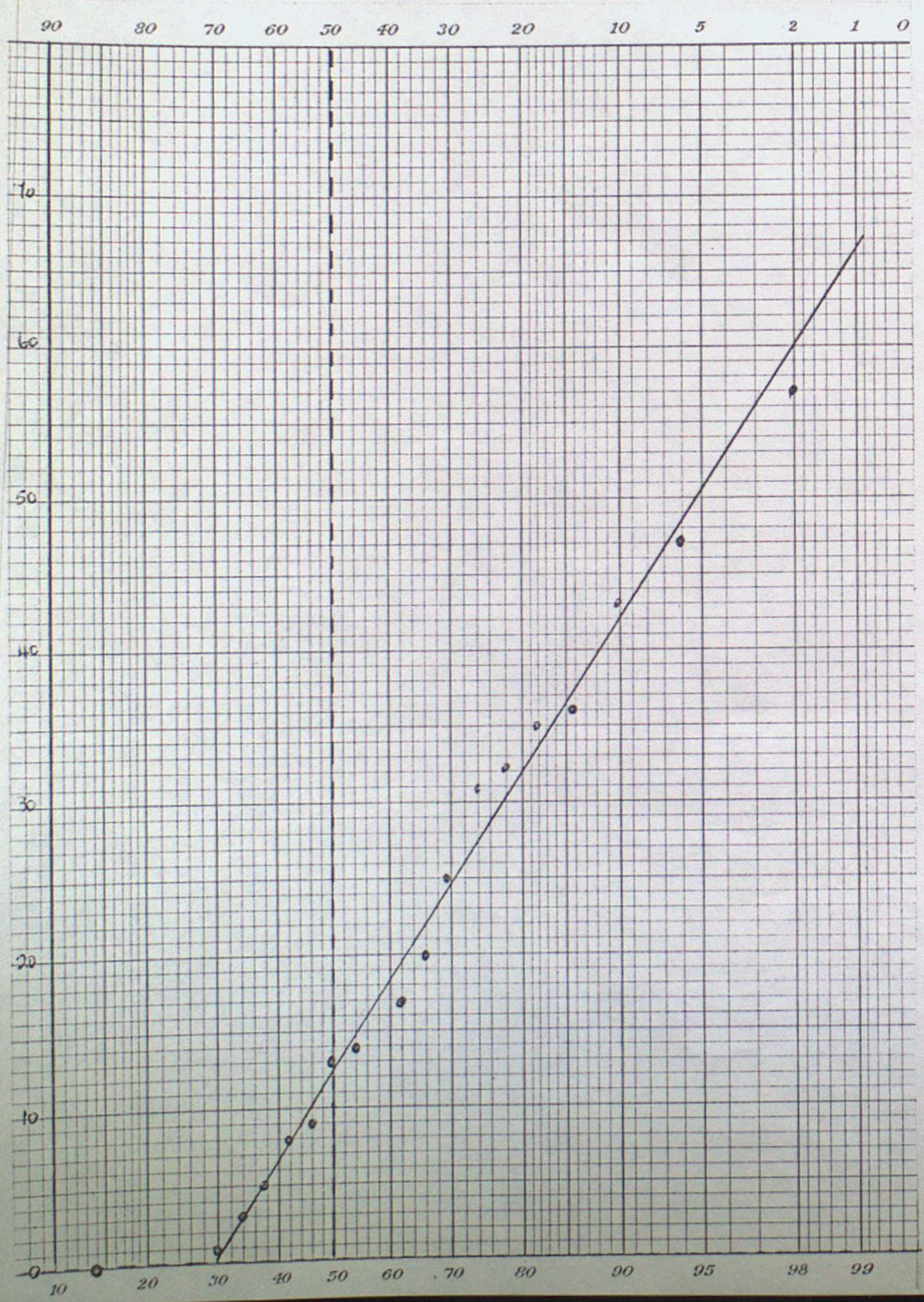


Fig 11 - Accumulated frequencies of actual equivalent headwinds, for three Atlantic routes combined, from Heathrow actual charts.

