# Forecasting Development

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## Providing Meteorological data to the Final Approach Spacing Tool

May 2001

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by

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# Providing Meteorological data to the Final Approach Spacing Tool

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This report describes work carried out with the aim of broadly satisfying the meteorological data requirements of a new air traffic management tool known as FAST (Final Approach Spacing Tool).

FAST is a ground-based Air Traffic Control tool that performs trajectory predictions for arrivals of aircraft in the airport radar manoeuvring area. It is designed to provide guidance for the turns from downwind to base leg and from base leg to ILS (Instrument Landing System) intercept leg.

FAST needs wind predictions that give a positional uncertainty of 0.2nm (370m) or less. This is required for an aircraft travelling at 170kts (88ms$^{-1}$) over a distance of 12nm (22.2km). This equates to a timing error of 1 second in each minute of the descent. If the entire decent was at 170kts then this would result from a headwind error of 2.83kts (1.46ms$^{-1}$) or more when averaged over the entire descent.

This report describes the format of the required forecasts and the software that has been written to produce them. It also contains the results of verifying the forecasts against AMDAR (Aircraft Meteorological DAta Relay) reports in the vicinity of London Heathrow airport.

The results of the verification show a capability to meet the FAST accuracy requirement on over 75% of occasions in the Heathrow area. A number of suggestions for improving the system and the verification technique are also included in the report.

# Providing Meteorological data to the Final Approach Spacing Tool

# 1 Introduction

This report describes work carried out with the aim of broadly satisfying the meteorological data requirements of a new air traffic management tool known as FAST (Final Approach Spacing Tool). FAST is a ground-based Air Traffic Control tool that performs trajectory predictions for arrivals of aircraft in the airport radar manoeuvring area. It is designed to provide guidance for the turns from downwind to base leg and from base leg to ILS (Instrument Landing System) intercept leg.

The required forecasts and the software that produces them are described. The report also contains the results of verifying the forecasts against AMDAR (Aircraft Meteorological DAta Relay) reports in the vicinity of London Heathrow airport.

# 2 Background

We were approached in early 1999 to provide meteorological data for FAST to perform its trajectory prediction and deceleration detection. The user requirement was set out in Smith, 1998. The development work and subsequent service were considered too costly for the scope of the FAST project within NATS (National Air Traffic Services) so the work proposed was not funded in 1999. The funding for the work presented in this report has come from NATS as part of the 2000/2001 R&D programme with the aim of completing the development work required to provide such a service if future enhancements of FAST (or similar projects) require it.

The required data for FAST in the terminal area (as specified in 1999) were:

- Wind speed and direction for the next 10 minutes
- Temperature for the next 10 minutes
- QNH for the next 10 minutes (lowest mean sea level pressure in the altimeter setting region)

Data are needed on horizontal grids at the following 'heights':

| | |
|---|---|
| 1200ft | 97008 Pa*QNH/1013.25 |
| 3000ft | 90812 Pa*QNH/1013.25 |
| 4000ft | 87510 Pa*QNH/1013.25 |
| 5000ft | 84307 Pa*QNH/1013.25 |
| FL70 | 78185 Pa |
| FL100 | 69682 Pa |
| FL130 | 61943 Pa |

The current horizontal resolution of the Met Office mesoscale model (0.11° latitude) was considered more than adequate but the output is not on a regular latitude-longitude grid. Instead, a regular 9x9 grid covering 1.0W to 0.6E and 51.1N to 51.9N was chosen for FAST to coincide with the location of London Heathrow airport. The code could easily be adapted to serve any airport if mesoscale model fields were made available for that area.

Figure 1 shows how the levels change with surface pressure in the mesoscale co-ordinate system and the FAST co-ordinate system. The lowest line is a Gaussian representation of the surface pressure over a hill or at a low-pressure centre, for example.

Figure 2 shows a 9x9-point wind field overlaid on a temperature field in the mesoscale co-ordinate system (left) and the FAST co-ordinate system (right).

# 3 How the FAST forecasts are produced

The software used to produce these forecasts is described in some depth in Appendices A and B. The code uses forecast fields from the mesoscale model as input and regrids them onto the FAST co-ordinate system (and interpolates them to 10-minute intervals). The regular 9x9 grid was used for the first 3 forecast levels but was extended to 19x19 at 5000ft, 29x29 at FL70, 39x39 at FL100 and 49x49 at FL130, centred on 0.2W and 51.5N as before and with the same resolution so that the higher levels cover a larger area. This was done to allow greater coverage of ascents and descents for verification purposes and did not form part of the original user requirement. The code has been designed so that additional levels up to FL410 could be easily added (increasing in coverage with height) with the aim of encompassing all possible continuous descent profiles into the airport.

It was assumed at the start of the project that regridded data would not be sufficiently accurate so an additional data assimilation scheme (WAFTAGE – Winds Analysed and Forecast for Tactical Aircraft Guidance over Europe) has been used to nudge the forecasts towards observations received in the half-hour before the forecast is due.

## 4 Verification

The forecasts for FAST are expected to give a positional uncertainty of 0.2nm (370m) or less. This is required for an aircraft travelling at 170kts (88ms$^{-1}$) over a distance of 12nm (22.2km). This equates to a timing error of 1 second in each minute of the descent. If the entire decent was at 170kts then this would result from a headwind error of 2.83kts (1.46ms$^{-1}$) or more when averaged over the entire descent.

The simplest way to judge the forecasts would be to compare them with aircraft observations for descents into the London airports. This method would lead to numerous difficulties partly because of the poor calibration of the Inertial Navigation Systems (INS) onboard after long haul flights and partly because of problems sorting the data when the aircraft does not make a continuous descent. It was decided to use ascents instead of descents to reduce the positional errors of the observations and to provide a continuous profile to verify.

The ground speed of each aircraft is not known so this has been approximated using the distance between the 3-dimensional positional observations and the time taken according to the observation. Unfortunately, the observed time is given only in minutes not seconds so a pseudo-observation of time (in minutes and seconds) is calculated for each point in the ascent using the approximated ground speed. Spacing errors and timing errors are calculated using the observed and forecast winds using the approximated ground speed and pseudo-observations of time.

AMDAR reports received from commercial aircraft on 30 days between 11/9/2000 and 7/12/2000 have been used for the verification.

Three types of forecast have been verified:

- It seemed too subjective to judge the accuracy of the FAST forecasts without comparing them with other forecasts designed for the same purpose. One such forecast is the wind information available on a Form 214 (UK Spot Wind Chart) an example of which is reproduced in Figure 3. He chart gives altitudes, wind direction, wind speed and temperature at 6 heights for 18 locations in the UK and surrounding area. The Form 214-style forecasts used in the verification are not identical to those issued operationally. The model fields used are from the mesoscale rather than preliminary global model and the vertical interpolation method is different to make it more consistent with the method used in the FAST forecasts. The Form 214 is only issued 4 times per day so observations have been compared with it if they occur in the 3 hours before or after the validity time. This aspect of the Form 214 means that a different number of observations have been verified against this forecast type than against the other two forecast-types.
- It was thought necessary to judge if the addition of WAFTAGE made a significant improvement to the forecasts. With this aim, regridded data from the mesoscale fields have also been verified. Observations have been compared with a forecast field if they occur 5 minutes before or after the validity time
- Finally the FAST forecasts with WAFTAGE data assimilation have been verified. Again, observations have been compared with a forecast field if they occur 5 minutes before or after the validity time

Figure 4,Figure 7 and Figure 10 are scatter plots showing the distribution of errors for the three forecast types

The errors marked with a + are calculated using the forecast for the level *above* the observation for each observation in the ascent. The errors marked with a x are calculated using the forecast for the level *below* the observation. The errors marked with a □ are calculated using the mean of the forecasts for the levels *above* and *below* the observation. The errors marked with a ◊ are calculated by interpolating the forecasts for the levels *above* and *below* the observation to the height of the observation. All these methods are represented so that advice can be given on how to use the data (provided on discrete levels) within FAST.

The information in the scatter plots is also summarised in histograms (Figure 5, Figure 6, Figure 8, Figure 9, Figure 11 and Figure 12). Table 1 to Table 6 summarise the information in the histograms. They show the mean and root mean square (RMS) of the errors and the percentage of errors that were considered acceptable by the FAST project.

