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Semigeostrophic Moist Frontogenesis in a Lagrangian model

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Abstract

A geometric technique for solving the Lagrangian conservation form of the semigeostrophic equations is extended to study moist frontogenesis . Model elements are required to conserve θ_E and the frontogenesis is forced by a deformation flow . The boundary layer elements ahead of the surface front are potentially unstable but initially unsaturated . As they are forced to ascend at the surface front these elements saturate and appear at a new equilibrium position after implicitly releasing latent heat . This leads to the formation of a 'lens' of moist air at mid-levels some two hundred km. behind the surface front . In terms of potential vorticity a positive anomaly has been created in the region where the elements saturate and a corresponding negative anomaly is created at the site of the lens . The inclusion of evaporational cooling using a simple model is found to have a significant effect on the overall evolution of the moist front .

Introduction.

There has been in recent years a concerted effort to determine the effect of latent heating on the evolution of a cold front , and to develop conceptual models to understand the dynamics of observed features of the circulation of a mature cold front . In particular the concept of line convection at the surface front and the associated warm conveyor belt has been described by Browning and Harrold (Browning and Harrold (1970), Harrold (1973)); and the presence of rainbands ahead and behind the surface cold front may be described by the theory of conditional symmetric instability (CSI) , (Bennetts & Hoskins 1979). More recently dynamical understanding has been based on the concept of potential vorticity , described by Hoskins et al (1985) , and moist potential vorticity calculated using equivalent potential temperature. Observations suggest that in regions of significant latent heat release at cold fronts the moist PV is near zero ; that is the atmosphere is neutrally stable to saturated disturbances , (Emanuel 1985) .

The work presented here uses a Lagrangian semigeostrophic model to describe possible balanced states arising in a frontal zone after moist slantwise convection has taken place , on a timescale of order f^{-1} .

Previous numerical simulation of moist frontogenesis has concentrated on the use of two-dimensional finite-difference models , both primitive equation and semigeostrophic , representing the moist processes with varying degrees of sophistication. (e.g. Hsie , Anthes and Keyser 1984 ; Thorpe and Emanuel 1985) In such models motions on scales smaller than the grid size must be parametrised , and if this is not done correctly the model may produce spurious circulations on the grid scale which must then be filtered out by 'diffusion'. The Lagrangian model used here constructs a sequence of states in thermal wind balance, without requiring explicit 'diffusion' . Thus the only energy lost from the system as the flow evolves is the energy of the unbalanced motions , which are not permitted by the semigeostrophic approximation .

Because the model used in this study imposes thermal wind balance the transient details of CSI cannot be described , but since the model is Lagrangian the air parcels are able to move directly to their equilibrium position without the model having to resolve details of the geostrophic adjustment . Thus the model can achieve directly the balanced state arising as a result of moist slantwise convection . This is important , because the transient flow as the air moves up to its equilibrium position may take the form of a thin 'plume' of rapidly ascending air , or may take the form of the roll circulations of CSI ,and such flows are poorly resolved even by high resolution 'mesoscale' forecast models .The interaction with other motions on the grid scale may totally destroy this part of the circulation in a finite difference model. The finite difference model including condensation heating which is most closely comparable with this study is that described by Thorpe and Emanuel (1985) . There the observed moist neutral state referred to earlier is explicitly included .

This paper describes the results of simulations of frontogenesis forced by deformation in a dry atmosphere ; in an atmosphere with a potentially slantwise-unstable boundary layer , and finally with a simple inclusion of the effects of evaporational cooling .

1 The Model

The geometric method of solution to the Lagrangian conservation form of the semigeostrophic equations was developed to study two-dimensional frontogenesis ; the technique grew out of a generalisation of a method of solution for zero potential vorticity (Cullen 1983). The mathematical theory behind the model was outlined by Cullen and Purser (1984) . The 2-D version of the Geometric Model constructs a sequence of cross-sections in the x-z plane , each of which is in thermal wind balance . After each step the prescribed forcing may be applied , and the new balanced solution calculated . The model does not describe the transient flow occurring between the two balanced states , however the change in position of fluid elements during the step defines the effect of the ageostrophic circulation averaged over the timestep .

A dry version of the model has been used to study the large-scale evolution of a sea breeze , and to study balanced flow over topography (Shutts et al 1988 b). An axisymmetric version has been developed and used to simulate the formation of an 'eye-wall' front by penetrative convection in a vortex (Shutts et al 1988a) . The underlying theory has been further developed by Chynoweth , (1987) , who demonstrated a link with catastrophe theory.

The two-dimensional Geometric Model works by discretising the given initial state into elements , each with a constant potential temperature θ , constant absolute momentum M and a specified area . The elements are then arranged to produce a convectively and inertially stable solution in thermal wind balance , such that the interfaces between elements satisfy Margules' relation and the elements fill the specified domain.

The requirement for convective and inertial stability is that potential temperature must increase upwards , and absolute momentum must increase in the direction of x increasing . This requirement may be succinctly expressed in terms of the modified geopotential P , which is defined by :

$$P = \phi + \tfrac{1}{2} f^2 x^2 \qquad (1)$$

where ϕ is the ordinary geopotential. We then have the relations

$$\partial P/\partial x = M f \qquad \text{(cross front geostrophy condition)} \qquad (2)$$

$$\partial P/\partial z = g\ \theta/\theta_0 \qquad \text{(hydrostatic)} \qquad (3)$$

The condition for convective and inertial stability - that the potential vorticity be positive - is equivalent to requiring the surface P(x,z) to be convex when viewed from below . Because the Geometric Model uses piecewise constant data for M and θ , the Geometric approximation to the surface P(x,z) will be a piecewise planar bowl .The construction of this forms the basis of the Geometric Model. Details of how this is achieved are given by Chynoweth (1987).

It has been shown (Cullen and Purser 1984) that given a set of values for M , θ and area there is a unique arrangement of elements. However in the moist case , in which elements conserve equivalent potential temperature (θ_E) , unique solutions need not exist , and the evolution must be solved as an initial value problem. This is because we may have a case of potential instability - the unstable air may be either in its low level unsaturated position , or may be in its equilibrium position having realised its available potential energy. The method used by Cullen and Purser (1984) to construct a unique solution cannot be applied here ; we must also specify whether the element is initially at the lower or upper position . In the moist model we specify the initial element positions and

θ_E , and then step forward in time . At each time we are guaranteed a unique solution by the theorems of Cullen and Purser (1984) , however if an element saturates its value of θ may change from one step to the next. The way this is calculated in accordance with the moist pseudo-adiabatic process is described later.

1.2 Potential Vorticity and the Geometric Model diagnostics.

Potential Vorticity (PV) in two dimensions is given by the expression :

$$PV = \partial M/\partial x \; \partial\theta/\partial z - \partial M/\partial z \; \partial\theta/\partial x = J\frac{(M, \theta)}{(x, z)} \qquad (4)$$

for constant density $\rho = 1 \text{ kg m}^{-3}$.

Computation of PV in the Geometric Model is not straightforward ,and as described above another important quantity in the model is the modified geopotential P . Nevertheless it is instructive to obtain information about the evolution of PV in the model . According to eq. (4) , as the model uses piecewise constant data , the PV is zero on each element and is concentrated into 'delta-function' points at the element intersections . Clearly this interpretation is of little practical use. Another approach is to consider the Jacobian expression of eq. (4) which in discrete form says that the PV is given by the ratio of the area associated with the element in data space (M-θ) to the area occupied in physical space . This can be useful in interpreting element pictures by eye , but in practice quantitative determination of the 'area' covered by an element in M-θ space is difficult .

