Meteorology Research and Development

Formulation of a balanced Exner pressure in the Lateral Boundary Conditions of the Unified Model

Technical Report No. 514

Peter Lean and Clive Wilson

email:nwp_publications@metoffice.gov.uk

Formulation of a balanced Exner pressure in the Lateral Boundary Conditions of the Unified Model

Peter Lean and Clive Wilson

Document Change History

First draft, Peter Lean	22/06/2007
Comments from Clive Wilson	04/03/2008
Approved by Stuart Bell	05/03/2008

Abstract

A new formulation of Exner pressure and density in the lateral boundaries conditions of limited area configurations of the Unified Model is presented. This new formulation is designed for use when the vertical grid of the limited area model is different to that in the model used to provide those boundary conditions. In these situations, the dynamically balanced state in the driving model can become disrupted by vertical interpolation onto the new vertical grid. The subsequent vertical accelerations in the lateral boundary regions resulting from this imbalance can lead to high dynamics solver iteration counts (increasing the run time of the forecasts) and, in some circumstances, a bias in the mean sea level pressure.

These problems were first noted in the operational NAE model after the introduction of a new vertical grid in the global model. This report summarises the idealised experiments conducted to identify the cause of these problems, the use of imposed vertical balance as a solution and finally the implementation of this solution in the operational limited area forecast models.

1 Background

Since the implementation of the non-hydrostatic New Dynamical core of the Unified Model in 2002 (Davies *et al.*, 2005), all operational forecast models at the Met Office have used the same 38 level vertical grid. This situation changed when in November 2005 the Global model increased to 50 levels which provided increased resolution (particularly in the stratosphere) and the model top was raised to allow assimilation of more satellite radiance observations. This change in setup meant that the global model had to perform vertical interpolation in order to provide lateral boundary conditions on the lower resolution vertical grid in the limited area models (LAMs).

Problems were immediately noticed when the run-time of the North Atlantic and European (NAE) model increased by around 25%. This run-time increase was found to result from an increased iteration count in the dynamics Helmholtz solver. The problem was temporarily resolved by imposing hydrostatic balance in the Exner pressure and resetting density using the equation of state. While this helped to reduce the iteration count to within operationally acceptable levels, an undesirable side effect was the introduction of a 2hPa negative bias in the pressure at mean sea level (PMSL), as shown in Figure 1.

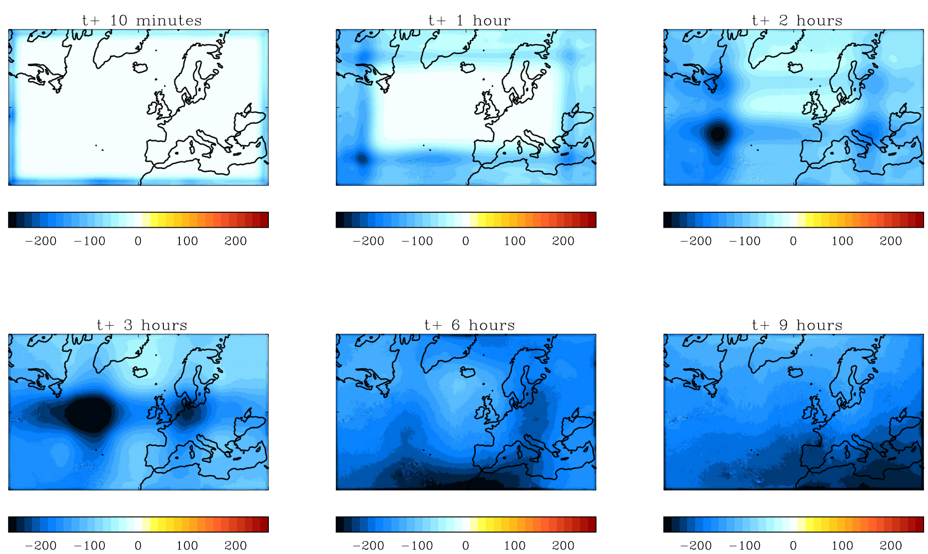


Figure 1: *Example of onset of the PMSL bias in the NAE when imposing hydrostatic balance in the boundaries. Difference in PMSL [Pa] (NAE - global model) is shown during the first few hours of the forecast.*

Several years earlier, the development of the High Resolution Trial Model (HRTM) required a vertical resolution increase within the high-resolution nested model. It was decided that short-range forecasts of fog and low-cloud would benefit most from having more vertical levels in the troposphere and in particular near the surface. Therefore, a new vertical levels set was constructed with increased resolution near the surface, but lower resolution in the stratosphere where the impact was expected to be negligible. This model was then run using lateral boundary conditions (LBCs) provided by the standard 38 level mesoscale model. However, it was soon noticed that the PMSL in the 4km model was about 8hPa higher than in the 12km driving model. This large PMSL bias associated with the use of vertically interpolated LBCs appears to vary with the particular configuration of vertical grids used by the driving and driven models.