A problem with the INS positioning was discovered at a late stage of the verification. When studying particularly poor forecasts it was noticed that the ground positions of several aircraft did not correspond to major London airports. Some AMDAR units report positions in degrees and hundredths of a degree and others in degrees and seconds. Poor encoding had led to some aircraft reporting seconds as hundredths of a degree. The discovery of these faulty position reports substantially devalued the verification performed so all the statistics were recalculated using corrected positions and these are the results presented. Unfortunately, this fault was discovered at a late stage in the project and there was insufficient time to produce a large sample of WAFTAGE forecasts using corrected data. In some ways this is a realistic approach since intermittent faults do occur on AMDAR units so the data assimilation scheme will always receive some inaccurate data. However, this is a much larger source of error than most so should be borne in mind when judging the benefits of the WAFTAGE scheme from the statistics presented.

The statistics show that the Form 214 forecasts produce greater relative errors than the other two forecasts. This result was to be expected because of the coarse grid used in these forecasts.

Using WAFTAGE on the regridded data has reduced the negative bias of the relative errors and reduced the RMS error. The percentage of ascents that fall within the expected positional uncertainty rises when WAFTAGE is used. This rise is not as significant as the rise seen when moving from the Form 214 forecast to the regridded forecast.

There seems to be little accuracy gained by using exact interpolations to the height of the aircraft instead of the mean of the levels above and below. When WAFTAGE is not used on the regridded data, the mean forecast gives marginally better results than the interpolation. When WAFTAGE is used, the interpolation marginally lowers the RMS error and increases the percentage of ascents falling within the expected limits but it actually increases the negative bias. The only clear result is that both of these methods are better than using either the level above or the level below alone.

| | Abs Error < 1/60 (%) | Mean error | RMS error |
|---|---|---|---|
| using level above | 46.0 | -11.8x10-3 | 30.9x10-3 |
| using level below | 47.9 | -5.8x10-3 | 25.4x10-3 |
| using mean of 2 levels | 54.1 | -8.6x10-3 | 25.4x10-3 |
| using interpolation of 2 levels | 50.7 | -8.3x10-3 | 25.0x10-3 |

Table 1 Summary of statistics in Figure 5 (Form 214 relative spacing errors)

| | Absolute Error < 1/60 (%) | Mean error | RMS error |
|---|---|---|---|
| using level above | 46.4 | -11.4x10-3 | 29.7x10-3 |
| using level below | 49.7 | -5.9x10-3 | 24.3x10-3 |
| using mean of 2 levels | 54.1 | -8.4x10-3 | 24.3x10-3 |
| using interpolation of 2 levels | 53.4 | -8.2x10-3 | 24.0x10-3 |

Table 2 Summary of statistics in Figure 6 (Form 214 relative timing errors)

| | Absolute Error < 1/60 (%) | Mean error | RMS error |
|---|---|---|---|
| using level above | 56.9 | -10.6x10-3 | 21.2x10-3 |
| using level below | 63.5 | -1.2x10-3 | 18.4x10-3 |
| using mean of 2 levels | 67.8 | -5.8x10-3 | 17.4x10-3 |
| using interpolation of 2 levels | 67.1 | -6.3x10-3 | 17.4x10-3 |

Table 3 Summary of statistics in Figure 8 (Regridded mesoscale relative spacing errors)

| | Absolute Error < 1/60 (%) | Mean error | RMS error |
|---|---|---|---|
| using level above | 59.2 | -10.2x10-3 | 20.3x10-3 |
| using level below | 66.7 | -1.6x10-3 | 17.5x10-3 |
| using mean of 2 levels | 68.6 | -5.8x10-3 | 16.8x10-3 |
| using interpolation of 2 levels | 67.8 | -6.2x10-3 | 16.8x10-3 |

Table 4 Summary of statistics in Figure 9 (Regridded mesoscale relative timing errors)

| | Absolute Error < 1/60 (%) | Mean error | RMS error |
|---|---|---|---|
| using level above | 67.5 | -8.2x10-3 | 18.2x10-3 |
| using level below | 71.4 | 1.2x10-3 | 17.1x10-3 |
| using mean of 2 levels | 76.5 | -3.4x10-3 | 14.7x10-3 |
| using interpolation of 2 levels | 76.9 | -3.8x10-3 | 14.6x10-3 |

Table 5 Summary of statistics in Figure 11 (WAFTAGE relative spacing errors)

| | Absolute Error < 1/60 (%) | Mean error | RMS error |
|---|---|---|---|
| using level above | 70.6 | -7.8x10-3 | 17.4x10-3 |
| using level below | 72.5 | 0.7x10-3 | 16.2x10-3 |
| using mean of 2 levels | 77.6 | -3.4x10-3 | 14.2x10-3 |
| using interpolation of 2 levels | 78.8 | -3.8x10-3 | 14.0x10-3 |

Table 6 Summary of statistics in Figure 12 (WAFTAGE relative timing errors)

## 5 Possible areas for improvement

### 5.1 Improvements to the verification method

A definite weakness of this study is the use of data from ascents to judge the accuracy of a forecast tool that is designed for predicting descent trajectories. As explained in section 4, ascent data was used because of the more frequent occurrence of INS drift in descent data. There are two differences between ascent and descent data that could be significant:

- In general, an aircraft will ascend to a higher altitude over the first few minutes of its flight than the altitude from which it would start the last few minutes of its descent. In general, forecast wind errors would be expected to be higher at higher altitudes because the associated wind speeds are higher.
- In general, an ascending aircraft has a higher airspeed making it less susceptible to the wind than a descending aircraft.

The only way to produce reasonable verification statistics from descent data would be to devise a scheme for adjusting errors caused by INS drift. The positions would need adjusting to correspond to a major airport and the associated vector adjustment would also need to be applied to the wind measurements.

### 5.2 Improvements to WAFTAGE

The WAFTAGE data assimilation scheme has three major components: The observational data, the model data used as a background field and the form of the covariance functions that allow the observations to be assimilated. There is room for improvement in the quality of all three components:

- If an INS drift correction scheme could be devised to run in real time (rather than simply for verification) then both the WAFTAGE and the mesoscale data assimilation schemes could benefit from the improved quality of the observations.
- Many AMDARs are switched off in the Heathrow area so there is scope for increasing the quantity (if not the quality) of the observational data used by WAFTAGE and the mesoscale model.
- Full mesoscale model forecasts are run every 6 hours. Running the mesoscale model more frequently could increase the quality of the background fields used by WAFTAGE and therefore improve forecast accuracy.
- Very little work has been done in recent years to determine the optimum covariance functions to use for the particular combination of observation and model data used in this project (i.e. AMDAR data and mesoscale model data). The accuracy of the WAFTAGE scheme would improve if such a study was performed and the recommendations were implemented.

### 5.3 Improvements to the co-ordinate system

The tables in section 4 show that forecast accuracy is sensitive to the vertical interpolation method used for verification. An interpolation is also performed from the mesoscale co-ordinate system to the FAST co-ordinate system so the data being verified has in fact been interpolated twice. One way of avoiding the subsequent inaccuracies would be to use a co-ordinate system that corresponds to where the aircraft will be flying rather than a regular latitude-longitude grid. Wind and temperature could be forecast at regular intervals along a variety of glideslopes so that the majority of descents would lie close to a trajectory for which the timing can be predicted without additional interpolation.

