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Cloud Microphysical Processes – A Description of the Parametrization used in the Large Eddy Model

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1 Introduction

The microphysics scheme in place in the LEM uses the parametrization of precipitating water particles based on (Lin et al. 1983). Lin however does not detail the representation of cloud. The ice and liquid cloud processes are parametrized using a scheme based on (Cotton et al.1986). The purpose of this paper is to bring together a comprehensive set of conversion rates for a six variable *bulk water* scheme. The resulting scheme is the basis of most microphysics schemes used in 3-D cloud resolving models (Flatau, Tripoli and Cotton 1989), (McCumber, Tao and Simpson 1991) ...

The microphysical processes in a cloud are those processes leading to the formation, growth and depletion of the water particles. These particles can be liquid, ice or a combination of both and may have an irregular or regular shape. The model's scheme divides these particles into several categories commonly used in bulk water schemes: liquid cloud droplets(q_C), rain drops (q_R), ice crystals (q_I), snow crystals or aggregates (q_S), and graupel or hail (q_G). There is a further variable (N_I) which represents the number concentration of ice crystals. The mass mixing ratio *wrt* air of each water category is represented by a model variable. The size spectra of the precipitating particles (rain, snow and graupel) is assumed to be an inverse exponential distribution dependent on their mass mixing ratio. Cloud particles are assumed to form a monodispersive size spectrum of homogeneous droplets or crystals. The cloud droplets are assumed to have a constant number concentration which will depend on type of air mass, continental or maritime. The number concentration of ice crystals is modelled as a separate variable so that the various ice nucleation process can be parametrized.

2 Model Representation of water categories

In order to represent each water category with just the mass-mixing ratio, the size spectra must be prescribed functions, only depending on model variables. The fall speed and the

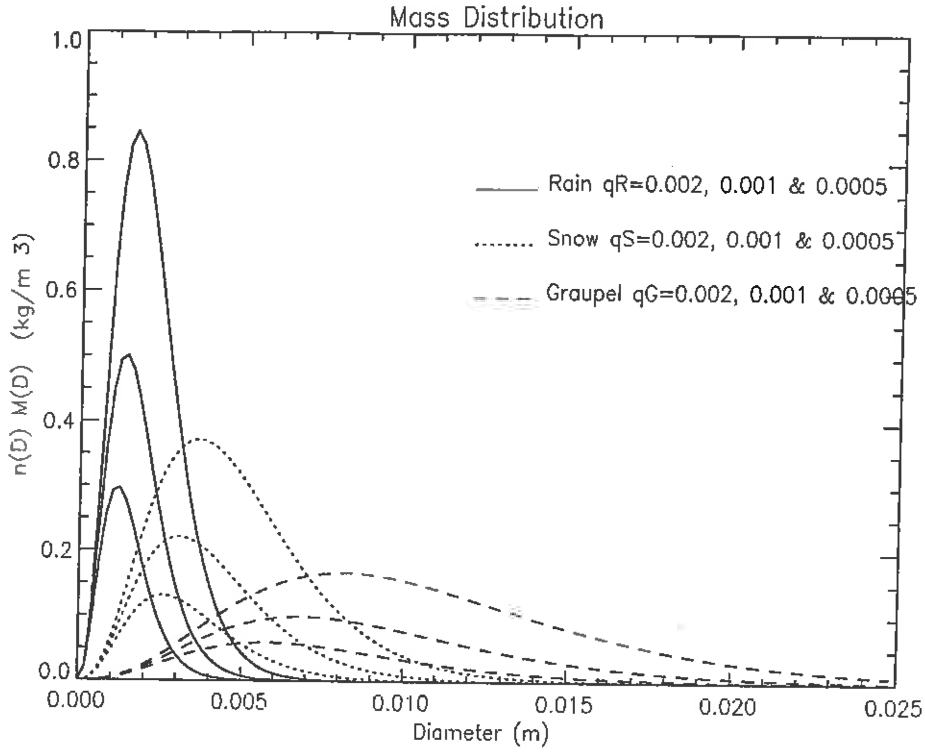

Figure 1: $\left(n(D)\frac{\pi D^3 \rho_w}{6}\right)$ against D for different mass mixing ratios of each hydrometeor.

rates of accretion, riming, evaporation, sublimation, condensation, melting and freezing depend on the size of the particles. Formulae for the conversion rates between water categories can be calculated by integrating the effect of a process over the spectrum of sizes.

Rain, snow and graupel are precipitating particles, those with non-zero fall velocity. Their size spectra is assumed to be inverse exponential:

$$n(D) = n_0 e^{-\lambda D}$$

The slope parameter, λ varies so that the mass mixing ratio is obtained by integrating the mass of each diameter of particle over the size spectra:

$$q = \frac{1}{\rho} \int_0^\infty n(D) \frac{\pi D^3 \rho_w}{6} dD$$

so

$$\lambda = \left(\frac{\pi \rho_w n_0}{\rho q} \right)^{0.25}$$

The intercept parameter, n_0 is assumed constant in time and space for a given simulation, and only varies with different applications of the scheme; eg for a given rain mixing ratio the rain drops are assumed fewer and larger in a tropical hurricane (Lord et al. 1984) than they will be in midlatitude frontal rainband (Rutledge and Hobbs 1983). This general equation describes the spectra of rain, snow and graupel:

$$n_R(D) = n_{0R} e^{-\lambda_R D}$$
$$n_S(D) = n_{0S} e^{-\lambda_S D}$$
$$n_G(D) = n_{0G} e^{-\lambda_G D}$$

and

$$\lambda_R = \left(\frac{\pi \rho_W n_{0R}}{\rho q_R} \right)^{0.25}$$
$$\lambda_S = \left(\frac{\pi \rho_S n_{0S}}{\rho q_S} \right)^{0.25}$$
$$\lambda_G = \left(\frac{\pi \rho_G n_{0G}}{\rho q_G} \right)^{0.25}$$

where the intercept parameters and particle densities are assumed constant. For mid-latitude deep convection simulations n_{0R}=8×10^{6}m^{-4}, n_{0S}=3×10^{6}m^{-4}, n_{0G}=4×10^{6}m^{-4}, ρ_W=1000kg/m^3, ρ_S=100kg/m^3 and ρ_G=500kg/m^3. Figure 1 shows the size spectra for rain, snow and graupel calculated using the values of the parameters given here and assuming a mass mixing ratio of q_R=q_S=q_G=1g/kg .

Ice crystals In addition to the five model variables representing liquid cloud and vapour combined, rain, snow, graupel and ice cloud a further variable is included to predict the number concentration of ice crystals. The main assumptions about ice crystals in the model are that the crystals are monodisperse and all of the same size so the mean crystal mass is $M_i = q_i/N_i$ where N_i is the number of crystals per unit mass of air (the new prognosed variable). The processes that initiate ice crystals are many and varied and are very dependent on the specific conditions. Once a pristine ice crystal is formed it quickly grows by deposition or by accreting liquid water so the modelling of both number concentration and mass mixing ratio is desirable.

Cloud water droplets in the model are assumed to have a constant number density but the only process that requires the mean cloud droplet radius is contact nucleation. For most processes the radius is assumed infinitely small. The effect of this approximation is to never have any supersaturation over water or any water cloud present in subsaturated air because rapid condensation and evaporation maintains thermodynamic equilibium. The conversion rate from cloud water to precipitating water due to coalescence of cloud droplets is highly parametrized without requiring a cloud drop size being defined.

This commonly used assumption allows water vapour and cloud water to be defined by just one model variable (q_T=q_V+q_C). The water vapour, the cloud water amount and the temperature are calculated every timestep from just two model variables, the total water mixing ratio and the liquid-water temperature (Shutts 1991).