For practical purposes it is necessary to return to the modified geopotential P . As described above the Geometric Model constructs a piecewise planar convex bowl by fitting tangent planes to the modified geopotential P(x,z) associated with the data. Values of P(x,z) may be read off from this construction onto a regular grid in x-z . (note that elements are irregularly placed in physical space) . Because we require information about the second derivatives of P, at each point on the regular grid a parabola is fitted to the P values at neighbouring points by a simple least squares method and the values of the first and second derivatives read off. This allows calculation of PV , M , θ , ζ ($=\partial M/\partial x$), etc. on a regular grid which can then be contoured using standard routines . However care must be taken in matching the regular grid to element size :- too large and information is lost ; too small and all the neighbours may fit in the same element , causing a bad fit for the parabola . It must also be remembered that the process inevitably smooths the data and if we really wish to know what is happening in the Geometric Model , for example at a discontinuity , we must return to the actual P-surface calculated by the model . The contoured cross sections presented in §2 have been produced in the manner described above.

1.3 The Moist Algorithm.

Calculation of latent heat release in the model is based on the conservation of equivalent potential temperature θ_E averaged over an element , and assumes that the element rises to its equilibrium position without mixing with the environment . The model specifies a value of θ and saturation height initially for each element , and from these θ_E is calculated using the formula of Bolton(1980):

$$\theta_E = \theta \exp(2675\ q/T_{SL}) \qquad (5)$$

where q is the humidity mixing ratio in Kg/Kg and T_{SL} is the temperature at the saturation level . Pressure in the model is related to the pseudoheight coordinate introduced by Hoskins and Bretherton , which is in their notation :

$$z = [1-(p/p_o)^{\kappa}]\ \gamma H_s/(\gamma-1) \quad (6)$$

Calculation of the saturated humidity mixing ratio q_s is carried out using

$$q_s = 0.622\ e_s/p \quad (7)$$

and a look-up table for $e_s(T)$.

Pressure and temperature are calculated at the element centroid heights , and an element is assumed to be saturated when its centroid is at or above the specified saturation level. The value of θ_E calculated for each element is held constant throughout the run , and when an element saturates equation (5) is solved iteratively for θ . For potentially unstable data the equilibrium height of the parcel is found by lifting the saturated parcel in turn to the elements above it along a surface of constant M , and comparing parcel θ with environment θ at that height . The parcel is placed in the layer below the level of negative buoyancy and is then allowed to adjust θ and z iteratively until eq. (5) is satisfied . In this way the compensating subsidence in the surrounding environment is automatically accounted for by the displacement of neighbouring parcels . The saturation height is then reset to be the current centroid height . Any subsequent descent is at constant θ.

1.4 Deformation Forcing

Frontogenesis in the model is forced by a barotropic deformation flow given by

$$u_G = -\ \alpha_D\ x$$

$$v_G = \ \alpha_D\ y$$

Any preexisting temperature gradient becomes concentrated along the y-axis

, and for two dimensional studies we consider the plane $y = 0$.

Hoskins and Bretherton (1972) showed using semigeostrophic theory that such flow is able to produce a discontinuity in a finite time.

For this flow the equations become :

$$dM/dt = -\alpha_D M$$

$$d\theta/dt = 0$$

and the continuity equation simplifies to :

$$dA/dt = - \alpha_D A$$

where A is the area of an element.

Thus we have the solution

$$M = M_o \exp(-\alpha_D t)$$

$$A = A_o \exp(-\alpha_D t)$$

and the values of M , θ and area are known for all time in terms of the initial values . For the dry case this allows the solution at any time to be determined immediately , but because the moist case must be treated as an initial value problem , the deformation forcing is applied progressively and the solution is stepped forward in time.

A deformation rate α_D of 2×10^{-5} s^{-1} is used with a timestep of 900 s .

2 Results

Results are presented first for the case of dry frontogenesis in the Geometric Model , then secondly for moist frontogenesis with data potentially unstable to moist slantwise ascent and finally the effect of including a simple model of evaporational cooling is described .

2.1 Dry Frontogenesis

The initial conditions used in the experiments are shown in figures 1a-d . The surface potential temperature profile is given by a tanh function , and an isentropic layer is shaded in figure 1.a . At the lateral boundaries $\partial M/\partial x \approx f$, which gives rise to a weak horizontal gradient of potential vorticity across the domain (0.2 PV units / 3000 km). The initial domain is from z = 0 to z = 8 km , and from x = -1000 to x = 2000 km .

In the absence of diabatic forcing , elements retain their initial values of θ and frontogenesis takes place by the collision of isentropes as the slopes of element interfaces are changed by deformation . In accordance with theory the front forms first at the surface and then extends into the domain . Cullen and Purser (1984) describe how this generates a line source of PV . Figure 2a shows the intrusion into the domain of a tongue of higher potential vorticity at the front , associated with the smoothing of the crease in the geopotential surface when the PV is calculated as described in §1.2 . In the Geometric Model , PV is conserved following an element , and no fluid crosses the frontal discontinuity so the line source of PV does not affect the flow .The presence of such a source of PV as a front forms in a finite difference model could feed back into the model dynamics ; the possible implications of this have not yet been fully explored . The question of conservation of PV in the presence of a true discontinuity is important , but is not central to the study of this paper. This is discussed further by Holt and Shutts (1988) .

2.2 Moist Frontogenesis

In the moist example the surface layer ahead of the cold front is given a saturation level corresponding to a relative humidity of ~ 90% . The prefrontal boundary layer is initially unsaturated but potentially unstable to moist slantwise displacement . As the frontogenesis proceeds this air is lifted at the surface front and becomes saturated . On saturation the air parcel is lifted directly to its' equilibrium level , conserving M and θ_E . The corresponding element positions are shown in figure 3a , the moist elements are shaded . Because the model imposes thermal wind balance it cannot describe the transient details of any possible CSI in the ascending air . The model shows the equilibrium position after such motions have occurred , and the ascending parcel must in reality be assumed to be connected to its surface position by a narrow plume of rapidly ascending moist air . This procedure implicitly assumes a separation of timescales, between the time taken in moist ascent of an air parcel and the timescale of the deformation forcing.

In terms of potential vorticity the presence of moist slantwise convection gives rise to a much stronger surface front , Fig 3b , primarily due to the extra convergence causing an increase in absolute vorticity there, with a lens-like region of lower PV aloft at the equilibrium position of the moist air . As we see in figure 3b this corresponds to a negative PV anomaly of only ~ 0.2 units relative to the background atmosphere . Because of the stratification of the background atmosphere the PV anomaly is broad and shallow , and so appears more in terms of a reduction in N^2 in the region ,rather than a reduction in absolute vorticity . However there is a small reduction in absolute vorticity in the region of the 'lens' - Fig 3c . In the moist Geometric Model the redistribution of PV takes place by the removal of mass from the layer in which the element saturates , and the dilution of the isentropic layer at the equilibrium level of the element , in the sense described by Haynes and McIntyre (1987). This may be quantified directly in terms of the Jacobian expression of eq(4).

As there is no explicit diffusion in the Geometric Model , there is no mixing of PV of the sort found in finite-difference models . The displacement of the air surrounding the moist 'lens' takes place on the scale of the Rossby radius based on the height scale of the convection , whereas in a primitive equation model the geostrophic adjustment process is carried out explicitly by radiation of gravity waves. The final state relies on these waves being adequately resolved by the grid .