### 5.4 Removal of biases

Table 4 shows a mean timing error regardless of how the regridded mesoscale data is interpreted in the vertical. Table 6 shows that this bias is still present after WAFTAGE has been used. The model data, the observational data or the verification method itself could have introduced this bias. Investigating the source of the bias and correcting for it would improve the forecast accuracy. Table 7 to Table 12 show how the accuracy could be improved if the biases were removed

| | Error - Mean Error < 1/60 (%) | Standard Deviation of errors |
|---|---|---|
| using level above | 50.0 | 28.6x10-3 |
| using level below | 52.7 | 24.7x10-3 |
| using mean of 2 levels | 56.2 | 23.9x10-3 |
| using interpolation of 2 levels | 55.5 | 23.6x10-3 |

Table 7 Summary of statistics in Figure 5 relative to mean. (Form 214 relative spacing errors)

| | Error - Mean Error < 1/60 (%) | Standard Deviation of errors |
|---|---|---|
| using level above | 50.3 | 27.4x10-3 |
| using level below | 55.1 | 23.6x10-3 |
| using mean of 2 levels | 56.5 | 22.8x10-3 |
| using interpolation of 2 levels | 58.2 | 22.6x10-3 |

Table 8 Summary of statistics in Figure 6 relative to mean. (Form 214 relative timing errors)

| | Error - Mean Error < 1/60 (%) | Standard Deviation of errors |
|---|---|---|
| using level above | 68.2 | 18.3x10-3 |
| using level below | 63.9 | 18.3x10-3 |
| using mean of 2 levels | 71.0 | 16.4x10-3 |
| using interpolation of 2 levels | 72.2 | 16.3x10-3 |

Table 9 Summary of statistics in Figure 8 relative to mean. (Regridded mesoscale relative spacing errors)

| | Error - Mean Error < 1/60 (%) | Standard Deviation of errors |
|---|---|---|
| using level above | 70.2 | 17.5x10-3 |
| using level below | 67.1 | 17.5x10-3 |
| using mean of 2 levels | 73.3 | 15.8x10-3 |
| using interpolation of 2 levels | 72.5 | 15.6x10-3 |

Table 10 Summary of statistics in Figure 9 relative to mean. (Regridded mesoscale relative timing errors)

| | Error - Mean Error < 1/60 (%) | Standard Deviation of errors |
|---|---|---|
| using level above | 71.8 | 16.2x10-3 |
| using level below | 71.4 | 17.0x10-3 |
| using mean of 2 levels | 78.4 | 14.3x10-3 |
| using interpolation of 2 levels | 79.2 | 14.1x10-3 |

Table 11 Summary of statistics in Figure 11 relative to mean. (WAFTAGE relative spacing errors)

| | Error - Mean Error < 1/60 (%) | Standard Deviation of errors |
|---|---|---|
| using level above | 75.7 | 15.5x10-3 |
| using level below | 72.2 | 16.2x10-3 |
| using mean of 2 levels | 79.6 | 13.7x10-3 |
| using interpolation of 2 levels | 81.2 | 13.5x10-3 |

Table 12 Summary of statistics in Figure 12 relative to mean. (WAFTAGE relative timing errors)

## 6 Conclusions

The statistics presented in this report show that the required accuracy for FAST can be met on over 75% of occasions in the Heathrow area using the system including WAFTAGE as described. If additional work were carried out to remove biases then this percentage could increase by approximately 2%. It is hard to quantify the benefits of implementing any of the other suggestions in section 5. However, finding ways of correcting for INS drift and making general improvements to WAFTAGE would have benefits for other projects so are certainly worth consideration.

## 7 References


Dharssi, I. and Forrester, D.A. "The improvement of Meteorological Data for Air Traffic Management Purposes", Stage 1 Report on CAA Contract No. 7D/S/998/1, October 1992.

Smith, C. "FAST – MET Data Requirements", NATS internal memo Ref:8RD/16/19/06, November 1998.


## 8 Glossary of acronyms

| | |
|---|---|
| AMDAR | Aircraft Meteorological DAta Relay (type of automated aircraft report) |
| FAST | Final Approach Spacing Tool |
| Form 214 | UK Spot Wind Chart (see Figure 3) |
| FL100 | The pressure level corresponding to 10 000 feet in the ICAO standard atmosphere |
| ICAO | International Civil Aviation Organisation |
| ILS | Instrument Landing System |
| INS | Inertial Navigation System |
| JCL | Job Control Language |
| MetDB | Meteorological Database |
| MSLP | Mean Sea Level Pressure |
| QNH | Lowest value of MSLP in altimeter setting region |
| RMS | Root Mean Square |
| WAFTAGE | Winds Analysed and Forecast for Tactical Aircraft Guidance over Europe |

# Appendix A - Description of code used to satisfy FAST user requirement.

Full documentation of the code exists elsewhere so only a brief description is presented in this report.

The code is executed from a single UNIX script. This script sets parameters such as the date, time and forecast hours required and creates a new directory for the data before executing 3 more scripts:

## Retrieve

This script submits JCL (Job Control Language) to the IBM mainframe to retrieve pp-fields of Wind, Temperature and Mean Sea Level Pressure (MSLP) for each of the required forecast hours from the latest operational fieldsfile. Ideally, these jobs should be given operational status but, since the code is still being being developed, they are currently submitted from the UNIX system.

## Regrid_fields

This script waits until 30 minutes before the first forecast is due and waits for the relevant pp-fields to be produced before running a Fortran executable

The executable is responsible for regridding the meteorological data from the mesoscale co-ordinate system to the FAST co-ordinate system. Firstly, a value of MSLP is extracted for the airport from each of the mesoscale model fields of MSLP. These values are then interpolated to 10-minute intervals for the six-hour period they cover. Ideally the QNH forecast should be fed to this program from the operational forecast of QNH so this part of the code would be redundant. The QNH is used to calculate the pressure levels of the lower heights required.

The next stage of the program is to form a cubic spline from the column of met data (Temperature, U or V-component of wind) at each gridpoint of the mesoscale grid. One added complication is that U and V are stored on a grid that is staggered relative to the surface pressure grid. The surface pressure is needed to establish the pressure of each level in the column so U and V are interpolated to the surface pressure grid before the cubic spline is formed. The data is also interpolated in time to 10-minute intervals before the splines are formed. The cubic splines are used to establish the value of T, U and V at every mesoscale gridpoint at the FAST heights at 10-minute intervals.

The final stage of the program is to interpolate the data from the mesoscale grid to the FAST grid and write each field to file.

## Waftage.script

This script waits for the regridded pp-fields to be produced by Regrid_fields and waits until each forecast is due before running the 3 Fortran executables described below.

- The first executable simply calls the MetDB (Meteorological Database) and updates the observations file to contain the most recent AMDAR (Aircraft Meteorological Data Relay) observations that have been received.
- The next executable prepares the observations for use in the main "WAFTAGE" executable. This involves pairing each observation with model data for its location in space and time. The interpolations are performed from mesoscale model data in much the same way as in the regridding routine. Observations which fail the Quality Control checks or which fall outside the 4-dimensional area of interest are discarded. Good observations are written to file in a new format.
- The final executable is the main "WAFTAGE" code and is described in Appendix B.

## Appendix B - Description of WAFTAGE code

WAFTAGE uses an optimal interpolation data assimilation scheme to nudge input fields towards recent observations. A thorough description of the scheme can be found in Dharssi and Forrester, 1992.

The basic principle is that each point in the input (or background) field will be adjusted according to the following optimal interpolation equation:

$$a_k = b_k + \sum_i Bgrd_{ki} \sum_j \left( B_{ij} + O_{ij} \right)^{-1} \left( o_j - b_j \right)$$

$a_k$ is the new value at the kth grid point, $b_k$ is the input value at the kth grid point, *o* is the vector of observed values, *b* is a vector of corresponding values interpolated from the background field, *O* is a symmetric matrix of observation error covariances, *B* is a symmetric matrix of background error covariances and *Bgrd* is a matrix of background error covariances between observations and gridpoints. The summations over *i* and *j* are both over the number of observations.

WAFTAGE also calculates new values at the observation points using a version of this equation (with *Bgrd* replaced by *B*). This is a useful tool for assessing how the scheme has reached its result but is not necessary for establishing the new values at the gridpoints.

The equation for *O* is given in 5 and an explanation of how matrix *B* is calculated is given in 6 and 7.

WAFTAGE has a time saving measure built in which makes the code vastly more complicated than it otherwise would be. Instead of performing all summations over the entire number of observations, it performs them only for the closest observations to the gridpoint (or observation) of interest. It does this by "pigeon-holing" all the observations and gridpoints into 4D boxes and forming arrays to store the indices of the nearest observations for each gridpoint or observation (see 1, 2 and 3).

WAFTAGE also updates the probability of a gross error in the observation throughout the assimilation process so that the influence of potentially bad observations with non-Gaussian errors can be diminished.

### Details of major subroutines

#### *1 MKIDX*

This subroutine takes the 4D locations of the observations and pigeon-holes them in a 4D grid. An upper limit is set on the number of "neighbours" an observation is allowed. The number of neighbours is found by looking at the surrounding boxes. The number of surrounding boxes to look in is initially set to a default in the horizontal, vertical and temporal directions. The subroutine checks if these are too large for each observation in turn and reduces them if they are. The actual number of surrounding boxes to look in for each box are then stored for later use.

#### *2 NEHBUR*

This subroutine is called by the main program for each observation in turn and by the subroutine GRIDS (8) for each gridpoint in turn. It takes the 4D location of the observation/gridpoint and the arrays created in MKIDX (1) and returns an array which stores the number of neighbours of the observation and the indices of those neighbours.