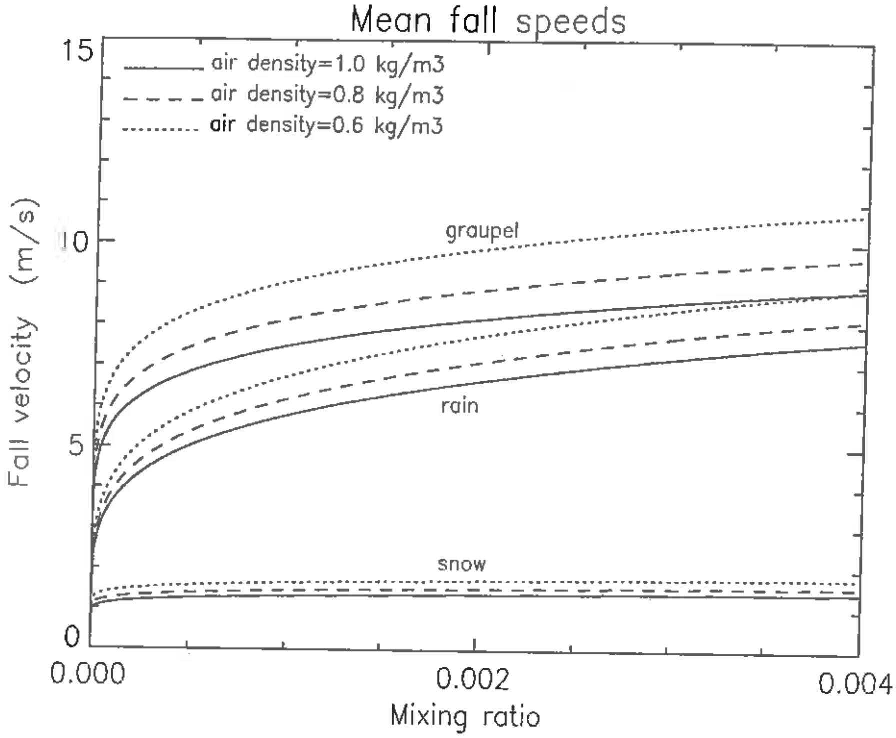

Figure 2: Mass-weighted mean terminal fall velocities for each hydrometeor.

3 Fall velocities

The terminal fall velocity of each type of precipitating particle,$v(D)$ can be expressed as a function of diameter:

$$v_{DR} = aD^b\left(\frac{\rho_0}{\rho}\right)^{0.5} \quad , \quad v_{DS} = cD^d\left(\frac{\rho_0}{\rho}\right)^{0.5} \quad , \quad v_{DG} = jD^k\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

The mass-weighted terminal fall velocity can therefore be defined by the following:

$$v = \frac{\int \frac{\pi D^3}{6} n(D) v_D dD}{q}$$

So the mass-weighted fall speeds of rain, snow and graupel, as shown on figure 2, are:

$$v_R = \frac{a\Gamma(4+b)}{6\lambda_R^b}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

$$v_S = \frac{c\Gamma(4+d)}{6\lambda_S^d}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

$$v_G = \frac{j\Gamma(4+k)}{6\lambda_G^k}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

4 Conversion Rates between water categories

The prognostic equations for each of the q variables are:

$$\frac{Dq_T}{Dt} = S_T$$

$$\frac{Dq_R}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_R v_R) + S_R$$

$$\frac{Dq_S}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_S v_S) + S_S$$

$$\frac{Dq_G}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_G v_G) + S_G$$

$$\frac{Dq_I}{Dt} = S_I$$

$$\frac{DN_I}{Dt} = S_N$$

S_T,S_R,S_S,S_G and S_I are source or sink terms due to transfer of water between the water categories, and their sum is zero. S_N is the source term for the number concentration of crystals. Each source term is the sum of all conversion rates to and from that water category.

$$S_T = P_{GSUB} + P_{SSUB} + P_{REVP} + P_{ISUB} \\ -(P_{SDEP} + P_{IDEP} + P_{GACW} + P_{RAUT} + P_{SACW} + P_{RACW} + P_{IFLT} + P_{IHAL} + P_{ICNT} + P_{IACW})$$

$$S_R = P_{GMLT} + P_{RAUT} + P_{GSHD} + P_{RACW} + P_{SMLT} + P_{IMLT} \\ -(P_{GACR} + P_{REVP} + P_{GFR} + P_{SACR} + P^g_{IACR} + P^s_{IACR})$$

$$S_S = P_{SAUT} + P_{SDEP} + P_{SACI} + P^s_{RACI} + P^s_{IACR} + P_{SACW} \\ -(P_{SSUB} + P_{GACS} + P_{RACS} + P_{GAUT} + P_{SMLT})$$

$$S_G = P_{GACI} + P_{GACW} + P_{GACS} + P_{GACR} + P_{SACR} + P_{RACS} P_{GAUT} + P_{GFR} \\ + P^g_{RACI} + P^g_{IACR} \\ -(P_{GSUB} + P_{GMLT} + P_{GSHD})$$

$$S_I = P_{IDEP} + P_{IFLT} + P_{IHAL} + P_{ICNT} + P_{IACW} \\ -(P_{SAUT} + P_{GACI} + P_{SACI} + P_{ISUB} + P_{IMLT} + P^g_{RACI} + P^s_{RACI})$$

$$S_N = [P_{IFLT} + P_{IHAL} + P_{ICNT}]/M_{I0} \\ - [P_{SAUT} + P_{GACI} + P_{SACI} + P^g_{RACI} + P^s_{RACI}] \times N_I/q_I$$

4.1 Autoconversion

Liquid cloud to rain: Autoconversion is the process whereby cloud water droplets form raindrops through collision and coalescence with each other. Following (Kessler 1969), it is parametrized as

$$P_{RAUT} = \alpha_C(q_C - q_{C0})$$

where the rate coefficient, $\alpha_C = 10^{-3}\mathrm{s}^{-1}$ and the threshold for autoconversion, q_{C0}=0.001 (deduced from airborne observations). (Weinstein 1970) showed from a sensitivity analysis that q_{C0} is the crucial parameter and order of magnitude variations in α_C do little to alter the overall microphysics. This is because once some rain is formed, the dominant process is accretion of cloud water by rain, and not collisions of cloud droplets.

Ice crystals to snow: The auto conversion of ice crystals to aggregates (snow) is due to collisions of the crystals with each other. The fall speed of an ice crystal is assumed to be

$$v_i = A_{vi}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

where the constants $A_{vi} = 0.04$.
The ice crystal diameter, D_i can be calculated from the mass diameter relationship $M_i(= q_i/N_i) = 0.0613D_i^2$. If it is assumed there is a variance in fall speed and that the efficiency of ice collisions making aggregates is constant ($E_{II} = 0.5$) then the autoconversion from ice crystals to aggregates (snow) is

$$P_{SAUT} = q_i N_i \frac{\pi}{6} D_i^2 v_i E_{II} X$$

where X is a constant proportional to the variance in fall speed.
The best estimate for X is by Passarelli and Srivastava (1979) of 0.25. Cotton et al.(1986) suggest that this be adjusted for different cases.