2.3 A Simple Inclusion of Evaporative Cooling

Preliminary results of observations in the FRONTS87 experiment have emphasised that the effects of evaporational cooling could be important in frontal circulations , as described earlier by (e.g.) Carbone (1982) . A simple model of evaporational cooling was therefore included in the moist Geometric Model to gain a qualitative understanding of the larger scale effects of such cooling . Briefly , the elements in figure 3a through which 'precipitation' fell were identified , and cooled . The immediate effect of this was to increase the convergence at the surface front so that more moist air ascended . The elements into which this extra precipitation would fall were then cooled in a similar manner in subsequent runs ; the final result is shown in figure 4a . The heavy shading denotes moist elements as in figure 3a , while the vertical hatching denotes those elements which have been cooled by 'precipitation'. A maximum cooling rate of 10K in 20 hours was used , in accordance with typical observed values . The cooling was applied progressively as the boundary layer elements saturated , so the applied cooling was nonuniform in both space and time . No adjustment was made to the relative humidity of the cooled parcels , which were undergoing unsaturated descent .

The main impact of the cooling may be seen in the PV , figure 4b ; the surface front has advanced about 100 km relative to figure 3b , corresponding to a mean increase in speed of the order of 5 km/hr . Because of the enhanced convergence at the surface front there has been more moist ascent , so the region of low PV anomaly is extended forward relative to its position in figure 3b . The cooling has resulted in a high PV anomaly below the frontal surface. The θ cross-section , Fig 4c , shows that in fact the applied cooling was not sufficient to produce a substantial cold dome at the surface , but (figure 4d) the effect of the slumping of cold air at constant M is to push lower values of M further ahead , increasing vorticity at the surface front , and to push relatively higher values of M to the left , producing a region of weaker vorticity in the vicinity of the cold dome .

Summary

A semigeostrophic Lagrangian model of moist frontogenesis has been described . The model has reproduced the main features of the observed mesoscale circulation of a mature cold front . Inclusion of a very simple parameterization of evaporational cooling has resulted in a marked change to the potential vorticity distribution , on the scale of the entire width of the cold front . While the basic sense of circulation induced by this cooling agrees with preliminary results of observations from the FRONTS87 project , nevertheless there are as yet few observations of potential vorticity on this scale for comparison. Such a comparison awaits further processing of the FRONTS87 cases .

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figure 1 Initial conditions

a) Element picture

domain $-1000km \leqslant x \leqslant 2000km$ $\qquad 0 \leqslant z \leqslant 8km$