These two problems (high solver iteration count and PMSL bias) have been found to be caused by the same process. When running a LAM on a different vertical grid from that of the driving model, the LBC data must be interpolated onto the required grid; this vertical interpolation of model variables can result in a new model state which is no longer dynamically balanced. Any imbalance generated by the action of interpolation manifests itself as vertical accelerations along the lateral boundaries. These strong vertical accelerations can cause a reduction in the surface pressure and an increase in the number of iterations required for the Helmholtz dynamics solver to converge on a stable solution.

While forecasts of the large scale flow out to one week (made by the global model) can be benefited by the use of extra satellite observations in the stratosphere and mesosphere (Walters *et al.*, 2007), a forecast of the formation of fog or low cloud (in convective-scale model) may benefit more by having more vertical levels within the boundary layer (Tardif, 2004; Bechtold *et al.*, 1996). Future Met Office NWP plans involve running operational forecast models on different vertical grids in order to maximise the performance of each model over their target time period. Consequently, a solution to the problems described earlier were required. This report describes the work related to identifying the cause of these problems, the proposed solution, followed by a summary of forecast trials leading to its operational implementation in the NAE after Parallel Suite 14 (March 2007).

2 Idealised experiments to identify the causes of the problem

The operational NAE and the operational global model differ considerably in their setup. Apart from the obvious difference in their horizontal resolutions, many of the options in the dynamical core and parameterisation of physical processes also differ, along with the choice of surface and climatological ancillary files, data assimilation systems and observations used. In a bid to isolate the factors causing this problem, a highly idealised approach was adopted. The aim of this experiment was to reproduce the negative PMSL bias found when running a limited area model on a different vertical grid from that of the global model and imposing hydrostatic balance in the LBCs.

2.1 Model setup

An idealised configuration of the Unified Model was used which shared the same dynamical core as the operational models, did not include parameterisation of sub-grid processes or moisture and ran from idealised initial conditions.

To mimic the operational setup, an idealised global model was used to drive an idealised limited area model which ran on a different vertical grid. To isolate the action of vertical interpolation in the generation of the LBCs as far as possible, the idealised LAM used exactly the same horizontal grid as that of the idealised global model, i.e. the LAM had the same grid length and did not use a rotated pole, such that grid points in the LAM were simply a subset of those in the global model. A horizontal grid spacing of 0.833x0.554 degrees (N216 equivalent) was used in both models which resulted in 432x325 grid points in the global model and a 150x150 subset of these, centred about the equator, in the LAM (see Figure 2).

A timestep of 20 minutes was found to give numerically stable results and so no horizontal or vertical diffusion was needed in these idealised simulations. LBCs were updated every timestep so that any effects from a low update frequency in the LBCs was avoided. In every respect, other than the use of LBCs and a different vertical grid in the LAM, the setup of both model was identical. An aqua-planet sphere was adopted to remove the complications introduced by the land surface and orography.

The global model used 50 vertical levels where the thickness increased quadratically with height (similar to in the operational models), whereas the limited area model used a 38 level equivalent (see Figure 3). The eta levels used in these idealised models were the same as those of the operational

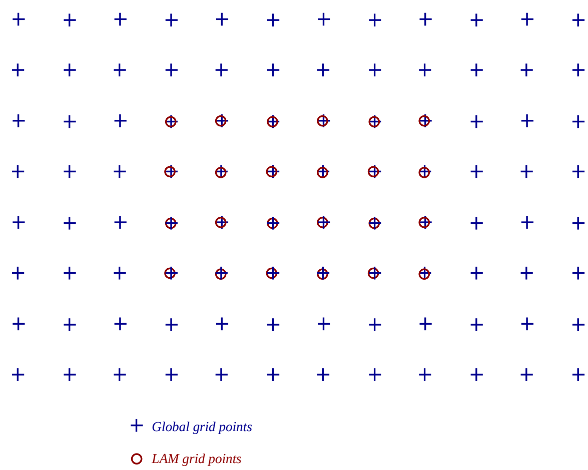


Figure 2: *Schematic to represent the grids of the global and limited area idealised models. Note that the LAM grid is effectively just a sub-set of the global grid points with the horizontal resolution and grid positions being identical.*