#### *3 MK_IDXOBS*

This subroutine loops through the observations creating an array which stores the number of correllated neighbours of the observation and the indices of those correllated observations (e.g. those with the same callsign/station number).

#### *4 ISANAL*

The aim of this subroutine is to calculate an array of WAFTAGE re-analysis values at the observation points using the optimal interpolation equation. This is done by setting $qnal = (B+O)^{-1}(o-b)$ and solving this iteratively using Newton's method. Firstly, the subroutine calculates *B* using BCK_ERR (6) and *O* using OBS_ERR (5). *O* is corrected during each iteration using the probablility that the observation does not contain a gross error. This probability is also updated at each iteration.

## 5 *OBS_ERR*

Preliminary calculation of *O* using: $$O(io, jo) = \frac{1}{2}\sigma^2 e^{-2\Delta t^2} + \frac{1}{2}\sigma^2 \delta_{iojo}$$

where $\Delta t$ is the difference in time between observation *io* and *jo* and *jo* is a correlated neighbour of *io* from MK_IDXOBS (3). $\delta_{iojo} = 1$ if *io* = *jo* otherwise it is zero (Kronecker delta function). *O* is zero for observation pairs which are uncorrelated.

## 6 *BCK_ERR*

This subroutine is called by ISANAL (4) for each observation point and by GRIDS (8) for each gridpoint. It loops through all the neighbours of that observation or gridpoint. It calculates the difference in x, y, z and t between the observation or gridpoint location and its neighbour and the average Coriolis parameter for their locations. It then calls UVCOR (7) to calculate the correlation between them and scales the output, using the neighbouring observation of temperature, to produce the covariance matrix.

## 7 *UVCOR*

This subroutine calculates the correlation between neighbouring points using the equations outlined in Appendix C.

## 8 *GRIDS*

The aim of this subroutine is to calculate an array of WAFTAGE re-analysis values at the grid points of the input analysis field using the optimal interpolation equation. (*qnal* has already been calculated in ISANAL (4).)

## Appendix C - Gaussian form of the error covariance function

Each Temperature-Temperature (T-T) element of the error covariance matrix is given by

$$B_{TT}(\Delta r) = \sigma_T^2 \exp\left(\frac{-\Delta r^2}{2}\right) \text{ where } \Delta r^2 = \frac{\Delta x^2}{L_T^2} + \frac{\Delta y^2}{L_T^2} + \frac{\Delta z^2}{Z_T^2} + \frac{\Delta t^2}{T_T^2}$$

$\Delta x$, $\Delta y$, $\Delta z$ and $\Delta t$ are the distances in 4 dimensions between 2 observations or an observation and gridpoint in a forecast field. $\sigma$ is the expected standard deviation of the errors in the observations or forecasts. $L_T$, $Z_T$ and $T_T$ are tuneable parameters used to determine the area over which the nudging has an effect. (T stands for Temperature in these equations and W stands for Wind in the subsequent equations). For the stream function ($\psi$) and velocity potential ($\chi$) of wind we have

$$B_{\psi\psi}(\Delta r) = \sigma_\psi^2 \exp\left(\frac{-\Delta r^2}{2N^2}\right) \quad \text{and} \quad B_{\chi\chi}(\Delta r) = \sigma_\chi^2 \exp\left(\frac{-\Delta r^2}{2N^2}\right)$$

$$\text{where} \quad \Delta r^2 = \frac{\Delta x^2}{L_W^2} + \frac{\Delta y^2}{L_W^2} + \frac{\Delta z^2}{Z_W^2} + \frac{\Delta t^2}{T_W^2}$$

$$u = -\frac{\partial \psi}{\partial y} + \frac{\partial \chi}{\partial x} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x} + \frac{\partial \chi}{\partial y}$$

so the u-u, v-v, u-v and v-u elements of the error covariance matrix are given by

$$B_{uu}(\Delta r) = -\left(\frac{\partial^2 B_{\psi\psi}(\Delta r)}{\partial y^2} + \frac{\partial^2 B_{\chi\chi}(\Delta r)}{\partial x^2}\right) = \sigma_\psi^2 \frac{1+\gamma^2}{N^2 L_W^2}\left[1 - \frac{\Delta y^2 + \gamma^2 \Delta x^2}{N^2 L_W^2 (1+\gamma^2)}\right]\exp\left(\frac{-\Delta r^2}{2N^2}\right)$$

$$B_{vv}(\Delta r) = -\left(\frac{\partial^2 B_{\psi\psi}(\Delta r)}{\partial x^2} + \frac{\partial^2 B_{\chi\chi}(\Delta r)}{\partial y^2}\right) = \sigma_\psi^2 \frac{1+\gamma^2}{N^2 L_W^2}\left[1 - \frac{\Delta x^2 + \gamma^2 \Delta y^2}{N^2 L_W^2 (1+\gamma^2)}\right]\exp\left(\frac{-\Delta r^2}{2N^2}\right)$$

where $\gamma^2 = \frac{\sigma_\chi^2}{\sigma_\psi^2}$. This is assumed to be approximately 0.01 ($\sigma_\chi < \sigma_\psi$)

$$B_{uu}(0) = \sigma_u^2 = B_{vv}(0) = \sigma_v^2 \quad \text{so} \quad \sigma_\psi^2 \frac{1+\gamma^2}{N^2 L_W^2} = \sigma_W^2$$

$$B_{uv}(\Delta r) = B_{vu}(\Delta r) = \left(\frac{\partial^2 B_{\psi\psi}(\Delta r)}{\partial x \partial y} - \frac{\partial^2 B_{\chi\chi}(\Delta r)}{\partial x \partial y}\right) = \sigma_W^2 \left[\frac{\Delta x \Delta y (1-\gamma^2)}{N^2 L_W^2 (1+\gamma^2)}\right]\exp\left(\frac{-\Delta r^2}{2N^2}\right)$$

The thermal wind equation gives an approximation for u and v in terms of T

$$\left(-\frac{\partial T}{\partial y}, \frac{\partial T}{\partial x}\right) = \frac{fT}{g}\left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}\right) = \frac{fT}{g}\left(-\frac{\partial^2 \psi}{\partial z \partial y} + \frac{\partial^2 \chi}{\partial z \partial x}, \frac{\partial^2 \psi}{\partial z \partial x} + \frac{\partial^2 \chi}{\partial z \partial y}\right)$$

so the T-u, u-T, T-v and v-T elements are given by

$$B_{Tu}(\Delta r) = B_{uT}(\Delta r) = \frac{fT}{g}\left(\frac{\partial^2 B_{\psi\psi}(\Delta r)}{\partial z \partial y}\right) = \sigma_W^2 \left[\frac{\Delta z \Delta y}{N^2 Z_W^2 (1+\gamma^2)}\right]\exp\left(\frac{-\Delta r^2}{2N^2}\right)$$

$$B_{Tv}(\Delta r) = B_{vT}(\Delta r) = \frac{fT}{g}\left(-\frac{\partial^2 B_{\psi\psi}(\Delta r)}{\partial z \partial x}\right) = \sigma_W^2 \left[\frac{-\Delta z \Delta x}{N^2 Z_W^2 (1+\gamma^2)}\right]\exp\left(\frac{-\Delta r^2}{2N^2}\right)$$