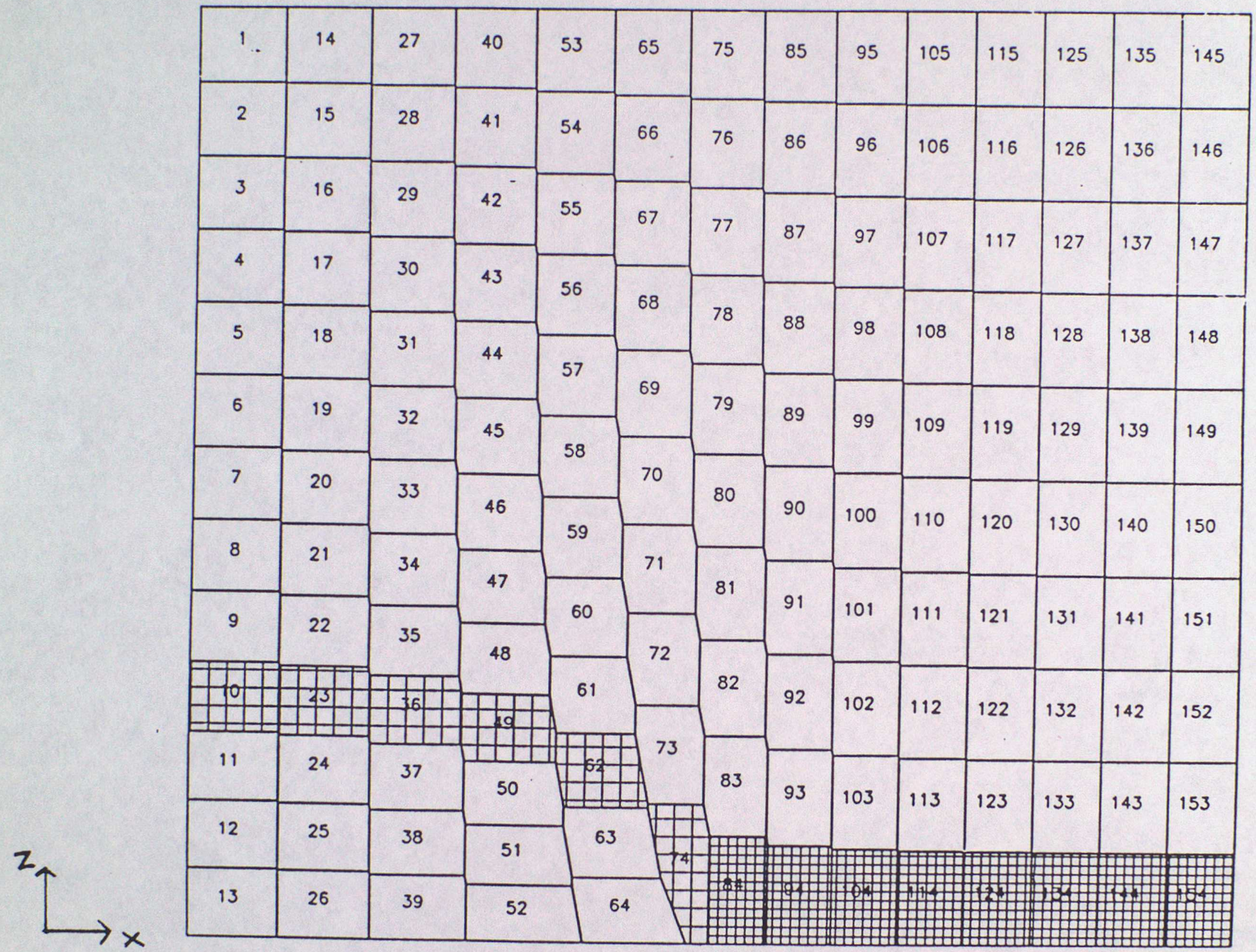


figure 1 Initial conditions b) θ contour interval 2K

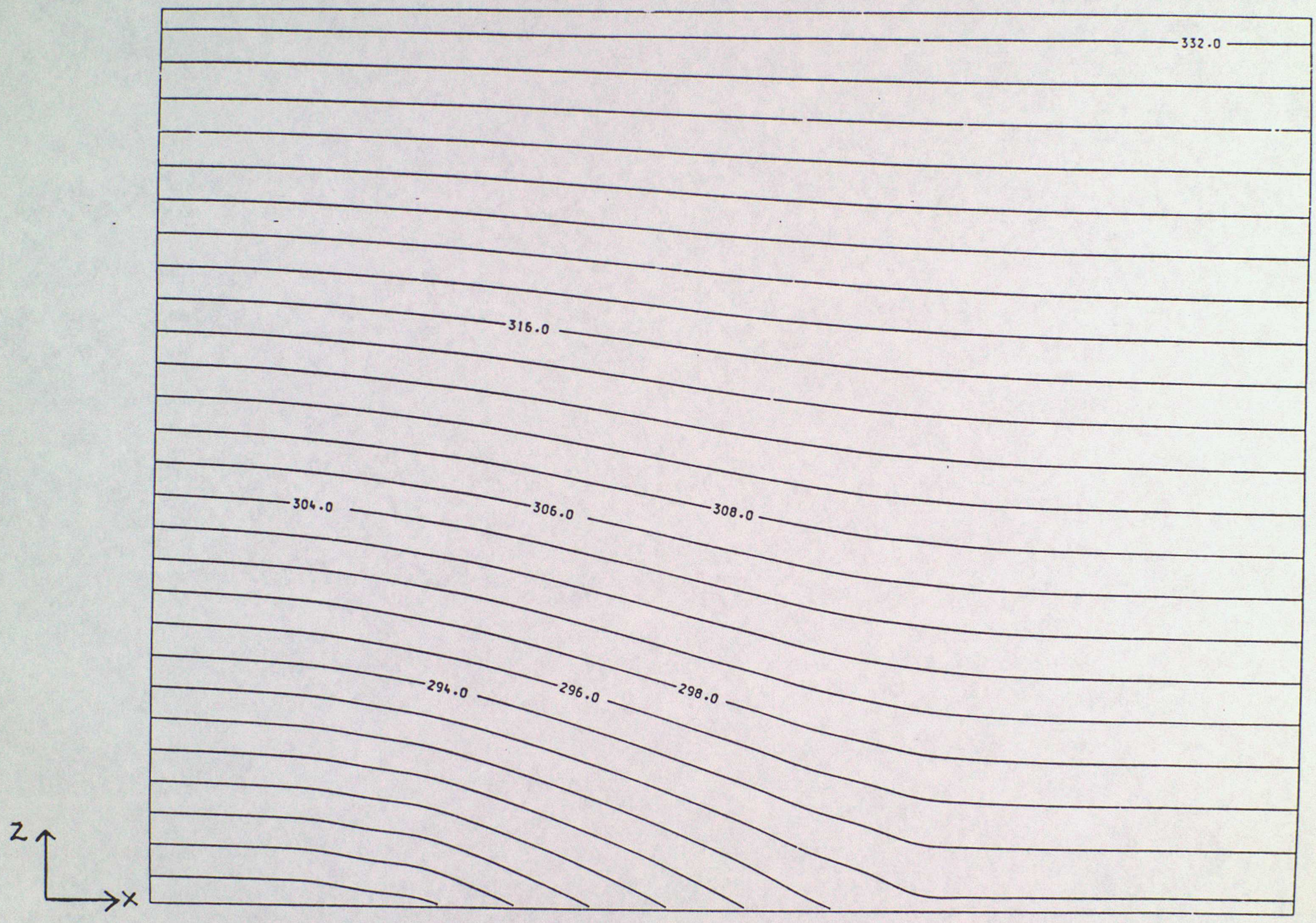


figure 1 Initial conditions c) Potential vorticity contour interval 0.1 PV units

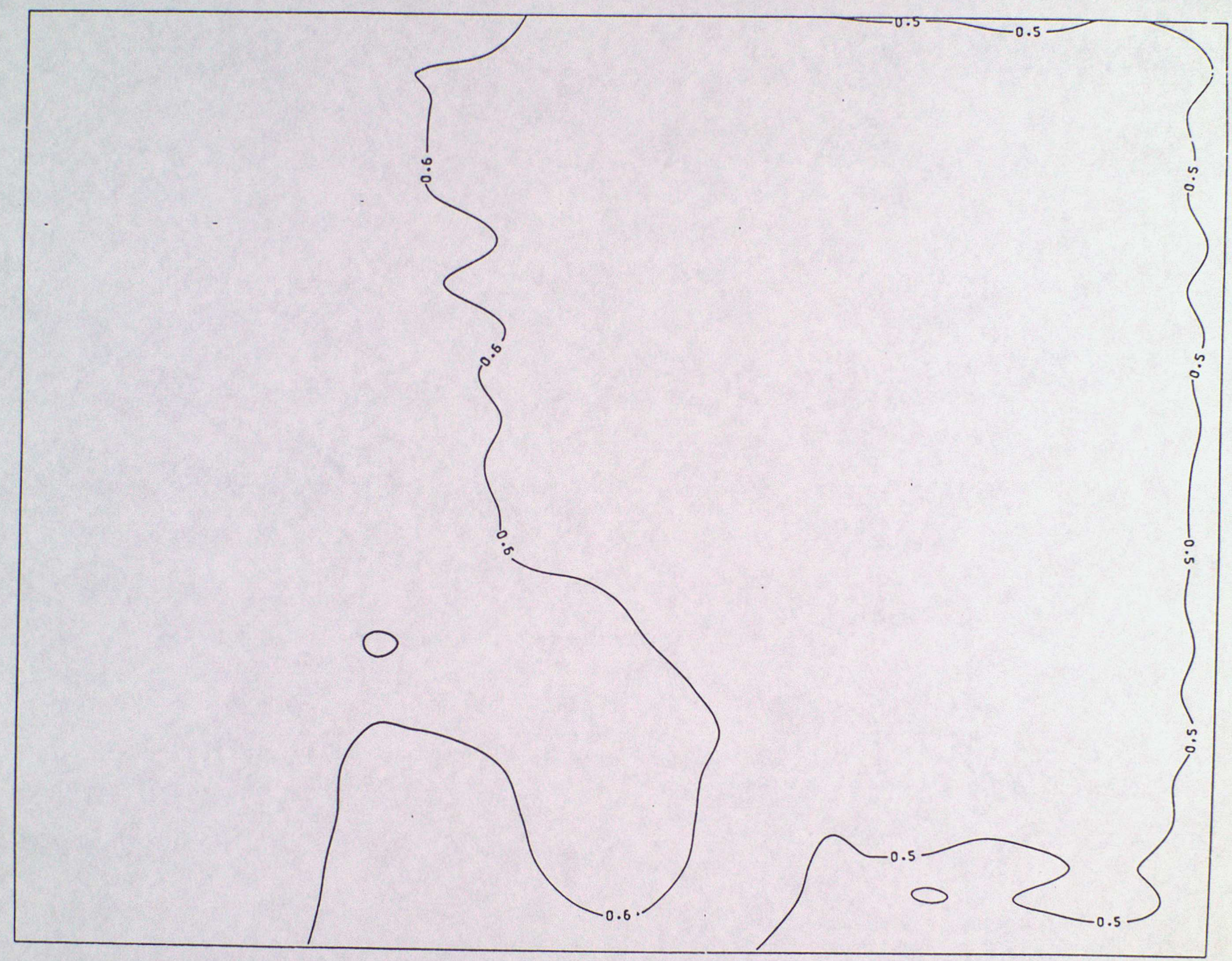


figure 2 Dry front after 20 hrs deformation a) PV $-237km \leqslant x \leqslant 474km$