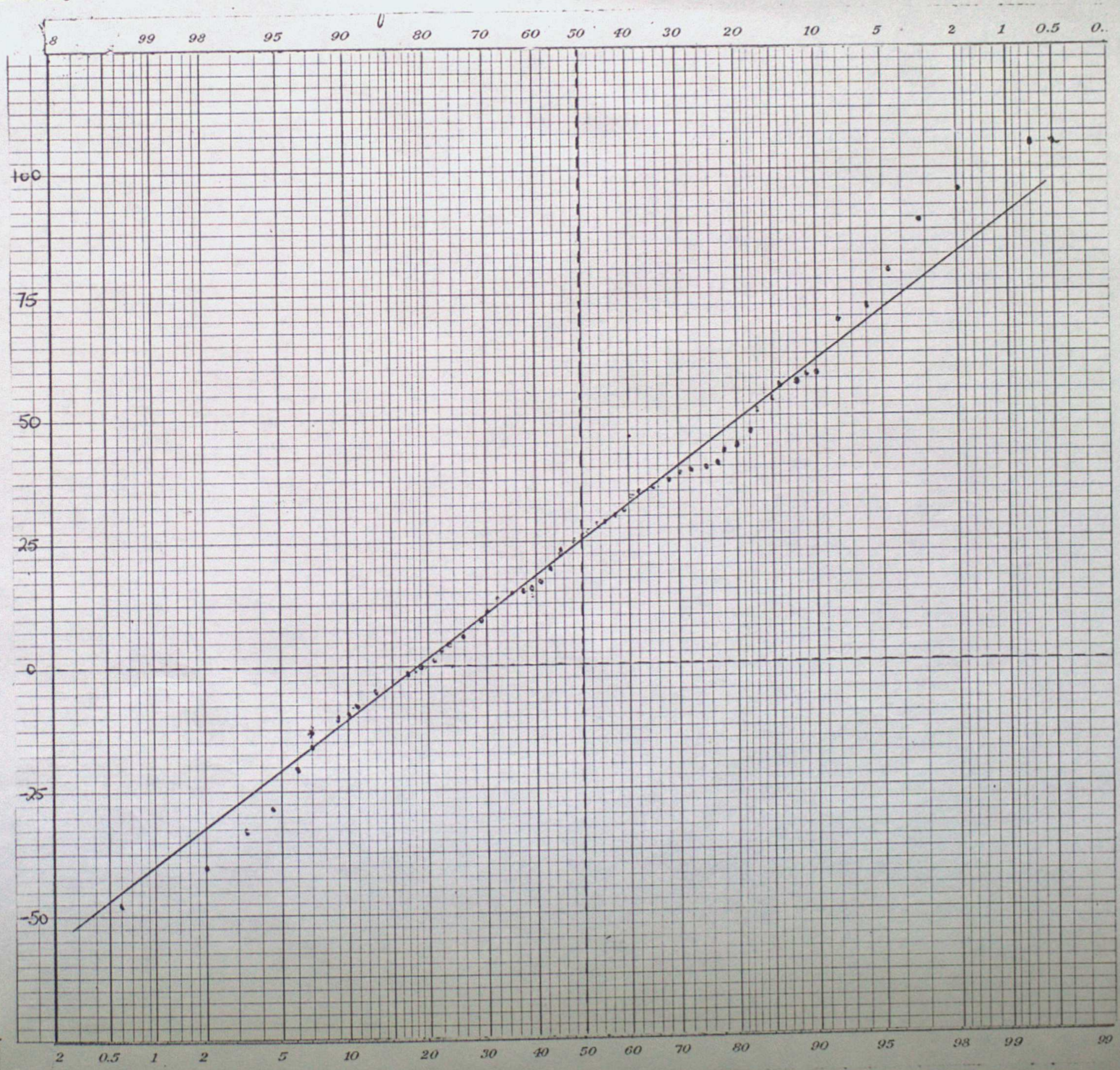


Table 1 - Equivalent headwinds based on reported winds, compared with equivalent headwinds based on measurements of contour gradients over 300 n.mile zones.

Zones Compared	Reports - Heathrow actual chart		C.F.O. actual chart - Heathrow Actual chart	
	Mean diff.	S.D.	Mean diff.	S.D.
1st zone Lindy L. - Aughton	-8.4	10.2	-5.3	9.5
3rd zone Lindy L. - Ship I	+0.8	10.9	-1.0	15.1
1st zone Rhumb L. - Camborne	-3.4	13.0	-5.3	11.3

Table 2 - Means and standard deviations of forecast errors, in terms of equivalent headwinds; 'Students' t probabilities that the mean differences of individual routes from the respective combined routes arise by chance.

Route	Subjective			Objective		
	Mean	S.D.	t-test	Mean	S.D.	t-test
London - N.York Lindy L.	+7.6	22.9	.52	-2.2	16.7	.98
Great C.	+14.4	18.7	.31	-2.7	15.2	.90
Rhumb L.	+9.4	18.8	.75	-1.2	14.9	.71
3 Atlantic routes combined	+10.6	19.7		-2.3	16.3	
Adjusted S.D. (Annex B)		(18.4)			(14.3)	
London - Oslo	-4.0	17.9	.47	-4.4	19.7	.13
-Warsaw	-0.5	15.1	.76	+6.9	18.7	.16
-Rome	-4.9	16.5	.29	+3.3	16.2	.60
-Madrid	+4.3	21.9	.19	-1.8	17.4	.33
4 European routes combined	-1.4	19.9		+1.6	19.9	
Adjusted S.D. (Annex B)		(17.4)			(17.4)	

Table 3 - Errors in forecasting for the least-time track. (see para 2b)

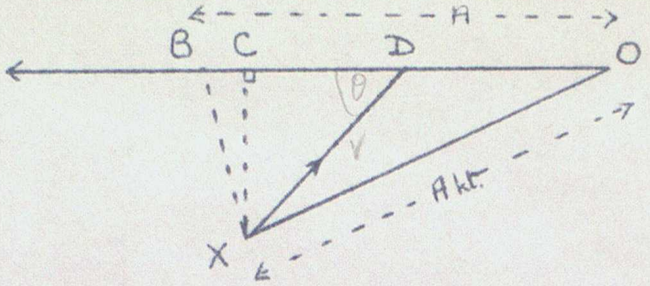
	Type of forecast	Mean	S.D.
Track selection error (Fig 8)	Subjective	+3.8	18.5
	Objective	+0.2	11.7
Timing error on least-time track (Fig 9)	Subjective	-1.2	17.3
	Objective	-3.0	12.5
Track selection advantage (over Great Circle) from Heathrow verifying actuals	- 25 cases incl. Great C.	16.4	17.1
	- 18 cases excl. Great C.	22.8	16.0

Table 4 - Comparison of Heathrow verifying winds with long period means 1949 - 1953.