# List of figures

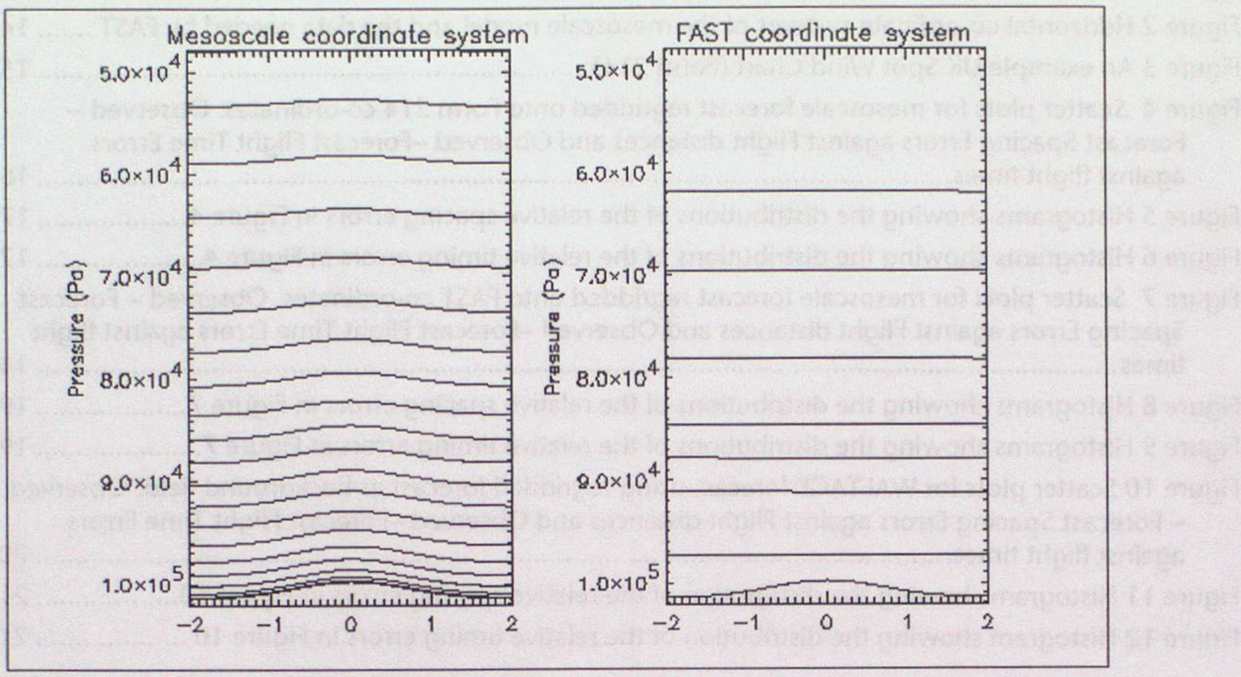


Figure 1 Vertical co-ordinate systems of the mesoscale model and the data needed by FAST

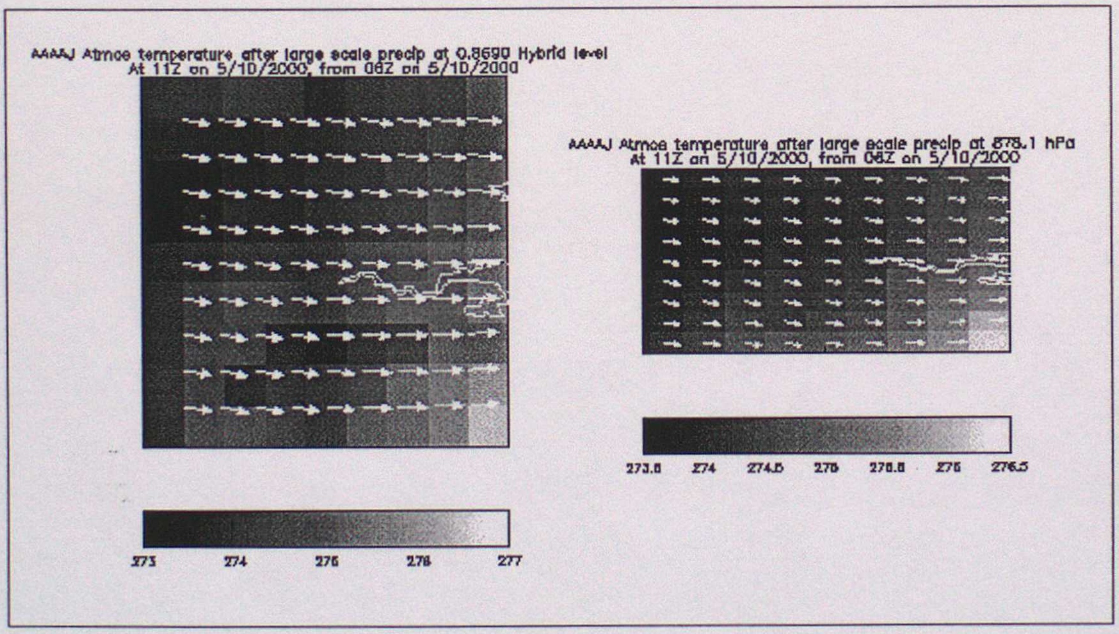


Figure 2 Horizontal co-ordinate systems of the mesoscale model and the data needed by FAST

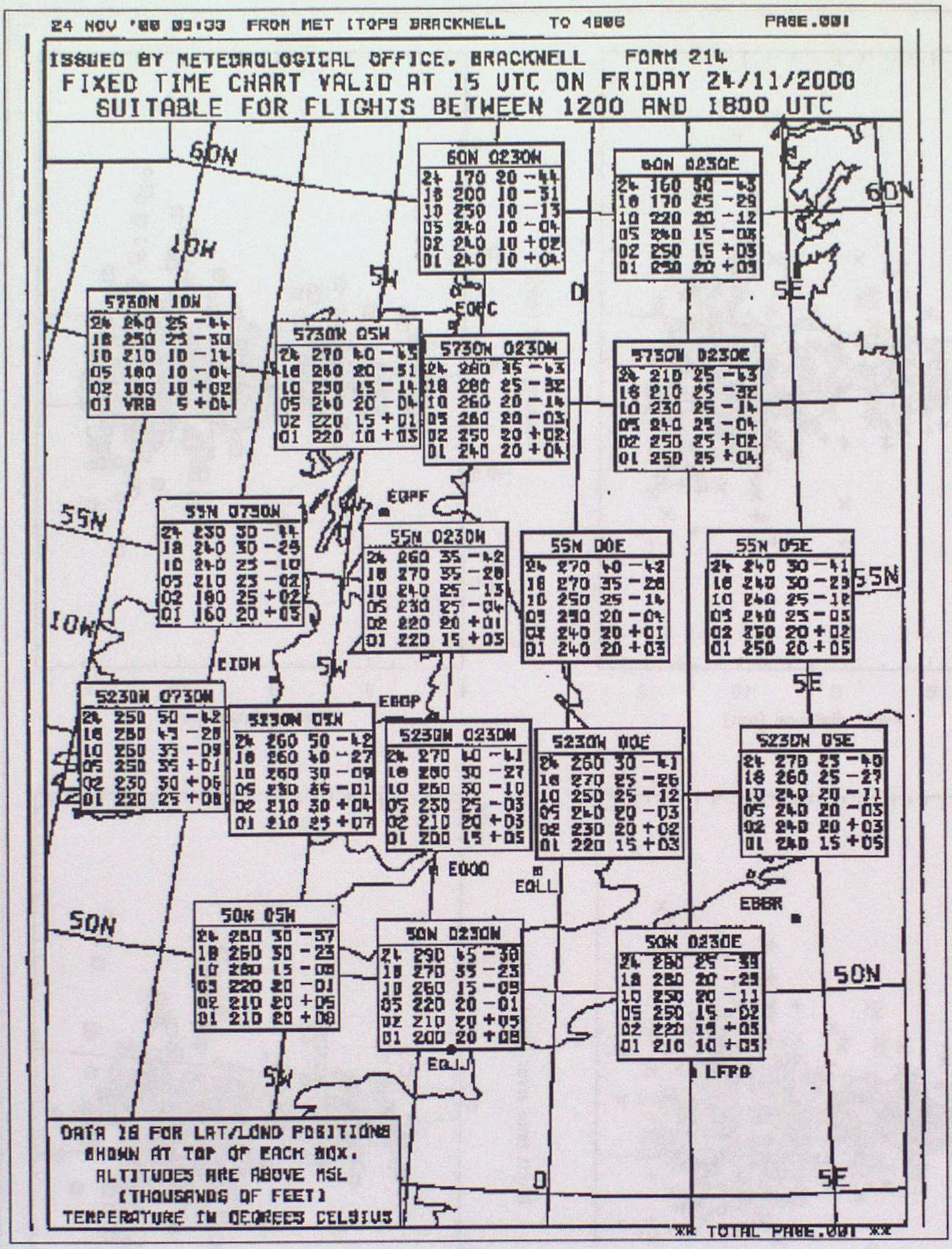


Figure 3 An example UK Spot Wind Chart (Form 214)