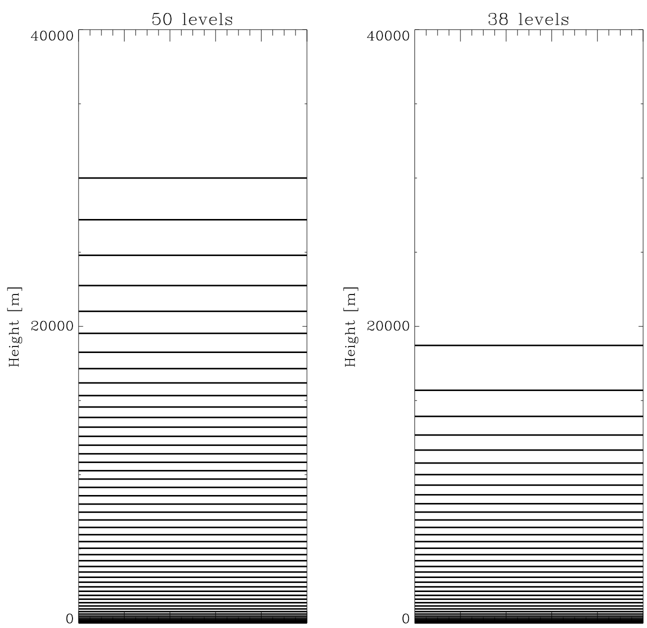


Figure 3: *Model θ levels used in the idealised global model (left) and limited area models (right).*

models; however the lids of each had to be lowered to allow the use of the geostrophically, hydrostatically balanced initial conditions.

It is worth noting that the coding of the reconfiguration had to be altered for these experiments to prevent hydrostatic balance from being imposed across the whole domain as this could complicate the interpretation of the results.

2.2 Initial conditions

A both hydrostatically and geostrophically balanced initial state was provided by a westerly jet following a cosine pattern, peaking at 50ms^{-1}, centred on the equator (see Figure 4) with an exponential decrease of pressure and an exponential increase of potential temperature (Figure 5) with height. This is, theoretically, a steady-state solution.

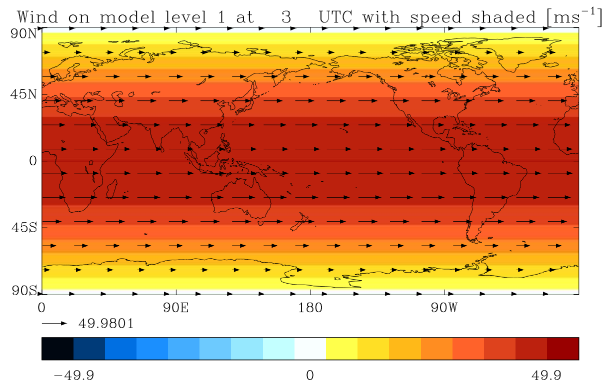
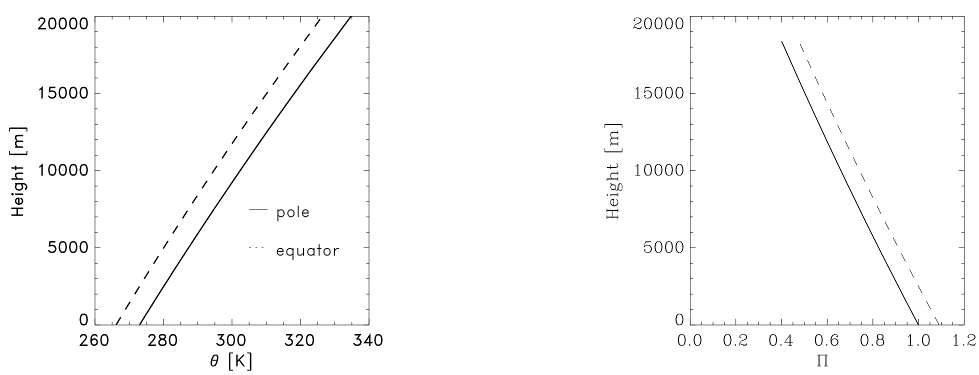