Route	25 Heathrow verifying winds		Period 1949-1953		Students t-tests
	Mean	S.D.	Mean	S.D.	
London - N.York Lindy L.	+16	27	+24	39	.14
Great C.	+31	31	+38	53	.26
Rhumb L.	+27	27	+34	51	.19
London - Oslo	+1	38	-13	30	.08
- Warsaw	-12	22	-21	25	.05
- Rome	-14	23	-20	24	.19
- Madrid	-8	28	+03	26	.06

Annex A - The effects on equivalent headwinds of errors in forecasting wind direction and wind speed.

1. The equivalent headwind may be derived as follows:-



OX is a vector with the magnitude A = true air speed and in the same direction as the aircraft heading

XD is the wind vector, speed v kt, at angle θ to the track.

ODCB is the track to be flown, where OD represents the ground distance made good and OB = OX.

Then the equivalent headwind E = BC + CD

$$BC = BO - CO$$

$$= A - \sqrt{A^2 - v^2 \sin^2 \theta}$$

$$CD = v \cos \theta$$

$$E = A - (A^2 - v^2 \sin^2 \theta)^{\frac{1}{2}} + v \cos \theta \qquad \text{------------} \qquad (1)$$

$$= A - A\left(1 - \frac{v^2}{A^2} \sin^2 \theta\right)^{\frac{1}{2}} + v \cos \theta$$

expanding the binomial:-

$$E = A\left(\frac{1}{2}\frac{v^2}{A^2} \sin^2 \theta - \frac{1}{8}\frac{v^4}{A^4} \sin^4 \theta + --\right) + v \cos \theta$$

and since $\frac{v}{A}$ will not normally exceed 0·5 for flights at 400 kt.

$$E = v \cos \theta + \frac{v^2 \sin^2 \theta}{2A} \qquad \text{------------} \qquad (2)$$

From (2) if $\theta = 0°$ (wind along the track) E (headwind) = v

if $\theta = 90°$ (wind normal to the track) E (beamwind) = $\frac{v^2}{2A}$

$$\frac{E\ (\text{beamwind})}{E\ (\text{headwind})} = \frac{v}{2A}$$

Since v rarely exceeds 160 kt at 300 mb and A = 400 kt, the ratio will not exceed 1/5.

2. Current forecasting techniques lead the upper wind forecaster to approach his problem in two steps, firstly a forecast of wind direction in terms of the contour height pattern, secondly a forecast of wind speed in terms of the contour spacing or the isotach pattern. Zone wind forecasts are usually given to the nearest 10° and 10 kt and the forecaster thinks of the accuracy of his forecasts in these terms, rather than in terms of equivalent headwind. Although the forecaster may know the tracks to be flown it is often difficult to visualise the equivalent headwind from the forecast wind. In particular the relative contributions of forecast wind speed and forecast wind direction vary with the angle between the wind vector and the track. This is readily seen from Fig 12 in which the equivalent headwind E is plotted against θ for several values of v. The family of curves are intersected by a dotted curve representing the the equality $\frac{-\partial E}{\partial \theta} = \frac{\partial E}{\partial v}$, where θ is in degrees and v in knots, and it is apparent that, for wind speeds exceeding about 60 kts there are wide ranges of θ in which the equivalent headwind is more sensitive

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to errors in wind direction than to errors in wind speed.

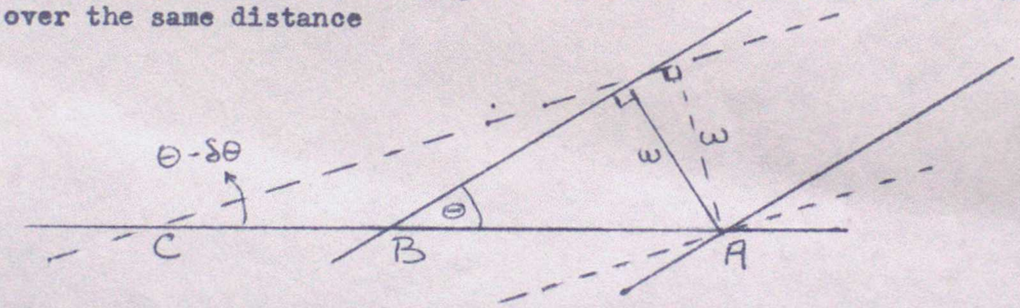
3. $\frac{\partial E}{\partial \theta^{o}}$ has a maximum value for each value of v when θ lies in the range 90° - 110°, the maximum being about 2·8 kt/degree when v is 160 kt. However, if it is assumed that the strongest jets are usually narrow, then both the narrowness and also the fact that the maxima of $\frac{\partial E}{\partial \theta^{o}}$ occur in beamwinds must must reduce to a minimum the time for which the aircraft is affected by extreme equivalent headwind errors arising from errors in θ. In certain circumstance smaller errors in equivalent headwind (due to errors in θ) may be more important than the maximum errors occurring in beamwinds because the smaller errors may affect a greater length of track. This might occur when the track intersects the flow at a smaller angle or when the wind flow is slower but broader.

4. The full effects of errors in v or θ may be seen if the corresponding times lost in flight are compared. Suppose a flight is crossing a strong wind belt w miles wide at an angle θ to the track. The distance along the track occupied by the wind belt is w cosec θ, the ground speed while crossing it is (A-E) kt. and the time taken to cross it is $\frac{w \operatorname{cosec} \theta}{(A-E)}$. The time taken to fly the same distance in calm conditions would be $\frac{w \operatorname{cosec} \theta}{A}$ and the time lost due to wind

$$= w \operatorname{cosec} \theta \left\{ \frac{1}{(A-E)} - \frac{1}{A} \right\} = \frac{E\, w \operatorname{cosec} \theta}{A\,(A-E)} \qquad \text{------------ (3)}$$

Using expression (3) it can be shown that, for a wide range of θ, the total effect on a flight of a directional error δθ (degrees) greatly exceeds the effect of a speed error δv (knots) of the same magnitude and the ratio of the errors in terms of time lost is materially greater than the ratio of the two equivalent headwinds for θ between 20 and 50°.