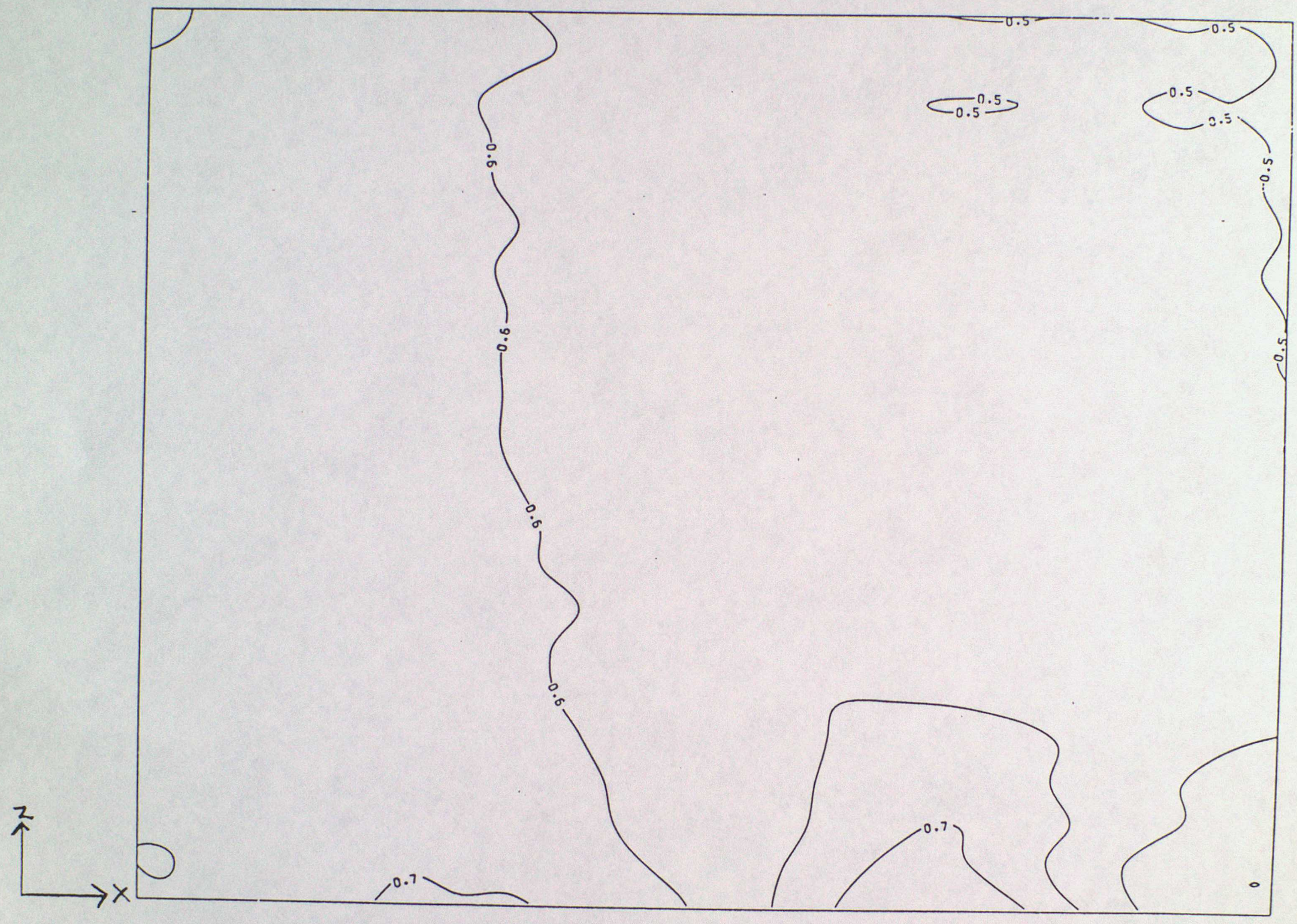


figure 2 b) Absolute vorticity contour interval $0.1 * 10^{-4}\ s^{-1}$

Dry front

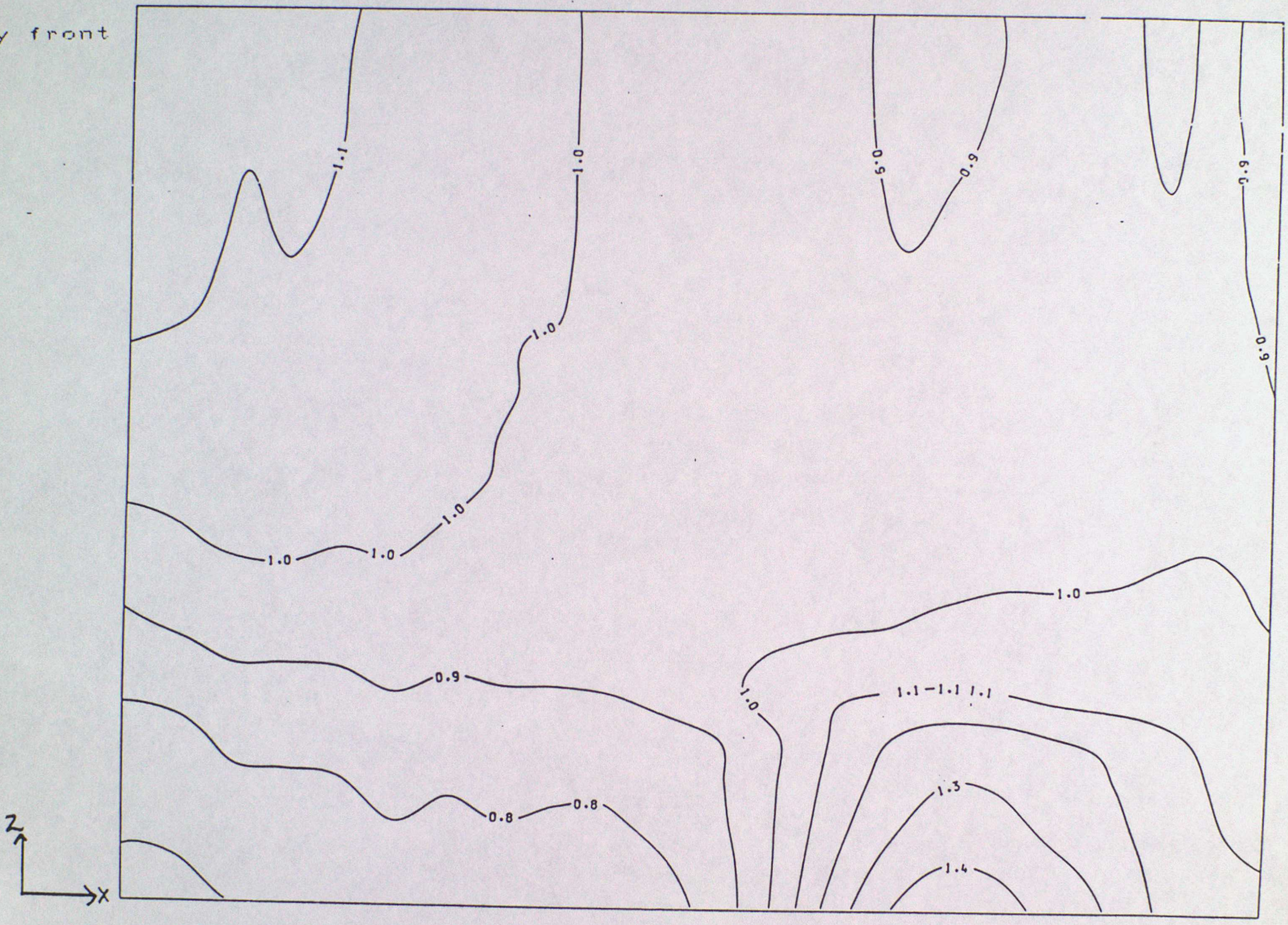