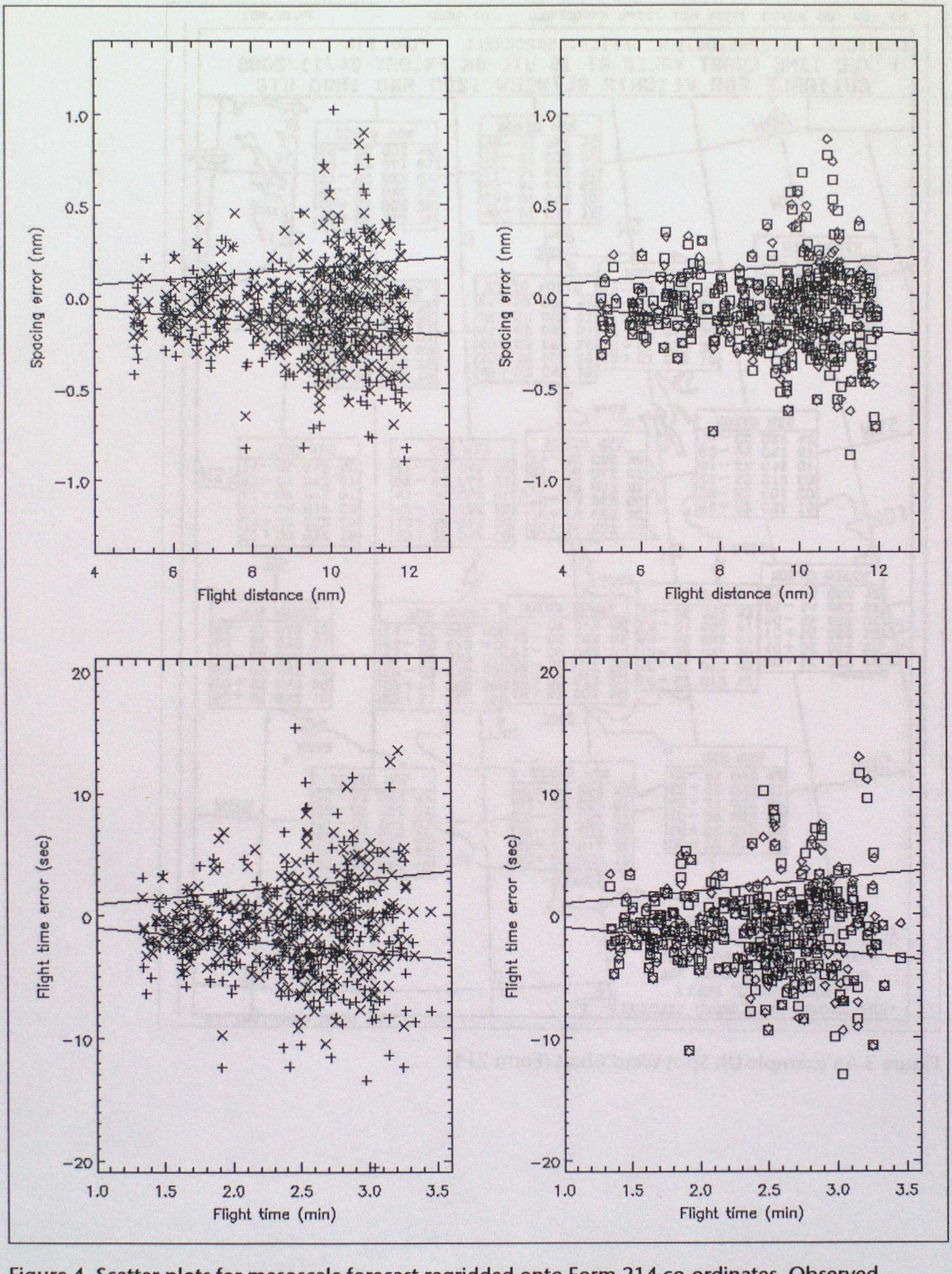


Figure 4 Scatter plots for mesoscale forecast regridded onto Form 214 co-ordinates. Observed – Forecast Spacing Errors against Flight distances and Observed –Forecast Flight Time Errors against flight times.

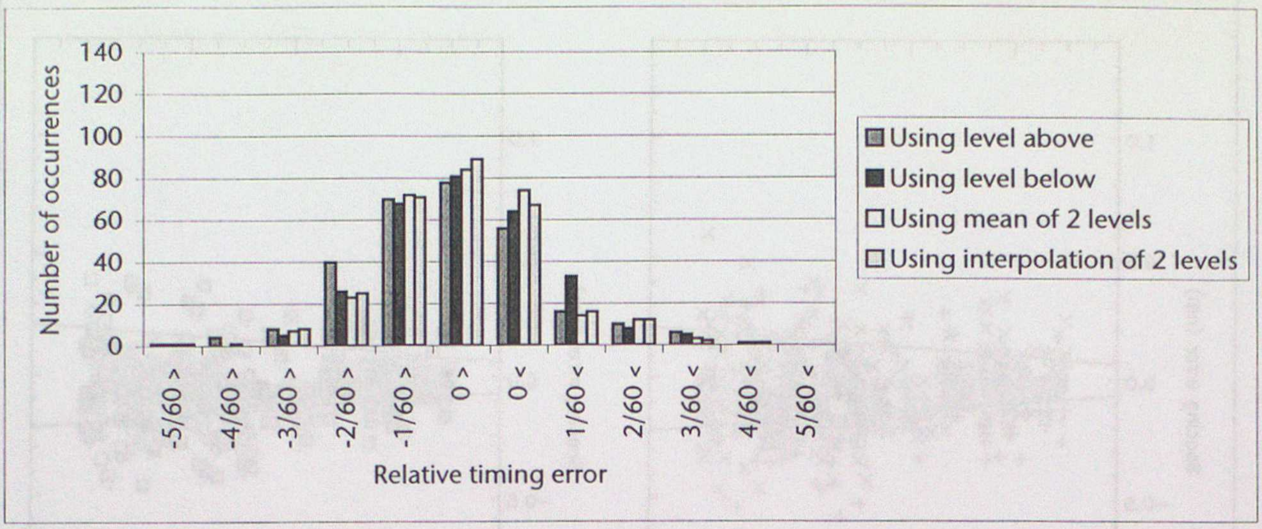


Figure 5 Histograms showing the distributions of the relative spacing errors in Figure 4

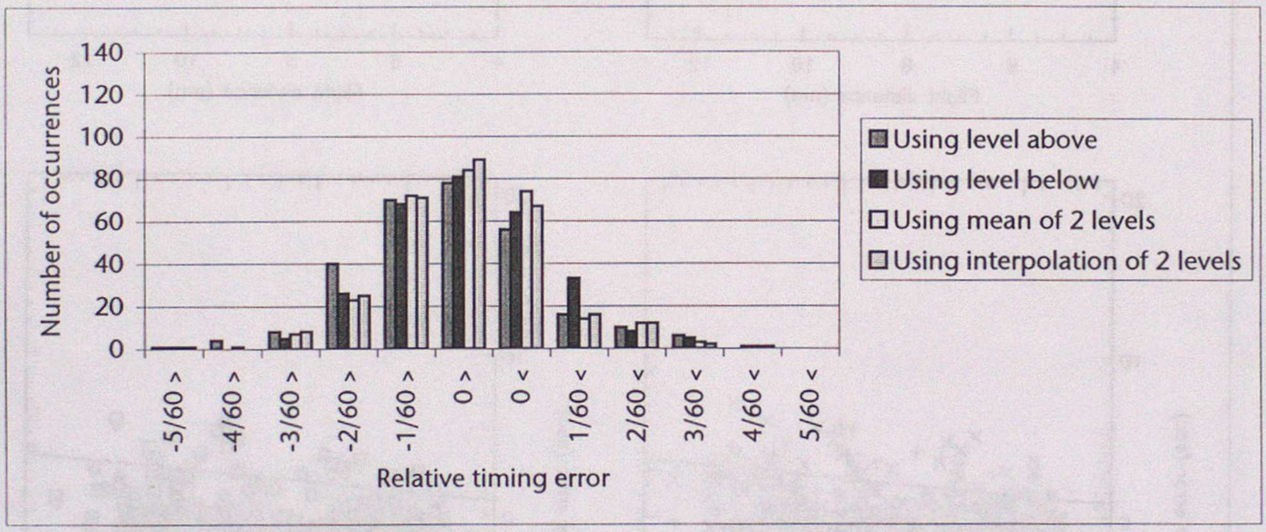


Figure 6 Histograms showing the distributions of the relative timing errors in Figure 4