5. In considering the full effect of an error - δθ in terms of time lost it is necessary to examine that part of the track on which the strong wind or jet stream is forecast to occur and to compare the forecast time lost with the actual time lost over the same distance



w = width of the jet stream, speed v, on verifying actual and forecast charts.
θ = angle of jet stream to track on verifying actual chart.
AB = w cosec θ = distance along track occupied by jet stream on verifying actual chart.
(θ-δθ) = angle of jet stream to track on forecast chart.
AC = w cosec (θ-δθ) = distance along track occupied by jet stream on forecast chart.
Using expression (3) in para 4, the forecast time lost along AC

$$\frac{E_{\theta-\delta\theta}}{A(A-E_{\theta-\delta\theta})} \cdot w \operatorname{cosec} (\theta-\delta\theta) \qquad \text{------------ (4)}$$

To arrive at the actual time lost along AC, from the verifying actual chart, it is necessary to make some assumptions about that part of the track BC which was not affected by the jet stream. The wind speed must be less than the jet speed v because BC lies outside the jet stream and, unless the angle of the wind flow to the track is much smaller than the angle θ in the jet stream, it is reasonable to assume that the equivalent headwind along BC cannot exceed the equivalent headwind along AB, due to the jet stream. Therefore the actual time lost along AC cannot exceed the value obtained on the assumption that a jet stream (wider than w) with speed v and at angle θ to the track covered the entire distance AC. From expression (3) the actual time lost along AC

$$\leqslant \frac{E_{\theta}\, w \operatorname{cosec} (\theta-\delta\theta)}{A\,(A-E_{\theta})} \qquad \text{------------ (5)}$$

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An estimate of the minimum forecast time error is obtained by subtracting expression (5) from expression (4). For a small increment $\delta\theta$, $(A-E_\theta) \simeq (A-E_{\theta-\delta\theta})$; therefore the forecast time error

$$\geqslant \frac{w \operatorname{cosec}(\theta-\delta\theta)}{A(A-E_{\theta-\delta\theta})} \cdot (E_{\theta-\delta\theta}-E_\theta) \qquad \text{------------ (6)}$$

6. If θ is now forecast correctly but an error $+\delta v$ kt. (equal in magnitude to $-\delta\theta^\circ$ in para 5) is made in forecasting v, then the jet stream occupies the same distance AB on both forecast and actual charts and, using expression (3),

the forecast time lost along AB $= \dfrac{E_{v+\delta v}}{A(A-E_{v+\delta v})} \cdot w \operatorname{cosec}\theta$ ------------ (7)

the actual time lost along AB $= \dfrac{E_v}{A(A-E_v)} \cdot w \operatorname{cosec}\theta$ ------------ (8)

and the forecast time error along AB is obtained by subtracting expression (8) from expression (7).

If it is now assumed that the forecast error δv in zone AB does not produce an error in the adjoining zone BC, then the forecast error due to δv along AB will apply to the longer sector AC.

i.e. Forecast time error along AC due to $\delta v = \dfrac{w \operatorname{cosec}\theta}{A(A-E_{v+v})} E_{v+\delta v} - \dfrac{w \operatorname{cosec}\theta}{A(A-E_v)} . E_v$

and, since for a small increment δv, $(A-E_{v+\delta v}) \simeq (A-E_v)$,

the forecast time error along AC $\simeq \dfrac{w \operatorname{cosec}\theta}{A(A-E_v)} \cdot \left[E_{v+\delta v} - E_v\right]$ ------- (9)

7. From expressions (7) and (9), the minimum value of the ratio of the forecast time error along AC due to $-\delta\theta^\circ$ (v correctly forecast) to the forecast time error along AC due to an error δv kt of equal magnitude (θ correctly forecast) is given by:-

$$\frac{\text{Time error due to } \delta\theta}{\text{Time error due to } \delta v} \geqslant \frac{\dfrac{w \operatorname{cosec}(\theta-\delta\theta)}{A(A-E_\theta)} \cdot (E_{\theta-\delta\theta} - E_\theta)}{\dfrac{w \operatorname{cosec}\theta}{A(A-E_v)} \cdot (E_{v+\delta v} - E_v)}$$

but E_θ and E_v are identical, and the expression reduces to:-

$$\geqslant \frac{\operatorname{cosec}(\theta-\delta\theta)}{\operatorname{cosec}\theta} \cdot \frac{(E_{\theta-\delta\theta} - E_\theta)}{(E_{v+\delta v} - E_v)}$$

Therefore the ratio of the time errors due to $\delta\theta$ and δv is greater than the ratio of the corresponding equivalent headwinds by a factor of not less than $\dfrac{\operatorname{cosec}(\theta-\delta\theta)}{\operatorname{cosec}\theta}$

Table 5 below shows the ratio $\dfrac{\operatorname{cosec}(\theta-\delta\theta)}{\operatorname{cosec}\theta}$ for various values of θ and it is apparent that the full effect of errors in θ on the time of flight must be considerably greater than the effect on equivalent headwind and must extend over wider ranges of θ than those contained within the horizontally shaded area in Fig. 12.