figure 3 Moist front   a) Element picture

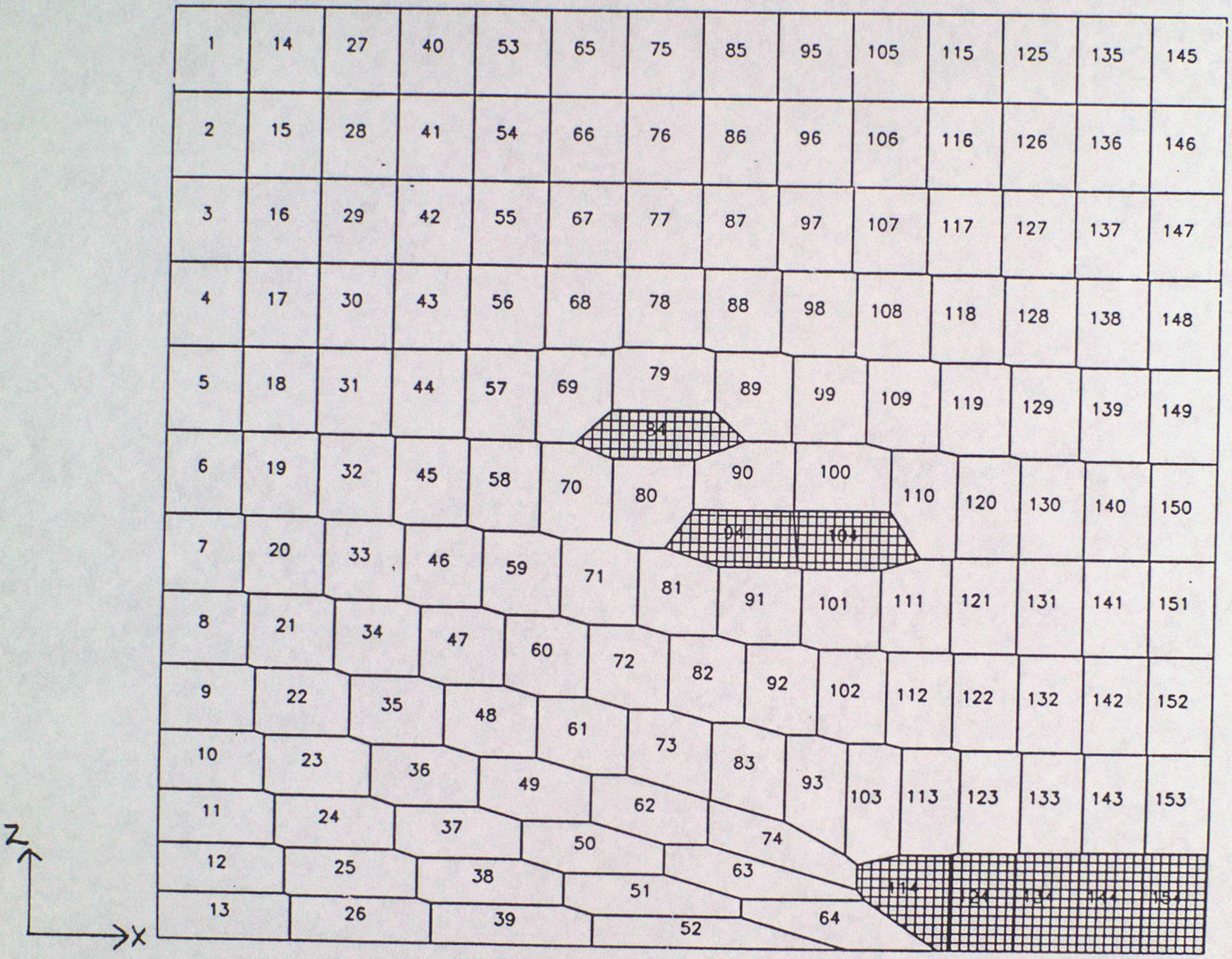


figure 3 Moist front b) PV

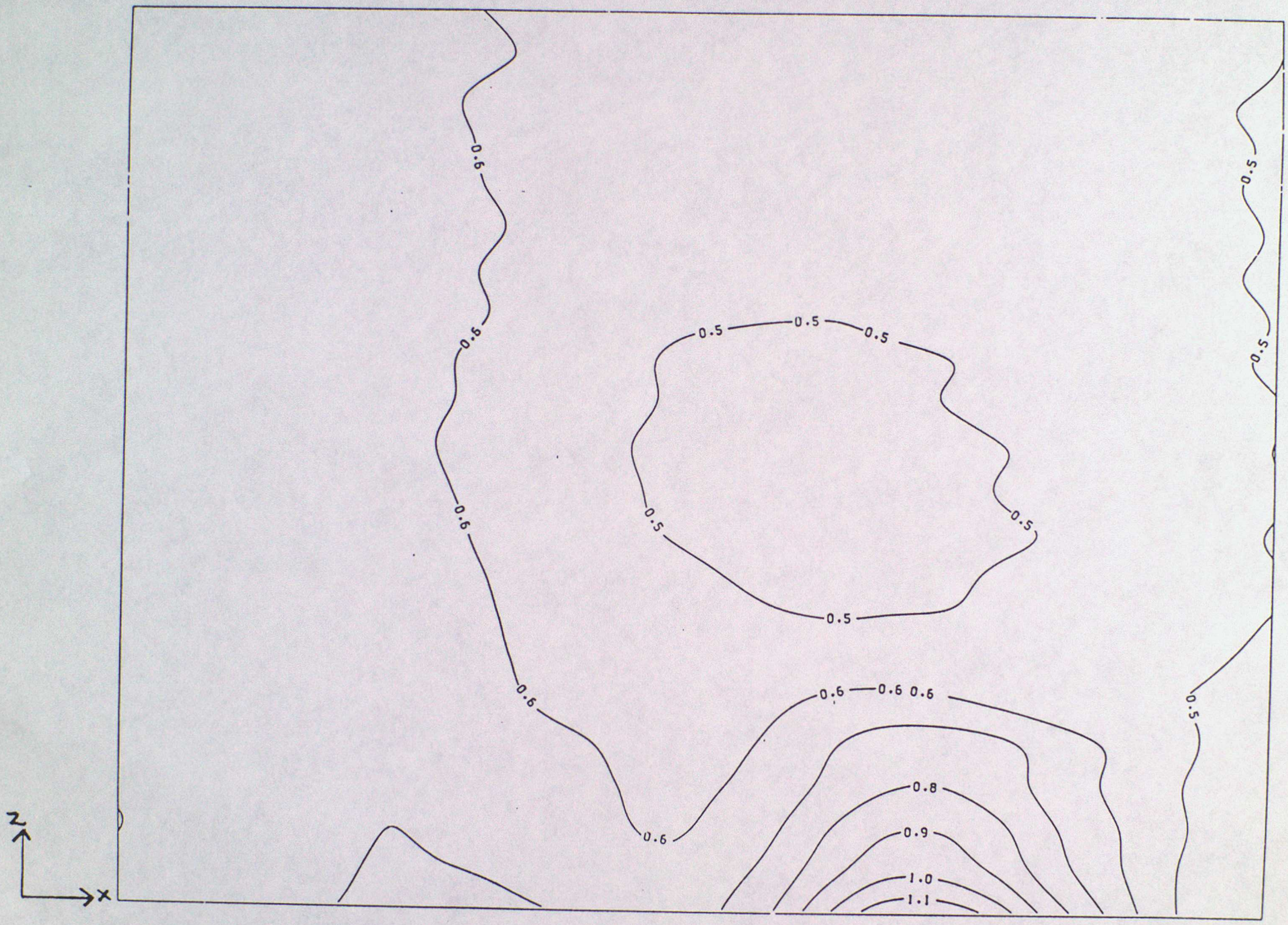


figure 3 Moist front c) Absolute vorticity