Snow to graupel: A new parameterization has been formulated for the autoconversion of snow to graupel. Some schemes such as (Lin et al 1983) parameterize this process in a similar way to that used for P_{RAUT} but this is unphysical as it allows graupel to form without the presence of super cooled liquid. The conversion of snow to form graupel dependent on the snow accreting liquid water. It is assumed that when snow rimes liquid cloud it increases its mass but the radius remains the same. A time scale Δt_{SG} can be calculated for all the snow to reach the density of graupel. So Δt_{SG} is the time for snow to accrete $(\frac{\rho_G}{\rho_S} - 1)$ of its own mass.

$$\Delta t_{SG} P_{SACW} = \left(\frac{\rho_G}{\rho_S} - 1\right) q_S$$

So the rate at which snow turns to graupel is $\frac{q_S}{\Delta t_{SG}}$ or

$$P_{GAUT} = \frac{P_{SACW}}{(\frac{\rho_G}{\rho_S} - 1)}$$

4.2 Collection processes

Accretion or riming by rain and snow Collection of ice or liquid cloud by rain or snow follows simple geometric sweep-out concepts. The proportion of air swept through by any type of precipitating particle in a second is:

$$\text{Sweepout rate} = \frac{1}{\rho}\int_0^\infty \frac{\pi D^2}{4} n(D)v(D)dD$$

Substituting in $v_R(D)$ and $n_R(D)$ gives the sweepout rate for rain:

$$\text{Sweepout rate}_{(RAIN)} = \frac{1}{\rho}\frac{\pi n_{0R} a}{4}\left(\frac{\rho_0}{\rho}\right)^{0.5} \int_0^\infty D^2 e^{-\lambda_R D} D^b dD$$

$$\text{Sweepout rate}_{(RAIN)} = \frac{1}{\rho}\frac{\pi n_{0R} a}{4}\left(\frac{\rho_0}{\rho}\right)^{0.5} \int_0^\infty D^2 e^{-\lambda_R D} D^b dD$$

$$= \frac{1}{\rho}\frac{\pi n_{0R} a \Gamma(3+b)}{4\lambda_R^{3+b}}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

Now the term for the accretion of water cloud by rain can be written as

$$P_{RACW} = \frac{\pi E_{RW} n_{0R} a q_C \Gamma(3+b)}{4\lambda_R^{3+b}}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

where the efficiency of coalescence for this process, $E_{RW} = 1$. This term always is a source of rain, even if the water is super-cooled. Accretion of ice-cloud by rain, however is a source term for either graupel or snow:

$$P_{RACI} = \frac{\pi E_{RI} n_{0R} a q_I \Gamma(3+b)}{4\lambda_R^{3+b}}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

The collection efficiency of rain for cloud ice, E_{RI} is assumed to be one. This accretion process involves three water species, so as well as P_{RACI}, the depletion of cloud ice, there is also a sink term for rain, P_{IACR}. This is calculated by assuming the ice crystals are monodispersive and of equal mass, $M_I = \frac{q_I}{N_I}$. Integrating over the spectrum of rain drop sizes gives the rate at which rain is accreted by ice:

$$P_{IACR}^{sorg} = N_I \int_0^\infty E_{RI}\frac{\pi D^2}{4} n_R(D) v_R(D) \frac{\pi D^3 \rho_W}{6} dD$$

$$P_{IACR}^{sorg} = \frac{\pi^2 E_{RI} n_{0R} a N_I \rho_W \Gamma(6+b)}{24\lambda_R^{6+b}}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

Since the cloud ice is small compared to rain droplets, the density of the new product will be determined by the size of the rain. If $q_R < 10^{-4}$, it is assumed that the rain drops will become less dense particles, so P_{RACI}^s and P_{IACR}^s are source terms for snow and $P_{RACI}^g = P_{IACR}^g = 0$. But if $q_R > 10^{-4}$ then P_{RACI}^g and P_{IACR}^g are source terms for graupel and $P_{RACI}^s = P_{IACR}^s = 0$.

The accretion of cloud ice by snow is an aggregation process which occurs if the temperature is less than T_0 (273K):

$$P_{SACI} = \frac{\pi E_{SI} n_{0S} c q_I \Gamma(3+d)}{4\lambda_R^{3+d}}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

where E_{SI} is the collection efficiency of snow for cloud ice. Similar to the rate coefficient for the autoconversion of snow from to cloud ice, E_{SI} is assumed to be temperature dependent and is expressed as

$$E_{SI} = e^{0.025(T-T_0)}.$$

The riming of cloud water by snow is,

$$P_{SACW} = \frac{\pi E_{SW} n_{0S} c q_L \Gamma(3+d)}{4\lambda_R^{3+d}} \left(\frac{\rho_0}{\rho}\right)^{0.5}$$

where E_{SW} is assumed to be 1 in this model. This conversion will increase the snow content at temperatures below freezing, and at temperatures above freezing it will increase the rain content via the assumption that frozen water will be shed from snow particles. The sensible heat associated with the accreted cloud water will also enhance the melting of snow (see the equation for the melting of snow).

The result of a double integration gives the rate at which rain is accreted by snow, P_{SACR} and the rate that snow is accreted by rain, P_{RACS}. Both of these processes produce graupel. The integration is made easier (or possible) by making the assumption that $| v_{DR}(D_R) - v_{DS}(D_S) | = MAX(v_R/4, | v_R - v_S |)$ for all values of D_R and D_S.

$$P_{RACS} = \pi^2 E_{SR} n_{0R} n_{0S} MAX(v_R/4, | v_R - v_S |) \left(\frac{\rho_S}{\rho}\right) \times \left(\frac{5}{\lambda_S^6 \lambda_R} + \frac{2}{\lambda_S^5 \lambda_R^2} + \frac{0.5}{\lambda_S^4 \lambda_R^3}\right)$$

$$P_{SACR} = \pi^2 E_{SR} n_{0S} n_{0R} MAX(v_R/4, | v_S - v_R |) \left(\frac{\rho_W}{\rho}\right) \times \left(\frac{5}{\lambda_R^6 \lambda_S} + \frac{2}{\lambda_R^5 \lambda_S^2} + \frac{0.5}{\lambda_R^4 \lambda_S^3}\right)$$

The collection efficiency for snow for rain and rain for snow, E_{SR} is one.

Accretion and riming by Graupel Graupel grows by accretion or riming of other water forms in either wet or dry growth mode. If all the liquid that is collected can not be frozen then wet growth results and water is shed from the graupel to form rain. The growth mode affects the efficiency of accretion of ice and snow, the efficiency is one if the surface of the graupel is wet but less if the graupel is in dry growth mode. The collection rates for liquid cloud, ice cloud, snow and rain can be written as:

$$P_{GACW} = \frac{\pi E_{GW} n_{0G} j q_L \Gamma(3+k)}{4\lambda_R^{3+k}} \left(\frac{\rho_0}{\rho}\right)^{0.5}$$

$$P_{GACI} = \frac{\pi E_{GI} n_{0G} j q_I \Gamma(3+k)}{4\lambda_R^{3+k}} \left(\frac{\rho_0}{\rho}\right)^{0.5}$$

$$P_{GACS} = \pi^2 E_{GS} n_{0G} n_{0S} MAX(v_G/4, | v_G - v_S |) \left(\frac{\rho_S}{\rho}\right) \times \left(\frac{5}{\lambda_S^6 \lambda_G} + \frac{2}{\lambda_S^5 \lambda_G^2} + \frac{0.5}{\lambda_S^4 \lambda_G^3}\right)$$

$$P_{GACR}^{dry} = \pi^2 E_{GR} n_{0G} n_{0R} MAX(v_R/4, | v_G - v_R |) \left(\frac{\rho_W}{\rho}\right) \times \left(\frac{5}{\lambda_R^6 \lambda_G} + \frac{2}{\lambda_R^5 \lambda_G^2} + \frac{0.5}{\lambda_R^4 \lambda_G^3}\right)$$

The efficiencies of these processes are E_{GW}=1, E_{GI}^{dry}=0.1, E_{GI}^{wet}=1, E_{GS}^{dry}=$0.2e^{0.08(T-T_0)}$, E_{GS}^{wet}=1 and E_{GR}=1. The term P_{GACR} is completely recalculated if the hail is in wet growth mode to take into account the shedding of water from graupel to form rain.