Figure 4: *Initial u-component of the wind [ms^{-1}] in the idealised global simulation.*



(a) **(b)**

Figure 5: *Initial profiles of (a) potential temperature and (b) Exner pressure, in the idealised global simulation at the pole (solid) and at the equator (dashed).*

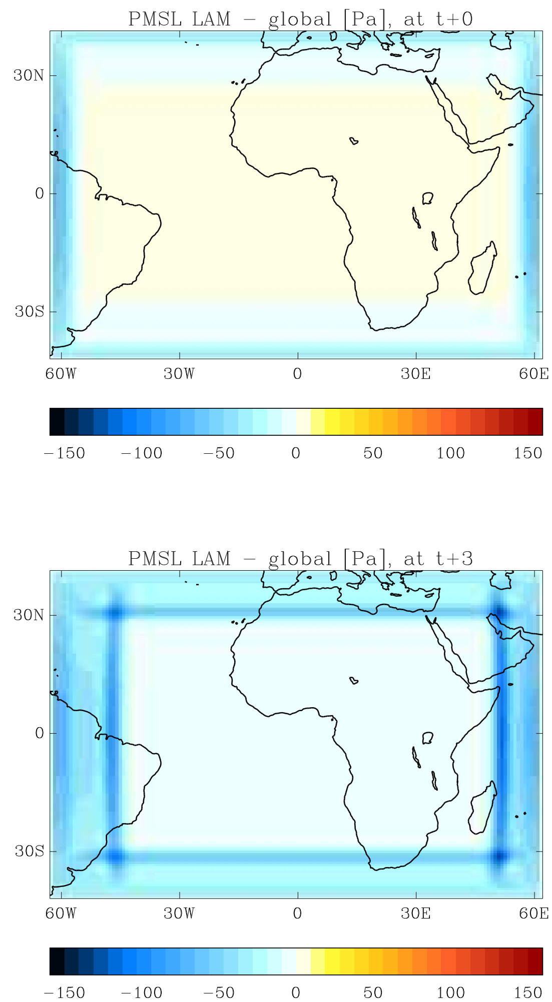


Figure 6: *PMSL difference [Pa] between the LAM and the global at the end of timestep 1 (top) and timestep 9 (bottom). This was from the simulation where hydrostatic balance was imposed in the LBCs, but not in the reconfigured start dump.*

2.3 Reproducing the problem

The initial aim of the experiment was to reproduce, in this idealised environment, the negative PMSL bias found in the operational forecasts when imposing hydrostatic balance in the LBCs. While the global model remained in a steady-state (as expected from theory), the LAM showed waves generated along the boundaries which spread across the domain. Figure 6 shows the difference in PMSL between the idealised limited area model and the global model. It can be seen that a negative PMSL bias is established around the boundaries which then spreads into the interior of the domain, in a very similar fashion to that in the operational forecasts (see Figure 1). The phase speed of these waves indicates that these are shallow water gravity waves. This experiment successfully reproduced the problem being investigated, allowing the causes to be diagnosed free from the complexities of the full operational setup.

As the source of the PMSL bias was evidently the boundaries of the domain, further attention was paid to these regions. Figure 7 shows a cross-section along the equator of the vertical velocity at the end of the first timestep. The dominant features are areas of ascent in the blending zones of the LBCs. Since hydrostatic balance was imposed on the LBCs this was, at first, a surprising result, especially as the vertical velocity was reset to zero at the beginning of the each timestep. All the vertical motion must have evolved during the first timestep.