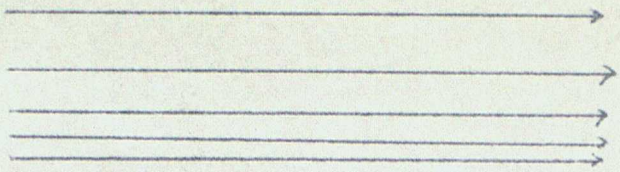
Table 5

θ°	Cosec θ	$\dfrac{\operatorname{cosec}(\theta-10)^\circ}{\operatorname{cosec}\theta}$
90	1.0000	1.02
80	1.0154	1.05
70	1.0642	1.09
60	1.1547	1.13
50	1.3045	1.19
40	1.5557	1.28
30	2.0000	1.46
20	2.924	1.97
10	5.759	-

The probable minimum extension of the area when errors are considered in terms of time lost is shaded diagonally.

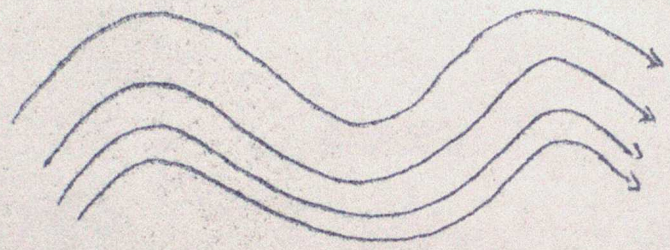
/8.

8. In addition to the effects already discussed there are also important differences in the way errors are correlated along a route, depending on the angle between the aircraft's track and the general wind direction. Consider first a broad straight flow with shear.



For a route making a small angle to the general direction of flow, the equivalent headwind depends mainly on the forecast wind speed and will be very sensitive to the positioning of the strongest flow with respect to the track flown. Forecast errors will tend to be positively correlated along the track and the total error in terms of equivalent headwind is unlikely to decrease rapidly with increasing route length. In contrast, for a route normal to the flow, errors tend to be negatively correlated and the equivalent headwind errors should decrease rapidly with increasing route length. Also, when the wind is mainly beamwind the equivalent headwind is less by a factor of at least five than for a similar wind directed along the track (para. 1).

9. Now consider a wave pattern superimposed on the broad flow



Equivalent headwinds for a track along the general flow must decrease as the amplitude of the wave pattern increases and it has been shown that the forecast wind direction is more important than the forecast wind speed unless the wind flow is almost entirely a headwind. For a route directed along the general flow the actual positions of troughs and ridges in the wave pattern are of little importance if the route is long compared with the wave length. For a track crossing the flow the angle θ between the track and the wind direction depends critically on the position of the pattern. If positive and negative errors in forecasting the wave pattern are equally likely then, for tracks across the mean flow, the mean error for a long series of equivalent headwind forecasts should tend towards zero but the standard deviation will be large. Any tendency to underestimate the amplitude of the wave pattern will underestimate the equivalent headwind and restrict the gross errors.