The criteria for wet growth of hail is

$$P_{GACR} + P_{PGACW} + P_{GACI}^{wet} + P_{GACS}^{wet} > P_{GWET}$$

where P_{GWET} is the amount of liquid that the graupel can freeze without shedding liquid. The equation for wet growth of hail is based on (Musil 1970) and modified to include the accretion of snow:

$$P_{GWET} = \frac{2\pi n_{0G}(\rho L_v(q_{0sat} - q_V) - K_a T_c)}{\rho(L_f + C_w T_c)} \times$$

$$\left[0.78\lambda_G^{-2} + 0.31 S_c^{\frac{1}{3}} \Gamma\left(\frac{k+5}{2}\right) j^{0.5} v^{-0.5} \left(\frac{\rho_0}{\rho}\right)^{0.25} \lambda_G^{-\left(\frac{k+5}{2}\right)}\right]$$

$$+(P_{GACI}^{wet} + P_{GACS}^{wet})\left(1 - \frac{C_i T_c}{L_f + C_w T_c}\right)$$

If the condition for wet growth is satisfied then P_{GACR} is redefined. If this would become negative then the shedding of water by graupel, P_{GSHD} becomes a source of rain.

$$P_{GACR}^{wet} = \mathrm{MAX}(0, P_{GWET} - P_{PGACW} - P_{GACI}^{wet} - P_{GACS}^{wet})$$

$$P_{GSHD} = \mathrm{MAX}(0, P_{PGACW} + P_{GACI}^{wet} + P_{GACS}^{wet} - P_{GWET})$$

It can be seen that shedding of water from graupel occurs when P_{GACW} is greater than $(P_{GWET} - P_{GACI}^{wet} - P_{GACS}^{wet})$. This shedding mechanism can cause rapid transformation of cloud water to rain in the 0 to $-10°C$ temperature range.

Riming by Icecloud The collection of liquid cloud droplets by ice crystals is calculated in a similar manner to the aggregation of ice crystals to form snow. The fall speed of an ice crystal is assumed to be

$$v_i = A_{vi}\left(\frac{\rho_0}{\rho}\right)^{0.5}$$

The ice crystal diameter, D_i can be calculated from the mass diameter relationship $M_i (= q_i/N_i) = 0.0613 D_i^2$. If it is assumed the efficiency that colliding ice and liquid droplets will rime is constant then the collection of water by ice crystals is calculated assuming a simple geometric sweepout:

$$P_{IACW} = q_L N_i \frac{\pi}{6} D_i^2 v_i E_{iw}$$

where the efficiency, $E_{iw} = 1$.

4.3 Melting and Freezing

All ice crystals ,graupel and snow upon melting will contribute to rain. The melting of ice crystals is assumed instant at temperatures above T_0. The equation for the melting rate of snow or graupel is based on heat balance considerations, with the cooling associated with the melting being balanced by the combined effects of conduction and convection of heat to the particle surface and the latent heat of sublimation and deposition. The snow or graupel melted per unit time from a single particle is given by

$$L_f \frac{dM_{melt}}{dt} = -2\pi D K_a T_c F' + 2\pi D L_v \psi \rho (q_V - q_{0sat}) F'$$

where q_{0sat} is evaluated at $0°C$ (the temperature of the particle surface) and F' is the ventilation factor which is given by (Thorpe and Mason 1966).

$$F' = 0.78 + 0.31 S_c^{\frac{1}{3}} R_e^{\frac{1}{2}} \quad , \quad R_e = V(D) D\rho / v$$

Integrating over all snow diameters, and applying a corrections due to the sensible heat associated with accreted liquid water the melting of snow to form rain is given as:

$$P_{SMLT} = \frac{2\pi}{\rho L_f}(K_a T_c - L_v \psi \rho (q_V - q_{0sat})) n_{0S}$$

$$\times \left[0.78 \lambda_S^{-2} + 0.31 S_c^{\frac{1}{3}} \Gamma \left(\frac{d+5}{2} \right) c^{0.5} v^{-0.5} \left(\frac{\rho_0}{\rho} \right)^{0.25} \lambda_S^{-\left(\frac{d+5}{2}\right)} \right]$$

$$+ \frac{C_w T_c}{L_f} (P_{SACW} + P_{SACR})$$

and the melting of graupel to form rain is given by:

$$P_{GMLT} = \frac{2\pi}{\rho L_f}(K_a T_c - L_v \psi \rho (q_V - q_{0sat})) n_{0G}$$

$$\times \left[0.78 \lambda_G^{-2} + 0.31 S_c^{\frac{1}{3}} \Gamma \left(\frac{k+5}{2} \right) j^{0.5} v^{-0.5} \left(\frac{\rho_0}{\rho} \right)^{0.25} \lambda_G^{-\left(\frac{k+5}{2}\right)} \right]$$

$$+ \frac{C_w T_c}{L_f} (P_{GACW} + P_{GACR} - P_{GSHD})$$

The equation for raindrop freezing is based on the work of (Bigg 1953) and represents the formation of graupel from rain due to immersion freezing, as explained in (Wisner et al 1972). It may be written as:

$$P_{GFR} = 20\pi^2 B_B n_{0R} \left(\frac{\rho_W}{\rho} \right) \left(e^{-A_B T_c} - 1 \right) \lambda_R^{-7}$$

where A_B and B_B are parameters in the freezing process as determined from laboratory experiment.

The freezing of liquid cloud droplets to ice cloud is instantaneous below T=-40°C. Above this temperature the Bergeron process is the main method of converting liquid cloud to ice cloud.

4.4 Sublimation and Deposition

The growth rate by vapour deposition of an ice crystal is given by (Byers 1965) as

$$\frac{dM}{dt} = \frac{C\left(\frac{q_V}{q_{isat}} - 1\right)}{A'' + B''}$$

where C is a shape parameter. For spheres the shape parameter $C = D/2$ but for ice crystals, which are assumed to be Hexagonal plates $C = 4\bar{D}_I$ where D_I is the average diameter of the ice particles. From (Hobbs et al. 1972), the diameter of a hexagonal plate can be computed from the mass of the plate by $D_I = 16.3M_I^{0.5}$. We compute M_I from q_I/N_I where N_I is the ice crystal concentration. Therefore the growth rate of cloud ice by deposition is

$$P_{IDEP}(\text{or} - P_{ISUB}) = \frac{4_{\bar{D}_I}\left(\frac{q_V}{q_{isat}} - 1\right) N_I}{A'' + B''}$$

where

$$A'' = \frac{L_S^2}{K_a R_w T^2}\ ,\quad B'' = \frac{1}{\rho q_{isat}\psi}$$

The depositional/sublimation growth rate of snow P_{SDEP} is based on Byers' equation for spherical particles but modified with a ventilation coefficient due to the particle having a non-zero fall velocity (as used in the melting equations). Integrating over all snow particle sizes gives:

$$P_{SDEP}(\text{or} - P_{SSUB}) = \frac{2\pi\left(\frac{q_V}{q_{isat}} - 1\right)}{\rho(A'' + B'')} n_{0S}$$

$$\times\left[0.78\lambda_S^{-2} + 0.31S_c^{\frac{1}{3}}\Gamma\left(\frac{d+5}{2}\right)c^{0.5}\upsilon^{-0.5}\left(\frac{\rho_0}{\rho}\right)^{0.25}\lambda_S^{-\left(\frac{d+5}{2}\right)}\right]$$

In air that is super-saturated *wrt* ice P_{SDEP} is positive and P_{SSUB} is set to zero, and in dry air P_{SSUB} is positive and P_{SDEP} is set to zero. This term are not calculated if T>0°C as melting is by far the dominant process at these temperatures.