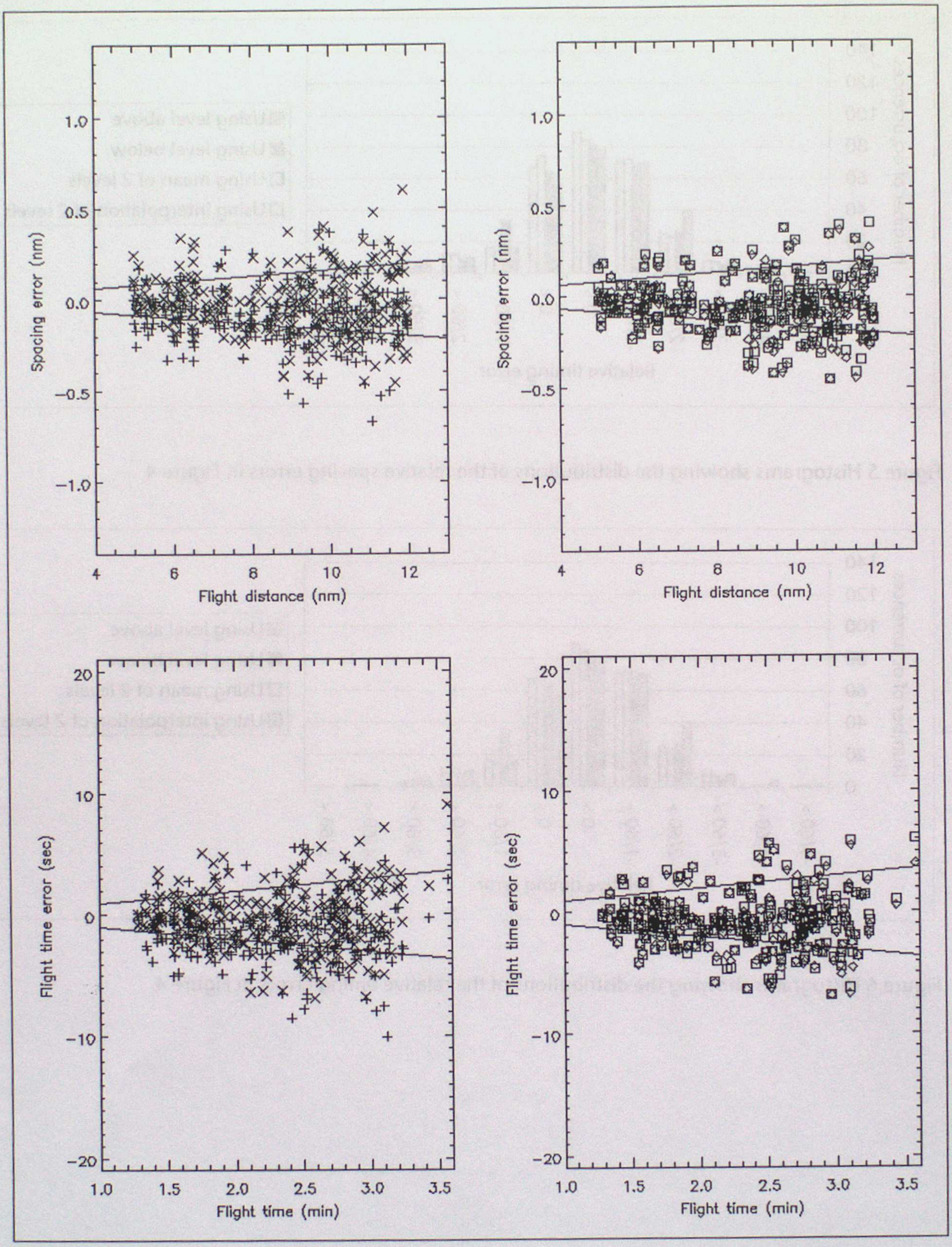


Figure 7 Scatter plots for mesoscale forecast regridded onto FAST co-ordinates. Observed – Forecast Spacing Errors against Flight distances and Observed –Forecast Flight Time Errors against flight times