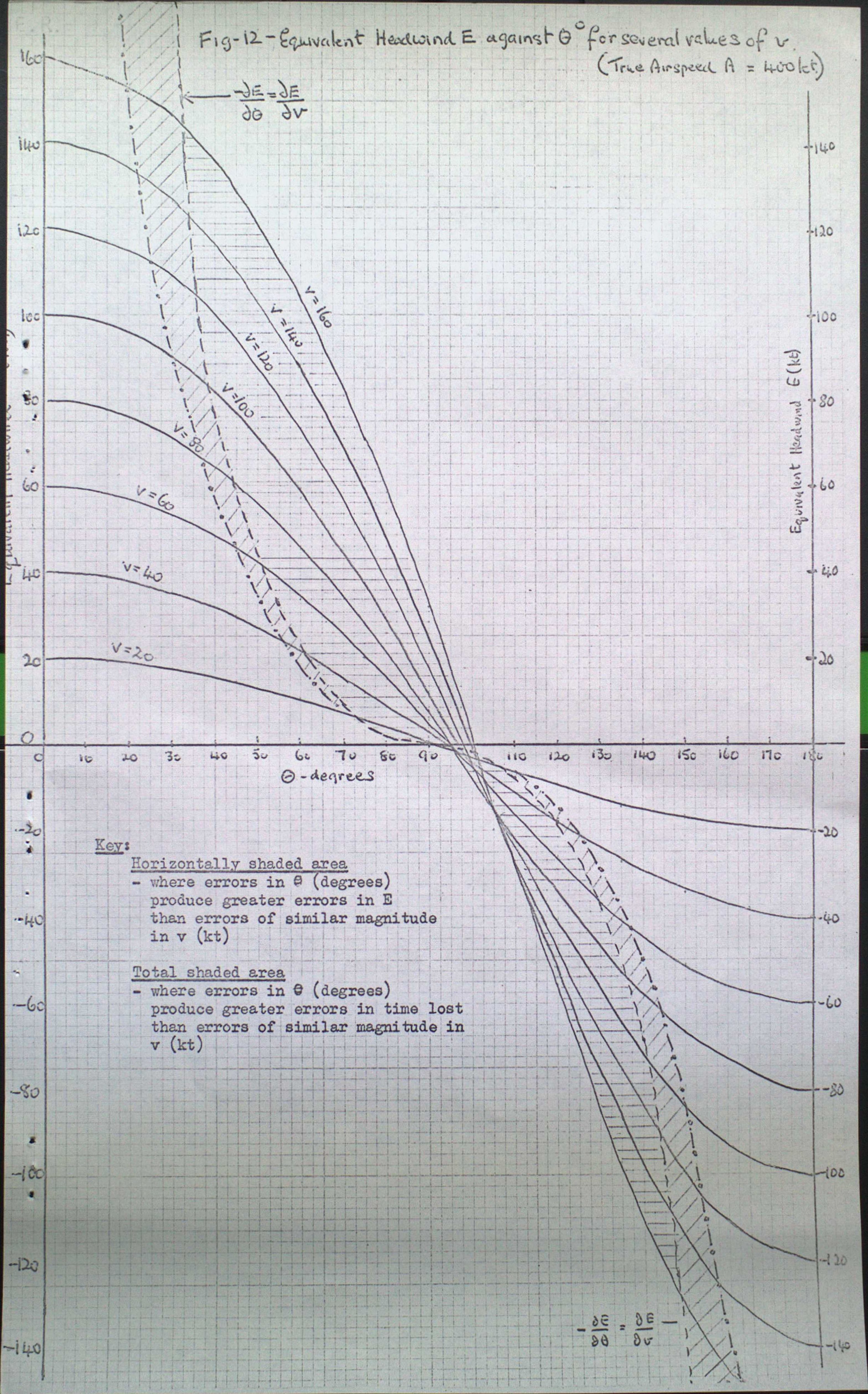


Fig-12 - Equivalent Headwind E against θ° for several values of v.
(True Airspeed A = 400 kt)

Annex B - Errors in chart construction and forecasting

1. The construction of both contours and isotachs is somewhat subjective, especially over the North Atlantic, and isotachs were constructed on the actual contour charts drawn at the Central Forecasting Office (CFO), to provide independent estimates of the verifying winds taken from the Heathrow charts. The existence of two independently drawn series of charts provided an opportunity to estimate the three variances of equivalent headwinds due to the use of the scale, contour construction, and contour plus isotach construction. These variances were applied as corrections to the variances of the differences between forecast charts and the verifying actual charts, to obtain better estimates of the variances due to forecast error.

2. Variance due to equivalent headwind measurement.

A knowledge of this variance is not essential, because it always occurs in combination with the variance(s) due to chart construction, but it is of interest because it represents the smallest possible variance likely to arise when two airline operators extract equivalent headwinds from the same forecast charts. In practice the variance between operators must exceed this because planning techniques differ and because bias is almost certainly introduced on occasions - for example when a planned flight time approaches the operating limit of the aircraft. The variance was estimated as follows. The same forecaster repeated his measurement of Heathrow actual charts for the three Atlantic routes (after an interval of three weeks and without referring to previous results). The variance of the two sets of measurements was calculated from differences between corresponding pairs (Fig 13). If it is assumed that equal skill was applied in both sets of measurements and there is no correlation between the corresponding errors, then the variances of each set from the (unknown) true value should be equal and the variance between the two sets should be twice this value. The main cause of measurement error is thought to be the difficulty in estimating the mean vector wind over each 300 n. mile zone. If equal skill is applied in measuring all the zones on the Atlantic routes then the variance for tracks of two zones should be 5/2 times the variance for tracks of five zones. From Fig 13 the five-zone standard deviation of 4·5 theoretically implies a two-zone standard deviation of 7·1 which compares well with the measured value of 7·2.

3. Variance due to contour construction and measurement

This was estimated by calculating the variance of the differences between corresponding pairs of geostrophic equivalent headwinds measured from the contour spacing on Heathrow and CFO verifying actual charts (Fig 14). In practice the variance of the pairs of winds will be an underestimate of the required variance because correlation of errors in contour construction may occur as a result of identical errors in observations on the pairs of charts.

4. Variance due to contour construction, isotach construction and measurement

The combined effect was estimated by calculating the variance of the differences between corresponding pairs of equivalent headwinds measured from the contours and isotachs on corresponding Heathrow and CFO verifying actual charts (Fig 15) As in para 3, correlations will probably reduce the differences between corresponding pairs of charts and lead to an underestimate of the required variance.

5. Variance between Heathrow subjective forecasts and the verifying actuals

This must depend on the forecast error and on the variance at para 4 above, because both the forecast charts and the verifying actual charts are isotached. A better estimate of the variance due to forecast error alone is obtained by subtracting the variance obtained at para 4.

6. Variance between Numerical objective forecasts and the verifying actuals

This must depend on the forecast error and on the variances at paras 3 and 4 because the actual chart is isotached but the forecast chart is not. A closer estimate of the forecast error was obtained by subtracting the average of the variances under paras 3 and 4.