The equation for the deposition and sublimation of graupel follows exactly the same principles:

$$P_{GDEP}(\text{or} - P_{GSUB}) = \frac{2\pi\left(\frac{q_V}{q_{isat}} - 1\right)}{\rho(A'' + B'')} n_{0G}$$

$$\left[0.78\lambda_G^{-2} + 0.31S_c^{\frac{1}{3}}\Gamma\left(\frac{k+5}{2}\right)j^{0.5}\upsilon^{-0.5}\left(\frac{\rho_0}{\rho}\right)^{0.25}\lambda_G^{-\left(\frac{k+5}{2}\right)}\right]$$

4.5 Evaporation and Condensation

The evaporation and condensation of cloud water is assumed to be instantaneous, so there is never any supersaturated air nor any cloud present in subsaturated air. Condensation of water vapour onto rain is therefore neglected, as supersaturation is required for this to occur.

The evaporation rate of rain is according to the concepts of diffusional growth originally developed by (Byers 1965) and is described in (Orville and Kopp 1977).

$$P_{REVP} = \frac{2\pi\left(1 - \frac{q_V}{q_{sat}}\right)}{\rho(A' + B')} n_{0R} \times \left[0.78\lambda_R^{-2} + 0.31 S_c^{\frac{1}{3}} \Gamma\left(\frac{b+5}{2}\right) a^{0.5} \upsilon^{-0.5} \left(\frac{\rho_0}{\rho}\right)^{0.25} \lambda_R^{-\left(\frac{b+5}{2}\right)}\right]$$

where

$$A' = \frac{L_v^2}{K_a R_w T^2}, B' = \frac{1}{\rho q_{sat} \psi}$$

This process only occurs in subsaturated air.

4.6 Initiation of Ice

Many models assume the number density of ice crystals is equal to the number density of natural ice nuclei as given by the Fletcher curve. But the Fletcher curve has been shown to be an underestimate of ice crystal concentrations by aircraft measurements with a 2-D cloud probe, (Heymsfield 1979).

The three ice initiation processes represented in the model are deposition onto natural ice nuclei (P_{IFLT}), ice splinter production during riming or the Hallet-Mossop effect (P_{IHAL}) and contact nucleation of water droplets by natural aerosols which is enhanced in subsaturated air (P_{ICNT}) . The latter two processes would be very difficult to model accurately without predictive equation for the ice number concentration because the parcel history is too important a factor.

These source terms for ice are now described. The ice number concentration source terms are these terms divided by M_{I0}, the pristine crystal mass.

Contact nucleation. This process is the freezing of cloud droplets due to collision with aerosols and is described by Young (1974). The collisions are due to Brownian motion so the rate depends on the whether the droplet is evaporating or growing. For an evaporating droplet the temperature gradient near the drop surface causes a thermophoretic force which increases the chance of collision, the gradient in water vapour concentration causes a diffusiphoretic force which reduces the chance of collision. The thermophesis dominates the diffusiphosis so contact nucleation is enhanced in subsaturated air where the water droplets are evaporating and colder than the surrounding air. Young's equations have been hugely simplified by assuming that the collisions per second can be expressed in the form

$$\frac{dn_a}{dt} = n_a n_c R_c \left(C_1 - (C_2 + C_3 R_c)\frac{\frac{q_V}{q_{sat}} - 1}{p}\right)$$

where n_a = aerosol concentration, n_c = cloud drop concentration (constant), R_c = cloud drop mean radius $=\left(\frac{3\rho q_L}{4\pi\rho_W}\right)^{\frac{1}{3}}$, and C_1, C_2 and C_3 are constants.
Assuming that the average aerosol particle is $0.2\mu m$ the constants can be interpolated from Young's curves.

$$C_1 = 5 \times 10^{-9} m^2 s^{-1} \quad , \quad C_2 = 1.5 \times 10^{-2} m^3 s\ kg^{-1} \quad , \quad C_3 = 6 \times 10^3 m^2 s\ kg^{-1}$$

The concentration of active contact nuclei is given by Young as

$$n_a = n_{a0}(270.16 - T)^{1.3}$$

where the constant n_{a0}=20000m^{-3}.
The growth of the cloud drops in the model is such that all the supersaturation is removed each timestep so the term $\left(\frac{q_V}{q_{sat}}\right)$ has to be diagnosed using Byers surface heat budget equation. The rate of change of mass of a single droplet due to evaporation or condensation is

$$\frac{dM}{dt} = \frac{4\pi R_c\left(\frac{q_V}{q_{sat}} - 1\right)}{A' + B'}$$

the total rate of change of liquid cloud mass is the rate of change of the saturation point minus the vapour transferred to the other water species:

$$n_c\frac{dM}{dt} = \rho\left(\frac{-dq_{sat}}{dz}w - P_{IDEP} - P_{SDEP}\right)$$

From the last two equations the supersaturation can be expressed as

$$\frac{q_V}{q_{sat}} - 1 = \frac{\rho(A' + B')}{4\pi R_c n_c}\left(\frac{-dq_{sat}}{dz}w - P_{IDEP} - P_{SDEP}\right)$$

The assumption is made that newly formed ice crystals have a mass M_{I0} so the conversion rate associated with this process is:

$$P_{ICNT} = \frac{1}{\rho}M_{I0}\frac{dn_a}{dt}$$

so the whole equation much simplified from Youngs original and put in terms of model variables is:

$$P_{ICNT} = \frac{1}{\rho}M_{I0}n_a n_c R_c\left[C_1 - (C_2 + C_3 R_c)\frac{\rho(A' + B')\left(\frac{-dq_{sat}}{dz}w - P_{IDEP} - P_{SDEP}\right)}{4\pi R_c n_c p}\right]$$

Deposition onto natural ice nuclei Following Cotton et al.(1991) Fletcher's equation predicting the number of natural ice nuclei has been modified to have a dependence on ice supersaturation and temperature. The equation used for the number concentration of ice nuclei is now

$$n_f = \frac{1}{\rho}n_{f0}e^{\beta T}\left(\frac{\frac{q_v}{q_{isat}} - 1}{\frac{q_{sat}}{q_{isat}} - 1}\right)^{4.5}$$

The rate of increase of this term following the motion gives the number of new crystals formed by deposition onto natural ice nuclei per second.

$$P_{IFLT} = M_{I0}\frac{dn_f}{dz}w$$

a numeric approximation makes this easier to calculate:

$$P_{IFLT} = M_{I0}\beta n_f\frac{dT}{dz}w$$

Ice splinter production by the Hallet Mossop Effect When water droplets are collected by large precipitating ice particles (graupel or snow) they can fracture causing ice splinters to be shed. This occurs because the outer shell of the droplet freezes first then as the centre freezes pressure builds up which shatters the freezing droplet. The speed at which the droplet freezes is critical for this process to occur. The necessary rate of freezing only occurs between -3°C and -8°C, peaking at -5°C. Approximately 350 ice splinters are formed for every milligram of rimed liquid at -5°C so

$$P_{IHAL} = 3.5 \times 10^8 M_{I0}(P_{GACW} + P_{SACW})f(T)$$

where f(T)=1 at -5°C and falls off to linearly to zero at -3°C and -8°C.

5 Validating and tuning the scheme

There are a number of physical values which can be varied in the scheme, to tailor it for modelling particular situations.. The observational work and modelling work that has been carried out to verify these has been biased toward tropical convection or continental deep convection, particularly in US. Using data from aircraft and radar these adjustable parameters can be either measured directly or inferred. Some of the assumptions of the parametrization can also be altered, without too much computational overhead, if observations show this to be beneficial.