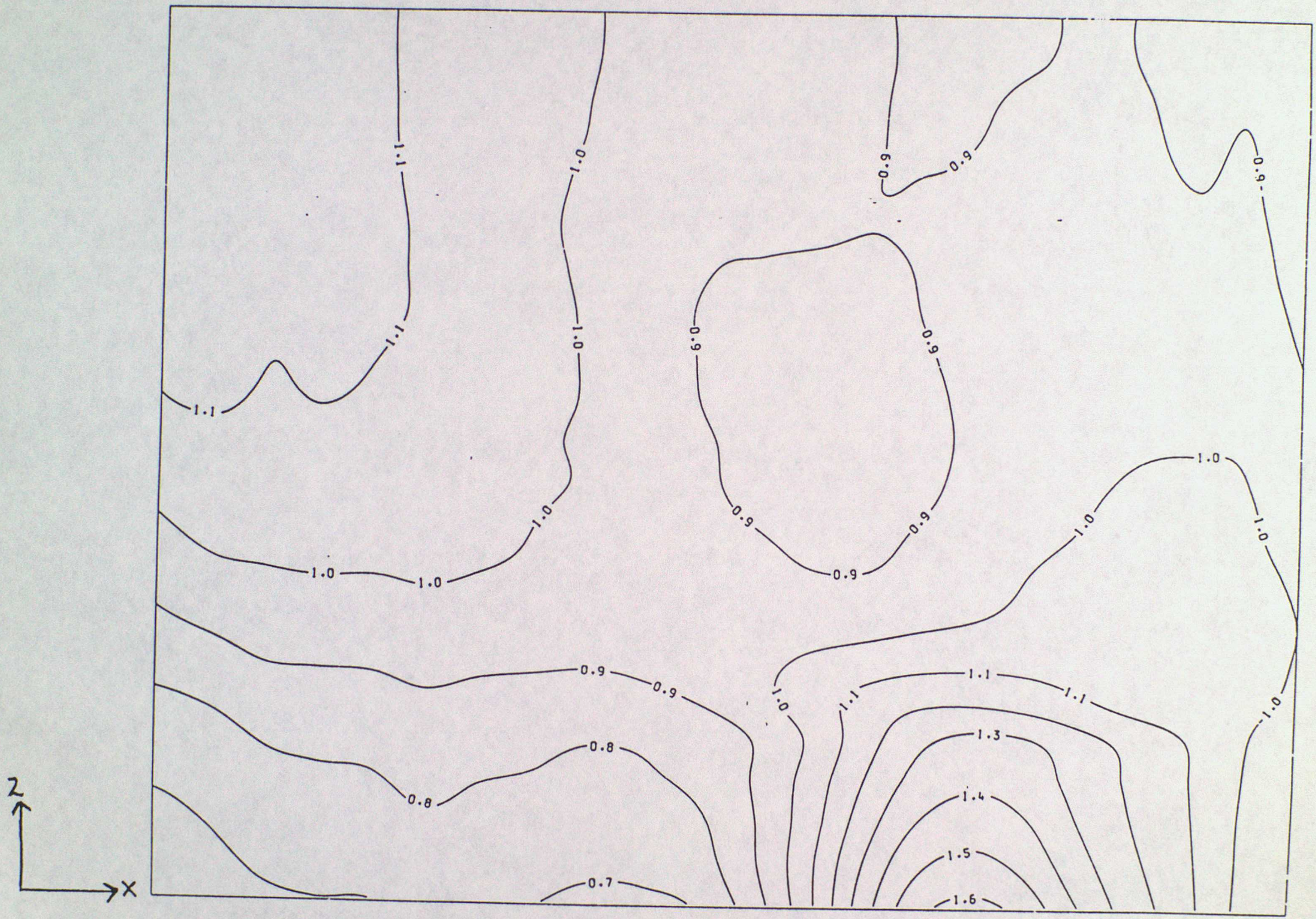


figure 4 Moist front with evaporational cooling

a) Element picture

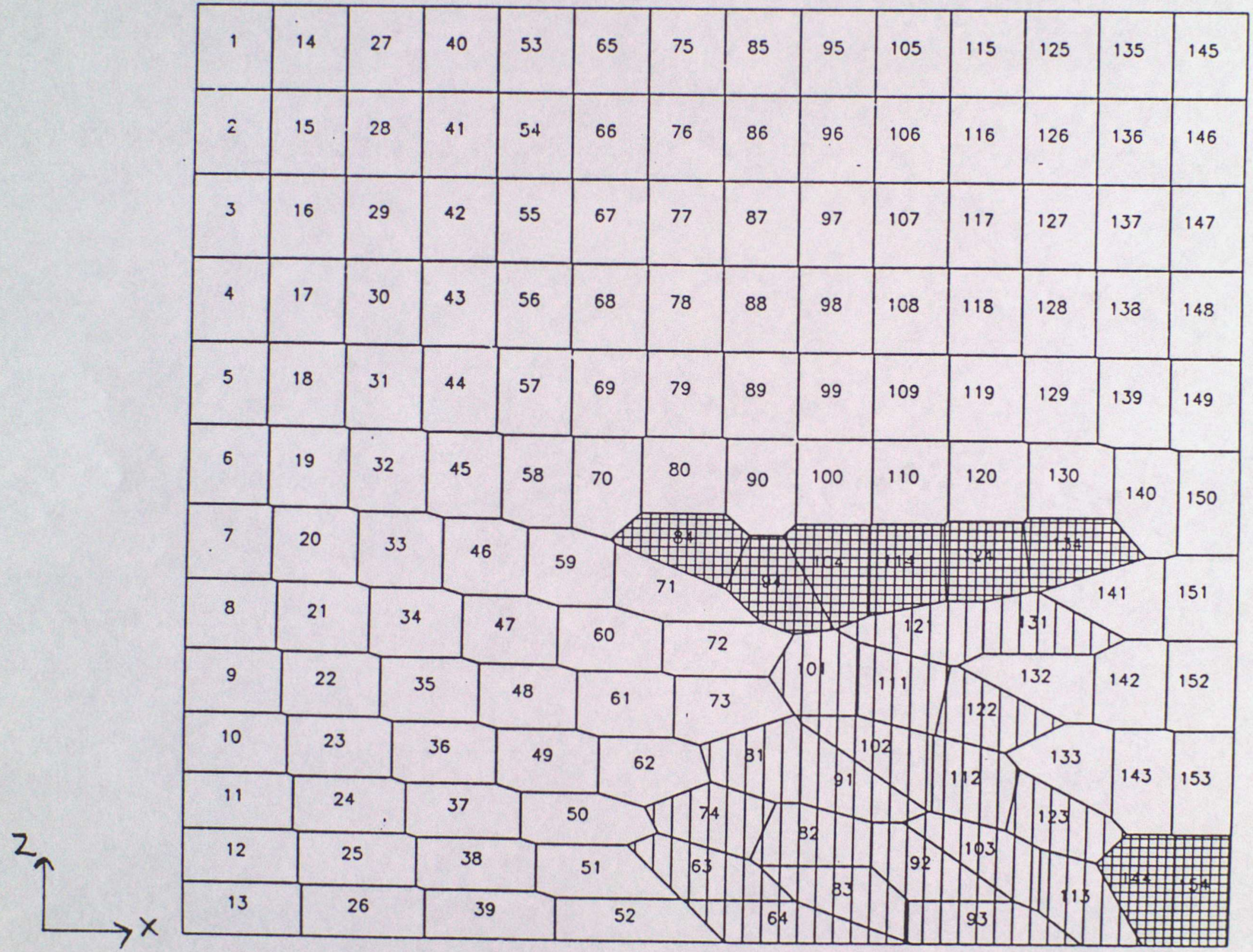


Figure 4 b) PV