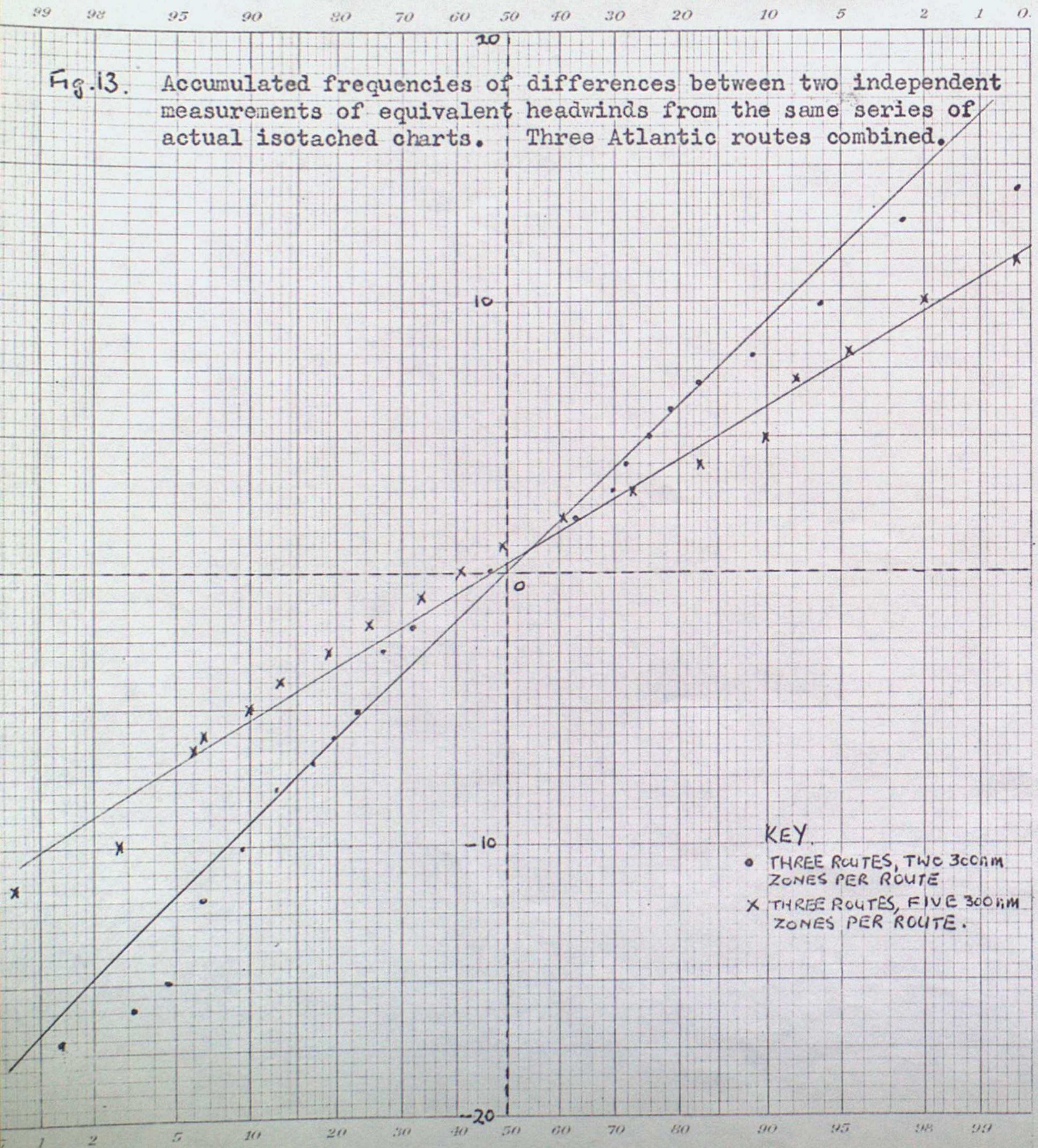


Fig. 13. Accumulated frequencies of differences between two independent measurements of equivalent headwinds from the same series of actual isotached charts. Three Atlantic routes combined.