5.1 Adjustable Parameters

A suitable starting point for investigation is to measure the parameters in the scheme that other modellers have varied for different applications. Particular emphasis should be put on those parameters which sensitivity studies show to be crucial to the accurate modelling of convective cloud. Many of the parameters in the following list are assumed constant, so measurements must be extensive to find a mean value or to determine them as a function of temperature or some model variable.

n_{0R},n_{0S},n_{0G} Intercept parameters used to describe the size spectra of rain, snow and graupel. Aircraft precipitation probe measurements can be used to estimate reasonable values of these for particular situations.

q_{C0},n_c The threshold mixing ratio for autoconversion of cloud water to rain will depend on the number concentrations of cloud droplets, which in turn depends on the number concentration of aerosols that act as condensation nuclei. This will be higher in continental air than in maritime air.

ρ_G,ρ_S Graupel particle and snow particle density. These are constants in the (Lin 1983) scheme. But the conversion rates and mass mean fall velocities could be recalculated with the particle density varying with particle size. Provision has now been made for the snow particle density to be in the form $\rho_S = AD_S{}^B$, where A and B are constant.

E_{xx} The efficiencies for collection or the probability that colliding particles will stick is assumed to be one if a liquid particle is involved in the collision. For ice-ice interactions, such as aggregation of crystals to form snow or for dry graupel to accrete snow, there is more uncertainty in the efficiency of collection. Aircraft or radar measurements that determine the time period that mixed particle types can coexist would be helpful in formulating new expressions for some of the efficiencies.

5.2 Assumptions of the parametrization

The assumption that the size spectra of precipitating particles can be described by an inverse exponential curve with constant intercept parameters: This is a widely used assumption in bulk-water schemes such as the one described. The results of detailed modelling of the size spectra of precipitating particles (Lee 1990) conclude that the mean drop size is larger for rain in dry air below the condensation level than it is inside a cloud, assuming the same mixing ratio. The consequence is to have the intercept parameter smaller at lower altitudes. Different approaches have been investigated. (Clark 1976) developed a warm-rain parametrization that assumed a log-normal size spectrum and (Cotton et al., 1986) assumed an inverse exponential size distribution, but with a constant mean particle diameter, as opposed to a constant intercept parameter, to describe the snow.

The assumption of constant density of graupel and snow particles: The scheme described assumed that every graupel particle has the same density (ρ_G=300kg/m^3). The graupel particle density will effect the fall speed and the sweepout rate of the falling graupel. Graupel is often the dominant precipitating particle in deep convection, so the evolution of convective cells is sensitive to its parametrization. Graupel or snow particle densities can be extracted from a combination of 2-D precipitating particle probe data and total water content measurements from aircraft. Extensive measurements would test the validity of this assumption and also show whether graupel and snow particle density might be better described as a function of particle size, this could be easily incorporated into the parametrization. (Cotton et al, 1986) had the density of aggregates (snow) varying as a function of diameter. This has now been included as an option in the LEM scheme.

Surface temperature of precipitating particles is assumed to be that of the environment: This assumption is made in the scheme, with the exception of graupel, which can be in wet growth mode at sub-zero temperatures. The surface temperatures of the particles can be evaluated by assuming that the release of heat by freezing or vapour deposition is balanced by the rate of diffusion of heat from the particle. The particles' surface temperature will affect the collection efficiencies of colliding particles. The mean surface temperature of water particles would be a useful quantity to diagnose if the model is running with a radiation scheme, as the long-wave emittance depends on this.

6 Conclusions

To represent the evolution of water in the atmosphere with only a handful of model variables, a number of assumptions must be made about the nature and size spectra of the hydrometeors involved. This approach is widely used and has shown to be adequate for modelling case studies and for predicting the severity of storms, but suffers in that assumptions must be made about the properties of the precipitation before a simulation is attempted. A more versatile and satisfying scheme would not require such assumptions, but rather predict these properties. Detailed parametrizations have been formulated describing the size spectra of each particle type with at least ten model variables, but computer constraints do not allow their use in 3-D simulations. As more aircraft and radar data is made available, together with data gathered specifically for the validation of this model, a comprehensive table showing how the adjustable parameters in the scheme vary with different situations can be compiled. It may then be possible to formulate a parametrization that predicts the adjustable parameters within the scheme.

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Appendix A
List of Symbols

Notation	Description	Value	Units	CODE
a	constant in formula for v_R		$m^{1-b}s^{-1}$	aF
A_B	constant in Bigg's freezing equation	0.66	K^{-1}	
A'	coefficient in diffusion equation for liquid drops		m s kg^{-1}	
A''	coefficient in diffusion equation for ice particles		m s kg^{-1}	
A_{vi}	coefficient in ice crystal fall speed equation	0.04	ms^{-1}	
b	coefficient in formula for v_R	0.8		bF
B_B	constant in Bigg's freezing equation	100	m^{-3} s^{-1}	
B'	coefficient in diffusion equation for liquid drops		m s kg^{-1}	
B''	coefficient in diffusion equation for ice particles		m s kg^{-1}	
c	coefficient in formula for v_S	4.84	m^{1-d} s^{-1}	cF
C_1	constant in approximation of Young's equation	5×10^{-9}	m^2s^{-1}	C1
C_2	constant in approximation of Young's equation	1.5×10^{-2}	m^3s kg^{-1}	C2
C_3	constant in apporoximation of Young's equation	6×10^3	m^2s kg^{-1}	C3
C_D	drag coefficient for graupel particle	0.6		CdF
C_i	specific heat for ice	2093	J kg^{-1} K^{-1}	CICE
C_p	specific heat for air at contant pressure	1005	J kg^{-1} K^{-1}	CP
C_w	specific heat for water	4187	J kg^{-1} K^{-1}	CWATER
d	constant in formula for v_S	0.25		dF
D	diameter of any water particle		m	
D_G	diameter of graupel particle		m	
D_R	diameter of rain drop		m	
D_S	diameter of snow crystal		m	
E_{IW}	aggregation efficiency for colliding ice particles	0.5		EII
E_{IW}	collection efficiency of ice for liquid cloud	1		EIW
E_{GI}	collection efficiency of dry graupel for cloud ice	0.1		EGI_DRY
E_{GR}	collection efficiency of graupel for rain	1		EGR
E_{GS}	collection efficiency of dry graupel for snow			EGS_DRY
E_{GW}	collection efficiency of graupel for liquid cloud	1		EGW
E_{RI}	collection efficiency of rain for ice cloud	1		ERI
E_{SI}	collection efficiency of snow for ice cloud			ESI
E_{SR}	collection efficiency of snow for rain	1		ESR
E_{SW}	collection efficiency of snow for liquid cloud	1		ESW
F'	ventilation factor for snow			
g	gravitational acceleration	9.81	m s^{-2}	G
j	In formula for v_G, $j = ((4g\rho_G)/(3C_D\rho_0))^{0.5}$		$m^{1-k}s^{-1}$	jF
k	coefficient in formula for v_G	0.5		kF
K_a	thermal conductivity of air	0.0243	m^{-1} s^{-1} K^{-1}	THCOND
L_v	latent heat of vaporization	2.5×10^6	J kg^{-1}	RLVAP
L_f	latent heat of fusion	3.336×10^5	J kg^{-1}	RLFUS
L_s	latent heat of sublimation	2.834×10^6	J kg^{-1}	RLSUB
M_I	mass of one cloud ice crystal $= q_I/N_I$	3.84×10^{-12}	kg	
M_{I0}	initial mass of ice crystal	10^{-12}	kg	MI0
n_a	number density of aerosols in Young's eqn.		m^{-3}	
n_{a0}	constant used in the calculation of N_a	20000	m^{-3}	
n_c	number density of cloud droplets		m^{-3}	S
n_f	number of natural active ice nuclei		m^{-3}	SCALNI
n_{f0}	constant in Fletcher curve	0.01	m^{-3}	nI0
N_I	number of ice nuclei per unit mass of air		kg^{-3}	RNI,QN

Appendix A
List of Symbols (cont.)