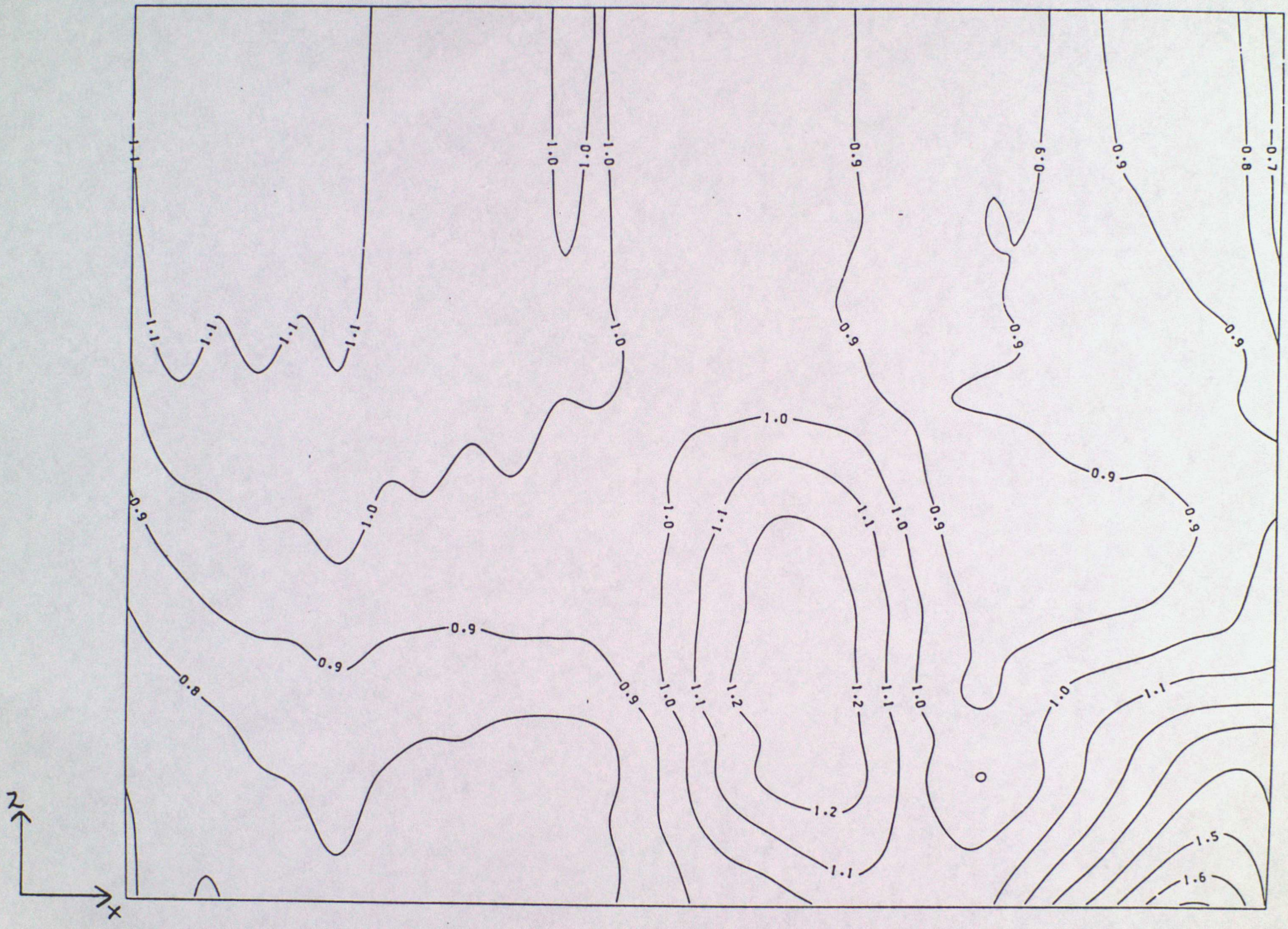
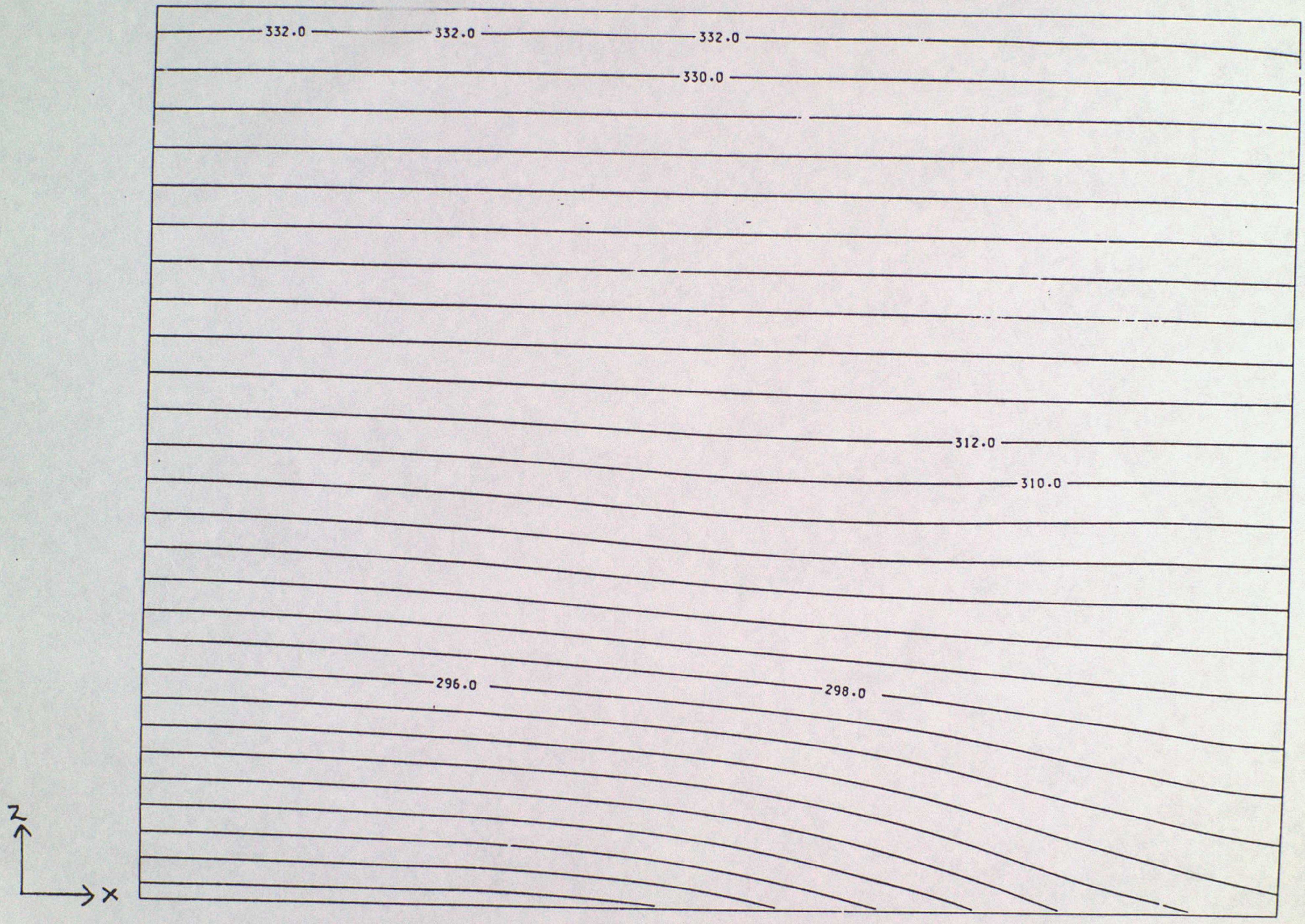


figure 4

c) contoured θ



d) Absolute Momentum   Dotted lines show contours without evaporational cooling.