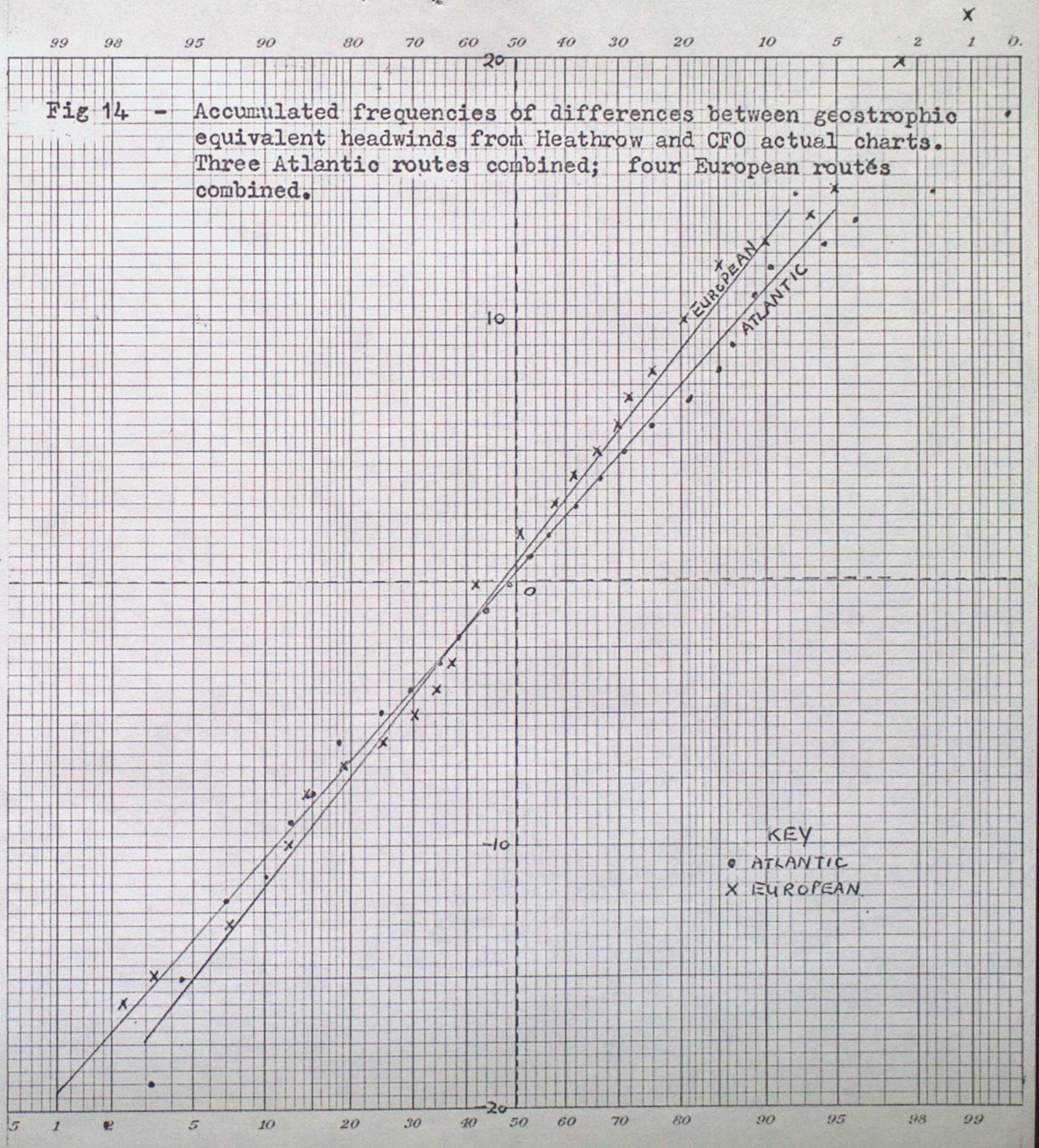
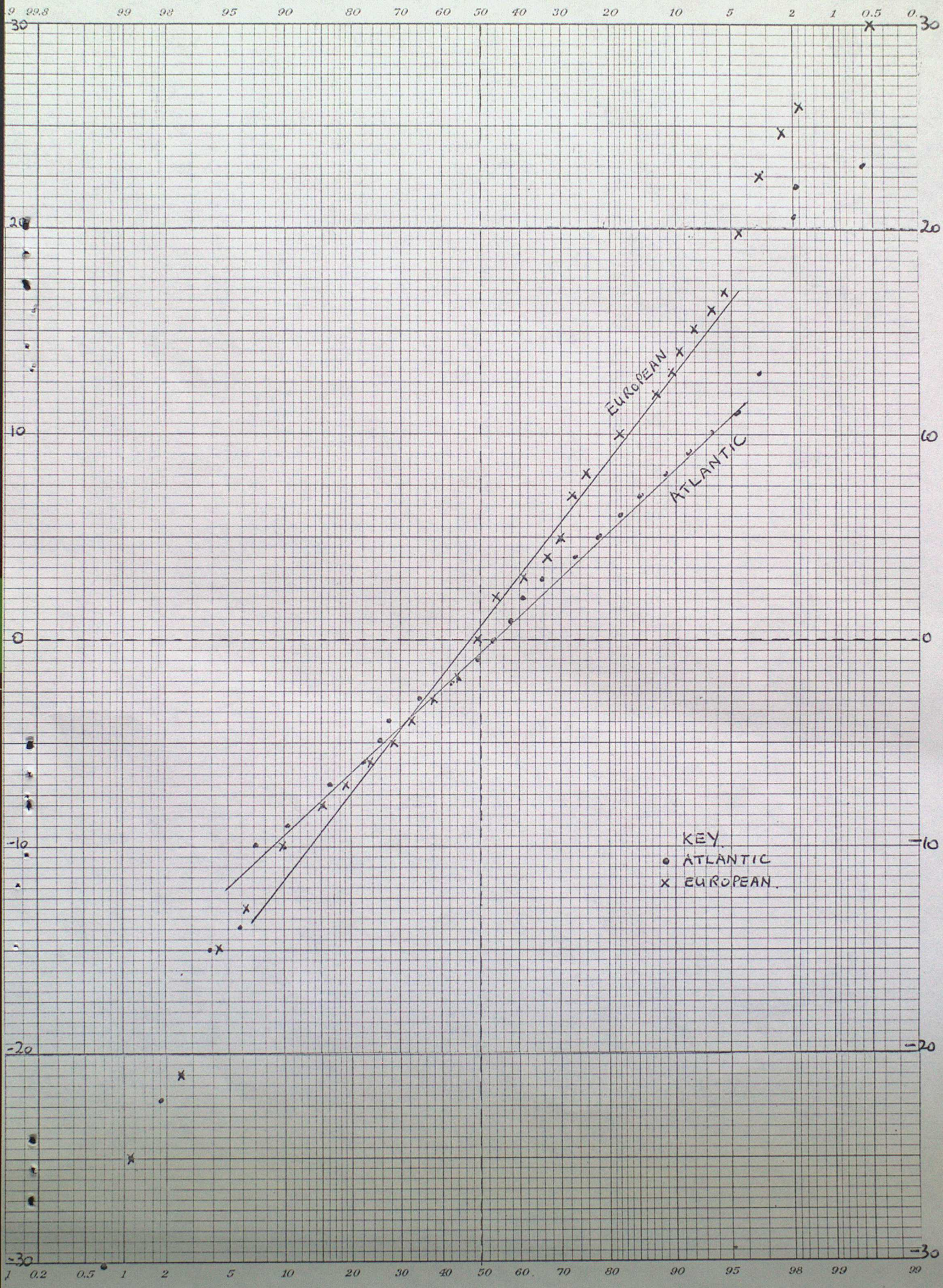


Fig 14 - Accumulated frequencies of differences between geostrophic equivalent headwinds from Heathrow and CFO actual charts. Three Atlantic routes combined; four European routes combined.

Fig 15 - Accumulated frequencies of differences between equivalent headwinds from isotachs on Heathrow and CFO actual charts. Three Atlantic routes combined; Four European routes combined.



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