Notation	Description	Value	Units	CODE
n_0	intercept parameter for particle size spectrum		m^{-4}	
n_{0R}	intercept parameter for rain	$\approx 8\times10^6$	m^{-4}	n0R
n_{0G}	intercept parameter for graupel	$\approx 4\times10^4$	m^{-4}	n0G
n_{0S}	intercept parameter for snow	$\approx 3\times10^6$	m^{-4}	n0S
p	atmospheric pressure		$kgm^{-1}s^{-2}$	PREFN
P_{any}	a conversion rate between water categories		s^{-1}	
P^s_{IACR}	rain accreted by ice cloud to form snow		s^{-1}	PIACR_S
P^g_{IACR}	rain accreted by ice cloud to form graupel		s^{-1}	PIACR_G
P_{IACW}	riming of liquid cloud by ice crystals		s^{-1}	PIACW
P_{ICNT}	contact nucleation of droplets to form ice		s^{-1}	PICNT
P_{IDEP}	deposition of vapour onto ice crystals		s^{-1}	PIDEP
P_{IFLT}	Fletcher nucleation of ice cloud		s^{-1}	PIFLT
P_{IHAL}	nucleation of ice by Hallet and Mossop effect		s^{-1}	PIHAL
P_{IMLT}	melting of ice cloud to liquid cloud		s^{-1}	PIMLT
P_{ISUB}	sublimation of ice cloud to vapour		s^{-1}	PISUB
P^s_{RACI}	ice cloud accreted by rain to form snow		s^{-1}	PRACI_S
P^g_{RACI}	ice cloud accreted by rain to form graupel		s^{-1}	PRACI_G
P_{RACS}	collection of snow by rain to form graupel		s^{-1}	PRACS
P_{RACW}	collection of liquid cloud by rain		s^{-1}	PRACW
P_{RAUT}	autoconversion of liquid cloud to rain		s^{-1}	PRAUT
P_{REVP}	evaporation of rain to vapour		s^{-1}	PREVP
P_{SACI}	collection of ice cloud by snow		s^{-1}	PSACI
P_{SACR}	collection of rain by snow to form graupel		s^{-1}	PSACR
P_{SACW}	collection of liquid cloud by snow		s^{-1}	PSACW
P_{SAUT}	autoconversion from ice cloud to snow		s^{-1}	PSAUT
P_{SDEP}	deposition of water vapour on to snow		s^{-1}	PSDEP
P_{SMLT}	melting of snow to rain		s^{-1}	PSMLT
P_{SSUB}	sublimation of snow to water vapour		s^{-1}	PSSUB
P_{GACI}	collection of ice cloud by graupel		s^{-1}	PGACI
P_{GACR}	collection of rain by graupel		s^{-1}	PGACR
P_{GACS}	collection of snow by graupel		s^{-1}	PGACS
P_{GACW}	collection of liquid cloud by graupel		s^{-1}	PGACW
P_{GAUT}	autoconversion of snow to graupel		s^{-1}	PGAUT
P_{GFR}	freezing of rain to form graupel		s^{-1}	PGFR
P_{GMLT}	melting of graupel to form rain		s^{-1}	PGMLT
P_{GSHD}	liquid water shed by graupel to form rain		s^{-1}	PGSHD
P_{GSUB}	sublimation of graupel to water vapour		s^{-1}	PGSUB
P_{GWET}	rate that graupel can freeze accreted liquid		s^{-1}	PGWET

Appendix A
List of Symbols (cont.)

Notation	Description	Value	Units	CODE
q	mass mixing ratio *wrt* air of any water substance			
q_C	liquid water mixing ratio			QL
q_G	graupel mixing ratio			QG
q_I	ice cloud mixing ratio			QI
q_S	snow mixing ratio			QS
q_T	vapour plus liquid cloud mixing ratio			QT
q_V	water vapour mixing ratio			QV
q_{0C}	threshold for autoconversion of cloud to rain	$\approx$0.001		QR0
q_{sat}	saturation vapour mixing ratio *wrt* water			QWS
q_{isat}	saturation vapour mixing ratio *wrt* ice			QIS
q_{0sat}	saturation vapour mixing ratio *wrt* water at 0^oC			QWS0
R_c	mean cloud droplet radius		m	Rlc
R_w	gas constant for water vapour	461.5	$J\ kg^{-1}\ K^{-1}$	RW
R_e	Reynolds number			
S_c	Schmidt number $[= v/\psi]$			
S_G	source term for graupel		s^{-1}	DQG
S_I	source term for ice cloud		s^{-1}	DQI
S_R	source term for rain		s^{-1}	DQR
S_S	source term for snow		s^{-1}	DQS
S_T	source term for liquid cloud and water vapour		s^{-1}	DQT
T	temperature		K	T
T_0	freezing temperature	273.15	K	
T_c	temperature in degrees Celsius		K	TDEGC
v	mass mean terminal fall velocity		$m\ s^{-1}$	
v_i	fall velocity of an ice crystal		$m\ s^{-1}$	
v_D	fall velocity of a particle of diameter D		$m\ s^{-1}$	
v_G	mass mean fall velocity of graupel		$m\ s^{-1}$	U_G
v_R	mass mean fall velocity of rain		$m\ s^{-1}$	U_R
v_S	mass mean fall velocity of snow		$m\ s^{-1}$	U_S
v_{DG}	fall velocity of a graupel particle of diameter D_G		$m\ s^{-1}$	
v_{DR}	fall velocity of a rain drop of diameter D_R		$m\ s^{-1}$	
v_{DS}	fall velocity of a snow crystal of diameter D_S		$m\ s^{-1}$	
α_C	autoconversion rate coefficient for P_{RAUT}	0.001	s^{-1}	alphaR
β	parameter in Fletcher's equation	0.6	K^{-1}	betaI
Δt	model intergration time step		s	DT
Δt_{SG}	time for snow to reach density of graupel by riming		s	
λ_R	slope parameter for rain size distribution		m^{-1}	
λ_S	slope parameter for snow size distribution		m^{-1}	
λ_G	slope parameter for graupel size distribution		m^{-1}	
ρ	air density		$kg\ m^{-3}$	RHO
ρ_0	surface air density		$kg\ m^{-3}$	rho0
ρ_G	density of a graupel particle	$\approx$500	$kg\ m^{-3}$	rhoG
ρ_S	density of a snow flake	$\approx$100	$kg\ m^{-3}$	rhoS
ρ_W	density of water	1000	$kg\ m^{-3}$	rhoR
v	kinematic viscosity of air	1.44×10^{-5}	$m^2\ s^{-1}$	VISAIR
ψ	diffusivity of water vapour in air	2.26×10^{-5}	$m^2\ s^{-1}$	DIFFWV