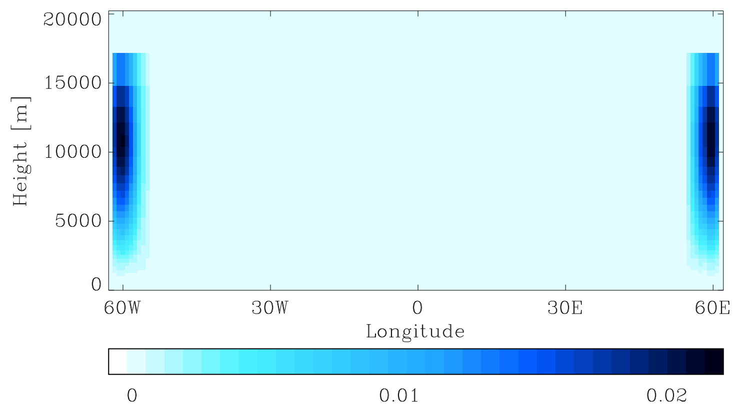


Figure 7: *Cross-section of vertical velocity [ms^{-1}] at the equator in the LAM at the end of timestep 1. This was from a simulation where hydrostatic balance was only imposed in the LBCs. Note that w in the boundary region was set to zero at the beginning of the timestep.*

3 Hypothesis of the cause of the problem

Vertical velocity increments are calculated using the vertical momentum equation (Staniforth *et al.*, 2003),

$$\underbrace{\frac{Dw}{Dt}}_{termA} = \underbrace{\frac{(u^2+v^2)}{r}}_{termB} + \underbrace{2\Omega u cos\phi}_{termC} - \underbrace{g}_{termD} - \underbrace{c_{pd}\theta_v\frac{\partial\Pi}{\partial r}}_{termE} + \underbrace{S^w}_{termF}$$

where *u, v and w* represent the wind components in directions of increasing longitude, latitude (ϕ) and radius (r) respectively. Π is the Exner pressure, Ω is the angular velocity of the Earth, θ_v is the virtual potential temperature, c_{pd} is the specific heat capacity of dry air at constant pressure, g the acceleration due to gravity and S^w are the increments of vertical velocity resulting from the parameterisation of sub-grid processes (which in this idealised case are zero). On a LAM grid with a rotated pole this becomes,

$$\frac{Dw}{Dt} = \frac{(u_{rot}^2+v_{rot}^2)}{r} - f_1 v_{rot} + f_2 u_{rot} - g - c_{pd}\theta_v\frac{\partial\Pi}{\partial r} + S^w \quad (1)$$

where,

$$f_1 = -2\Omega sin\lambda cos\phi_0$$
$$f_2 = 2\Omega\left(cos\phi sin\phi_0 - sin\phi cos\lambda cos\phi_0\right)$$

and where u_{rot} and v_{rot} are the components of the wind along the rotated grid.

The vertical velocities seen at the end of the first timestep *must* have resulted from an imbalance of the terms in this equation. In the operational and idealised setups, hydrostatic balance is imposed on the LBCs, i.e. the Exner pressure profile is adjusted such that the acceleration due to the vertical pressure gradient (term E) is equal and opposite to the acceleration due to gravity (term D). If the other terms are considered negligible then this can be expected to lead to state with no vertical accelerations. Appendix A shows that the accelerations due to the vertical component of the coriolis force (term C) and the metric term (term B) in equation 1 are not negligible in this case and lead to the spurious vertical accelerations. Apparently, these vertical accelerations lead to a reduction in the air density near the surface (due to continuity) and hence a reduction in pressure at mean sea-level (via the equation of state).

i.e. it is hypothesised that in a non-stationary, hydrostatically balanced atmosphere, the vertical coriolis and metric terms in the New Dynamics formulation can result in significant vertical accelerations. When these conditions are imposed in the LBCs of the UM this can lead to surface pressure biases and increase the number of iterations needed for the dynamics solver to converge on a stable solution.

3.1 Testing the hypothesis: imposing a more precise balance

If vertical accelerations are the cause for the PMSL bias and high iteration counts then the obvious solution would be to impose dynamical balance in the vertical to ensure that these accelerations do not occur, i.e. balance all the terms in the vertical momentum equation to prevent these accelerations from occurring (shown schematically in Figure 8).