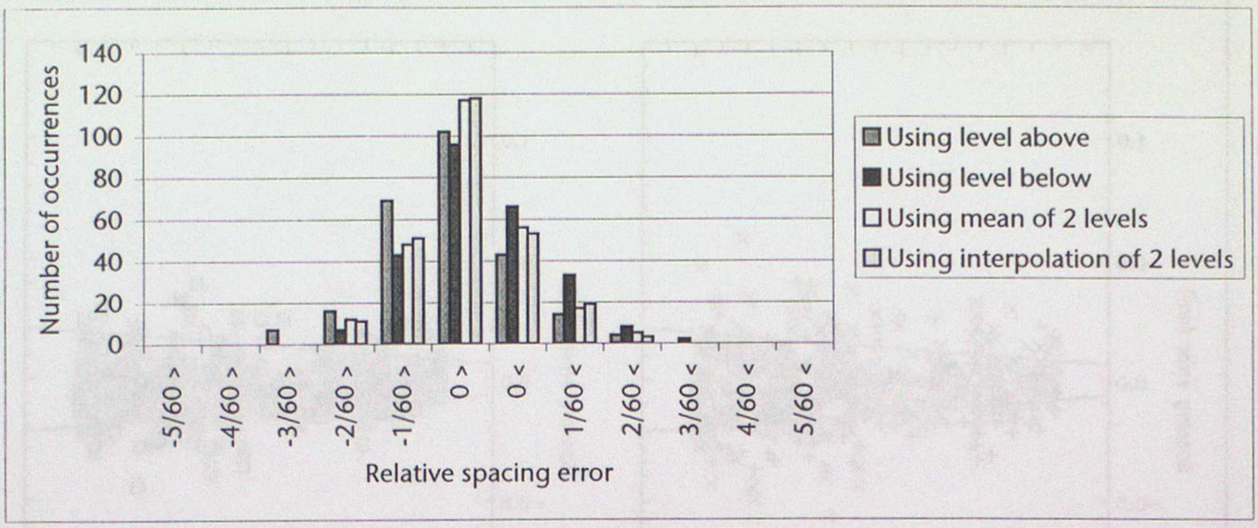


**Figure 8 Histograms showing the distributions of the relative spacing errors in Figure 7**

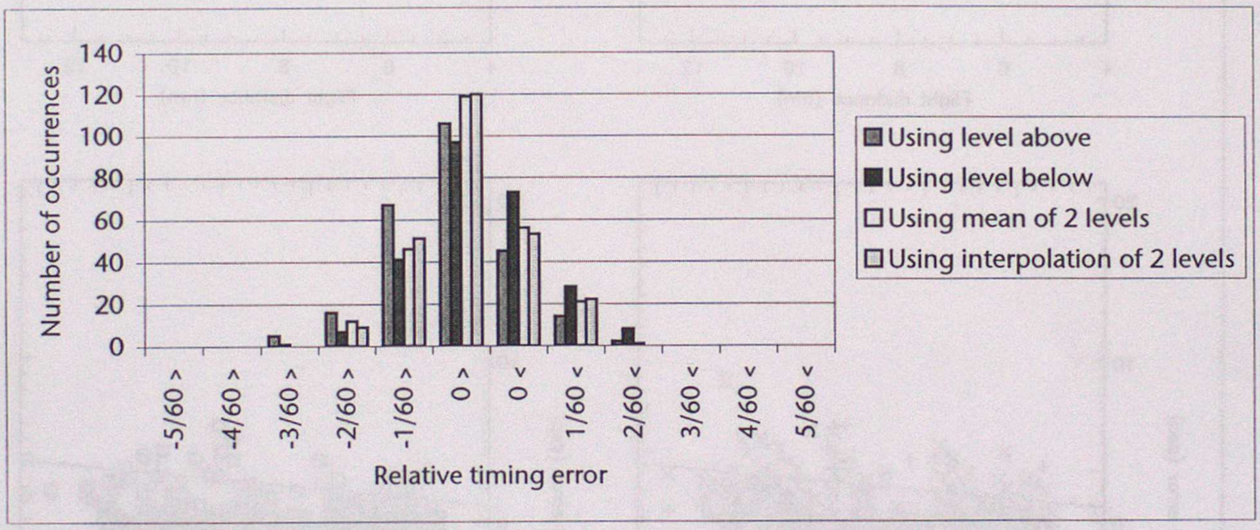


**Figure 9 Histograms showing the distributions of the relative timing errors in Figure 7**

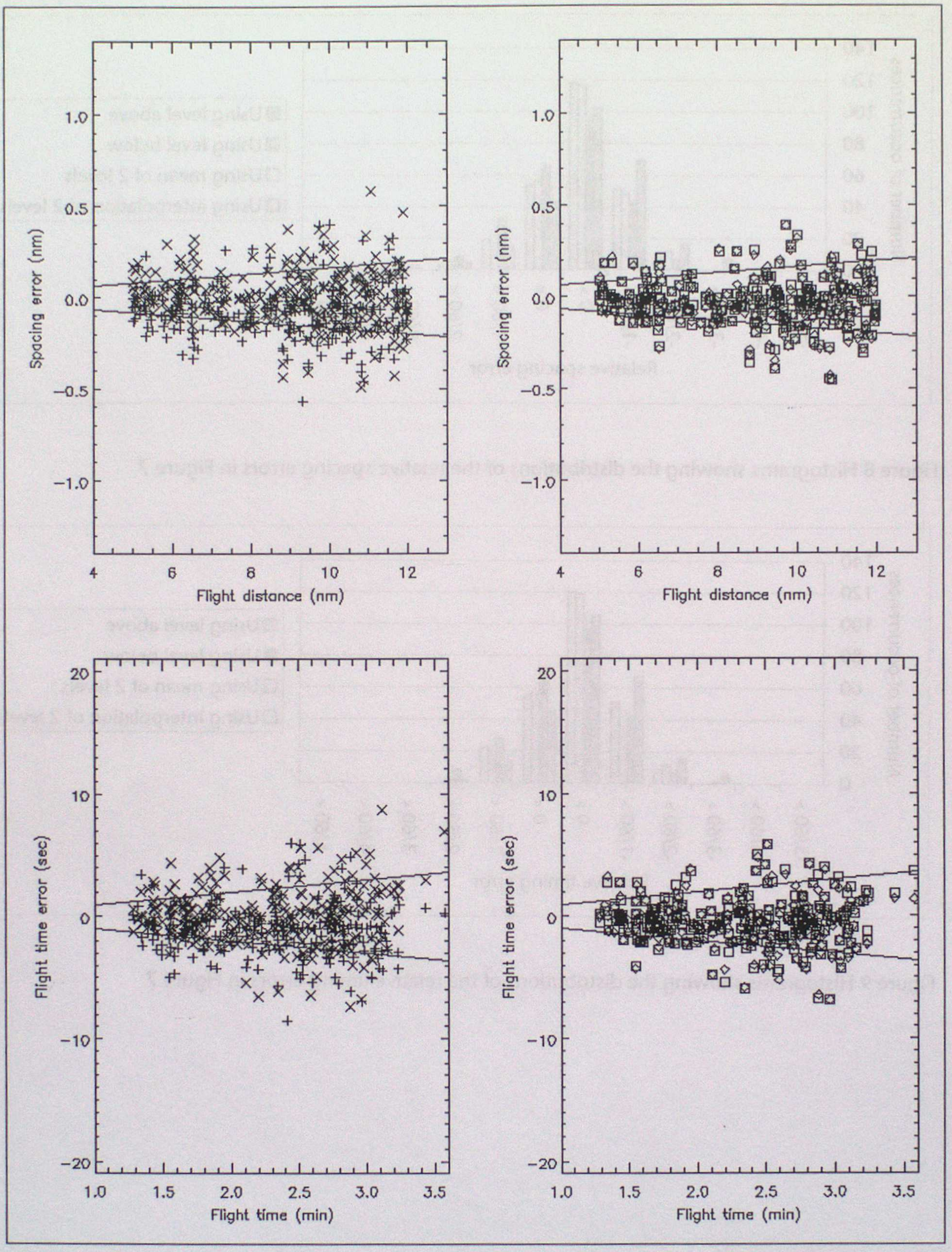


Figure 10 Scatter plots for WAFTAGE forecast using regridded forecast as background field. Observed – Forecast Spacing Errors against Flight distances and Observed –Forecast Flight Time Errors against flight times.

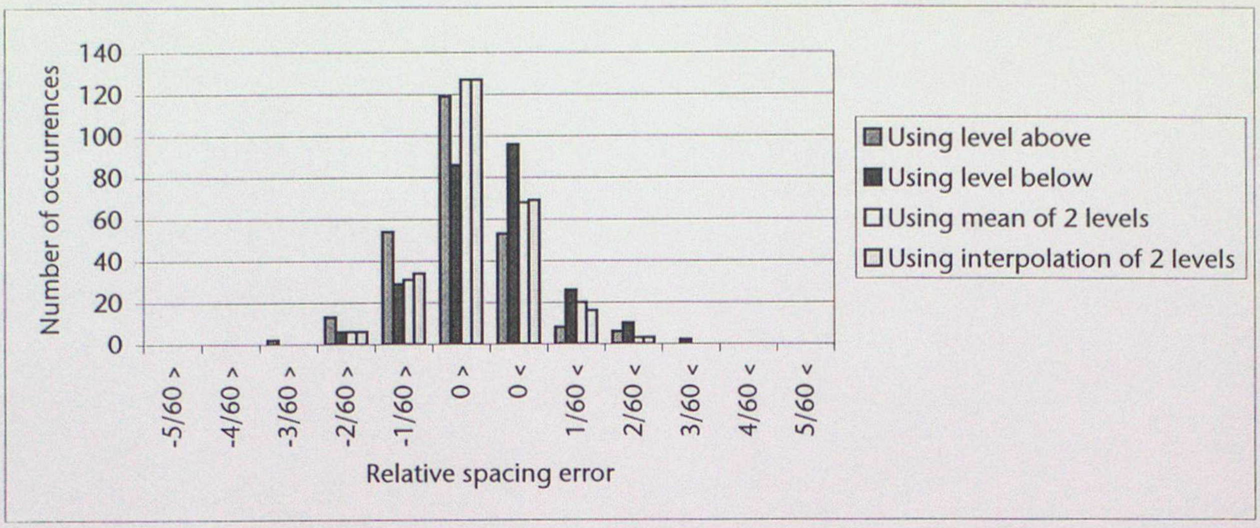


Figure 11 Histogram showing the distribution of the relative spacing errors in Figure 10

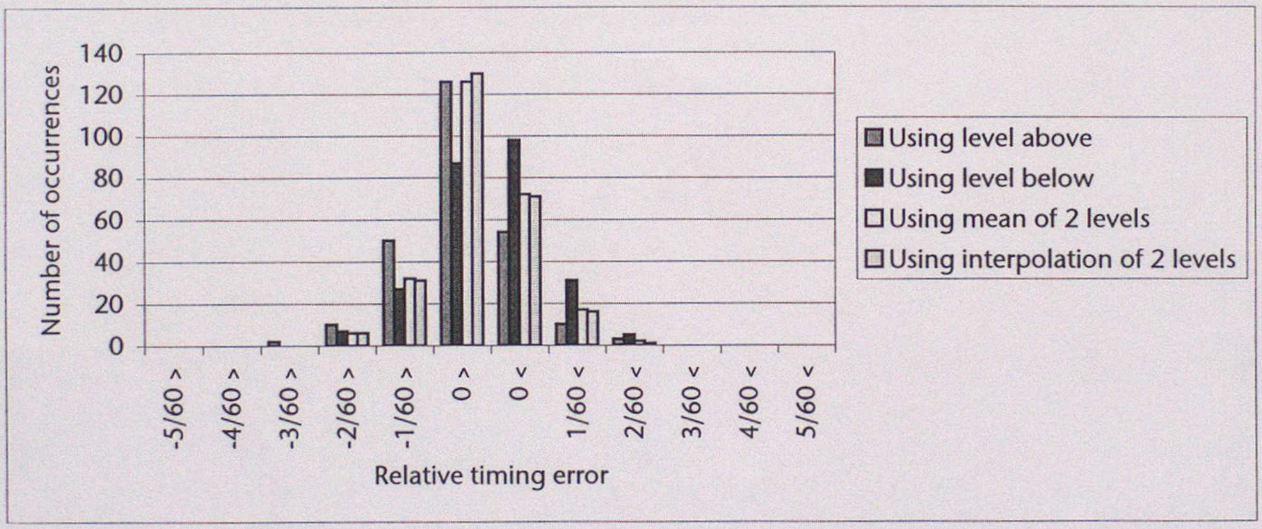


Figure 12 Histogram showing the distribution of the relative timing errors in Figure 10