The Conversion Rates Evaluated

The Model code that calculates the conversion rates is formulated by a front-end fortran program. This arrangment allows the code to be rewritten for any parameter values. The code here is formulated using the parameter values quoted in this paper. These conversion rates are only calculated for the gridpoints that either contain some water substance, or that are supersaturated. The DO loops and array indices have been removed.

```
.................................................................
 THE FOLLOWING BLOCK OF CODE IS GENERATED BY 'LINC.FOR' ON THE VAX
.................................................................

    RMI0=0.1000E-11

     IF(TDEGCjk.LT.0.)THEN
       FLTFAC=1.!((QVjk-QISjk)/(QWSjk-QISjk))
       IF(TDEGCjk.GT.-25.)THEN
         RNIFLT=0.01*EXP(-0.6*TDEGCjk)*FLTFAC**4.5/RHONk
       ELSE
         RNIFLT=32690.*FLTFAC**4.5/RHONk
       ENDIF
       PIFLT=MAX(0.,0.6000E-12*DTDTjk*RNIFLT)
       RNI=MAX(10.,MIN(QIjk/1.E-12,RNIFLT))
     ELSE
       RNI=10.
     ENDIF
     RNI=MAX(10.,MIN(QIjk/1.E-12,QNjk))

     u_R=0.2299E+02*RHONk**(-0.300)*QRjk** 0.200
     u_S=0.2029E+01*RHONk**(-0.438)*QSjk** 0.063
     IF(IGRAUP.EQ.1)
   & u_G=0.1206E+02*RHONk**(-0.375)*QGjk** 0.125

     ABITEMP=(1.-QVjk/QISjk)
   &  /(0.7162E+12*RHONk*TABjk**(-2)+0.4425E+05/QISjk)
   & PISUB=0.2538E+02*ABITEMP*RHONk*QIjk** 0.500
   &          *RNI** 0.500
     PSSUB=ABITEMP*0.1885E+08*
   & (0.2541E-04*(RHONk*QSjk)** 0.500+0.3056E-03
   & *RHONk** 0.406*QSjk** 0.656)
   & PGSUB=ABITEMP*0.2513E+08*
   & (0.9840E-05*(RHONk*QGjk)**0.5+0.2121E-03*
   & RHONk** 0.438*QGjk** 0.688)

     IF(QVjk.GT.QISjk)THEN
        PSDEP=-PSSUB
        PSSUB=0.
        PIDEP=-PISUB
        PISUB=0.
        PGSUB=0.
     ENDIF

     PGACS=MAX(u_G/4.,ABS(u_G-u_S))/RHONk*
   & (0.7269E+01*(QGjk*RHONk)**0.25*(QSjk*RHONk)** 1.500
   & +0.1810E+01*(QGjk*RHONk)**0.50*(QSjk*RHONk)** 1.250
   & +0.2815E+00*(QGjk*RHONk)**0.75*(QSjk*RHONk)** 1.000)
      PGACR=MAX(u_G/4.,ABS(u_G-u_R))*RHONk**0.75*
   & (0.1056E+01*QRjk**1.5 *QGjk*0.25
   & +0.5972E+00*QRjk**1.25*QGjk*0.5
   & +0.2112E+00*QRjk       *QGjk*0.75)

     GACTEMP=0.2911E+01*RHONk** 0.375*QGjk** 0.875
     PGACI=GACTEMP*QIjk
     PGACW=GACTEMP*QLjk

    EGI_DRYTEMP= 0.100
     EGS_DRYTEMP=0.2*MIN(1.,EXP(0.08*TDEGCjk))
     PGWET= MAX(0.,
   & ((0.3391E+06*(QWS0k-QVjk)-0.1459E+03*TDEGCjk/RHONk)
   & /(0.7968E+02+TDEGCjk))  *
   & (0.9840E-05*(RHONk*QGjk)**0.5+
   & 0.2121E-03*RHONk** 0.438*QGjk** 0.688)  +
   & (PGACI+PGACS)*(1.-0.4999E+00*TDEGCjk
   &                        /(0.7968E+02+TDEGCjk) ) )
     PGDRY=PGACW+PGACR
   &         +EGI_DRYTEMP*PGACI+EGS_DRYTEMP*PGACS
```

```
       PGACI=PGACI*EGI_DRYTEMP
       PGACS=PGACS*EGS_DRYTEMP
      ELSE
       PGACR=MAX(0.,PGWET-PGACW-PGACI-PGACS)
       PGSHD=MAX(0.,-PGWET+PGACW+PGACI+PGACS)
      ENDIF

      RACTEMP=MAX(u_R/4.,ABS(u_R-u_S))/RHONk
      PRACS=RACTEMP*
     & (0.1028E+02*(QRjk*RHONk)**0.25*(QSjk*RHONk)** 1.500
     & +0.1810E+01*(QRjk*RHONk)**0.50*(QSjk*RHONk)** 1.250
     & +0.1991E+00*(QRjk*RHONk)**0.75*(QSjk*RHONk)** 1.000)

      PSACR=RACTEMP*
     & (0.1697E+01*(QRjk*RHONk)**1.50*(QSjk*RHONk)** 0.250
     & +0.1542E+01*(QRjk*RHONk)**1.25*(QSjk*RHONk)** 0.500
     & +0.8761E+00*(QRjk*RHONk)      *(QSjk*RHONk)** 0.750)

      PRACW=0.3614E+01*RHONk** 0.450*QLjk*QRjk** 0.950
      PSACW=0.1641E+01*RHONk** 0.313*QLjk*QSjk** 0.813
      PRAUT=0.1000E-02*MAX(0.,QLjk-0.1000E-02)


       ABLTEMP=1./
     &     (0.5573E+12*RHONk*TABjk**(-2)+0.4425E+05/QWSjk )
      IF(QLjk.LT.1.E-8)THEN

      PREVP=MAX(0.,0.5027E+08*(QWSjk/QVjk-1))*
     & (0.4920E-05*(RHONk*QRjk)**0.5+0.1129E-03*
     & RHONk** 0.475*QRjk** 0.725)
     & *ABLTEMP
      ELSEIF(TABjk.LT.270.15)THEN
          Rlc=(RHONk*QLjk*0.1194E-11)**0.3333333

        SwrtLml=RHONk*(-DTDTjk*DQWSATDT(QWSjk,TABjk)
     &                      -PIDEP-PSDEP)
     &            /(0.2513E+10*Rlc*ABLTEMP)

        PICNT=MAX(0.,0.4000E+02*Rlc*(270.16-TABjk)**1.3
     &    *((5.E-9)-((1.5E-2)+(6.E3)*Rlc)*SwrtLml/PREFNk)) /RHONk
      ENDIF


      IF(TDEGCjk.LT.0.)THEN
FREEZING

        PIHAL=0.3500E-03*(PGACW+PSACW)
     &       *MAX(0.,(1.-ABS(TDEGCjk+5.)/2.5))
        AICE=0.1281E+02*(QIjk/RNI)** 1.000
     &                    *RHONk**0.5*RNI

       PIACW=0.2000E-01*AICE*QLjk

       PIFRW=MAX(0.,(-38.-TDEGCjk)*CP_LFREEZDT)

       PSAUT=0.1000E-02*AICE*QIjk
       PSACI=EXP(0.025*TDEGCjk)
     & *0.1641E+01*RHONk** 0.313*QIjk*
     & QSjk** 0.813

       PRACI=0.3614E+01*RHONk** 0.450*QIjk*QRjk** 0.950

       PIACR=0.3171E-02*RHONk** 1.200*RNI*QRjk** 1.700
       IF(QRjk.GT.0.0001)THEN
        PRACI_G=PRACI
        PIACR_G=PIACR
       ELSE
        PRACI_S=PRACI
        PIACR_S=PIACR
       ENDIF

       PGAUT=0.2500E+00*PSACW

       PGFR=0.9954E-04*
     & (EXP(-0.660*TDEGCjk)-1.)*RHONk** 1.250
     & *QRjk** 1.750
WARM
      ELSE
       PIMLT=TDEGCjk*CP_LFREEZDT

       PSMLT=MAX(0.,
     & (0.1373E+01*TDEGCjk/RHONk-0.3192E+04*(QVjk-QWS0k))*
     & (0.2541E-04*(RHONk*QSjk)** 0.500+0.3056E-03
     & *RHONk** 0.406*QSjk** 0.656)
     & +0.1255E-01*TDEGCjk*(PSACW+PSACR) )
       PGMLT=MAX(0.,
     & (0.1831E+01*TDEGCjk/RHONk-0.4257E+04*(QVjk-QWS0k))*
     & (0.9840E-05*(RHONk*QGjk)**0.5+0.2121E-03*
     & RHONk** 0.438*QGjk** 0.688)
     & +0.1255E-01*TDEGCjk*(PGACW+PGACR-PGSHD) )

      ENDIF

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