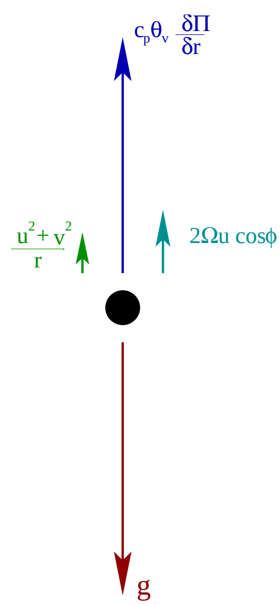


Figure 8: *Schematic to show the forces causing vertical accelerations in the New Dynamics formulation of the UM.*

We want $\frac{Dw}{Dt} = 0$, therefore equation 1 becomes,

$$\frac{(u_{rot}^2 + v_{rot}^2)}{r} - f_1 v_{rot} + f_2 u_{rot} - g - c_{pd}\theta_v \frac{\partial \Pi}{\partial r} = 0$$

$$c_{pd}\theta_v \frac{\partial \Pi}{\partial r} = \frac{(u_{rot}^2 + v_{rot}^2)}{r} - f_1 v_{rot} + f_2 u_{rot} - g \quad (2)$$

3.2 Implementation in the Unified Model

This solution was implemented in the UM as a new subroutine BALANCE_LBC_VALUES, which adjusts the Exner profiles in the LBCs at the beginning of each timestep. LBC data are output from the driving model as normal. These data are read in by the limited area model. At the beginning of each timestep, balance is applied to prevent spurious vertical accelerations. Details of the exact method of adjusting the Exner profiles is provided below. Finally, the model values along the boundaries are blended with the LBC values.

The UM grid is staggered on an Arakawa C-grid in the horizontal and a Charney-Phillips grid in the vertical. Equation 2 is discretised as follows,

$$c_{pd}\theta_v^k \frac{\Pi^{k+1} - \Pi^k}{r_\rho^{k+1} - r_\rho^k} = \frac{\left(\overline{\overline{u}}_{rot}^2 + \overline{\overline{v}}_{rot}^2\right)}{r_\theta^k} - f_1\overline{\overline{v}}_{rot} + f_2\overline{\overline{u}}_{rot} - g \qquad (3)$$

where horizontal and vertical interpolation of the winds onto θ points is carried out as follows,

$$\overline{\overline{u}}_{i,j} = w_1\left(\frac{1}{2}u_{i-1,j,k} + \frac{1}{2}u_{i,j,k}\right) + w_2\left(\frac{1}{2}u_{i-1,j,k+1} + \frac{1}{2}u_{i,j,k+1}\right).$$

Indices, *i, j, k*, indicate the horizontal and vertical location of a grid point, and w_1 and w_2 are interpolation coefficients. At the top model level, an upper boundary condition is applied, $\overline{\overline{u}}_N=\overline{u}_N$ and $\overline{\overline{v}}_N=\overline{v}_N$. Equation 3 can be re-arranged to allow a balanced Exner profile to be constructed by integrating from the lowest model level upwards, given the profiles of potential temperature and wind,

$$\Pi^{k+1} = \Pi^k + \frac{r_\rho^{k+1} - r_\rho^k}{c_{pd}\theta_v^k}\left(\frac{\left(\overline{\overline{u}}_{rot}^2 + \overline{\overline{v}}_{rot}^2\right)}{r_\theta^k} - f_1\overline{\overline{v}}_{rot} + f_2\overline{\overline{u}}_{rot} - g\right).$$

Issues arise when interpolating the u and v winds horizontally due to the way LBC data are stored in the UM. LBC data are stored in memory sequentially in four segments; north, south, east and west. For points near the edge of each segment, it is sometimes necessary to retrieve data from a different LBC segment (e.g. see Figure 9) and so care had to be taken in to ensure that the data was addressed correctly.

It is recognised that to impose a truly balanced state, the atmospheric state should be calculated such that it remains balanced after passing through the semi-implicit, semi-Lagrangian dynamics of the UM. The methodology adopted here would only be exact when used with an explicit time-stepping scheme, i.e. if the model dynamics used the same time-stepping scheme as the method used here. While this method will mean that the balance imposed is not exact, the results indicate that the inaccuracy of the method is smaller than the imbalance which is causing the problem and so provides a significant improvement. The implementation of a scheme to impose an exact balance matching the UM dynamics would prove substantially more computationally expensive than the relatively cheap scheme adopted here.

After the Exner pressure profiles have been changed, the density must also be balanced such that the equation of state is obeyed. i.e.

$$\rho = \frac{p_0}{\kappa_d c_{pd}\overline{\theta}_v \Pi^{\frac{\kappa_d - 1}{\kappa_d}}}$$

where $\kappa_d = \frac{R_d}{c_{pd}}$.

While the New Dynamics employs a linearised form of the equation of state, the differences resulting from this different definition were found to be negligible.

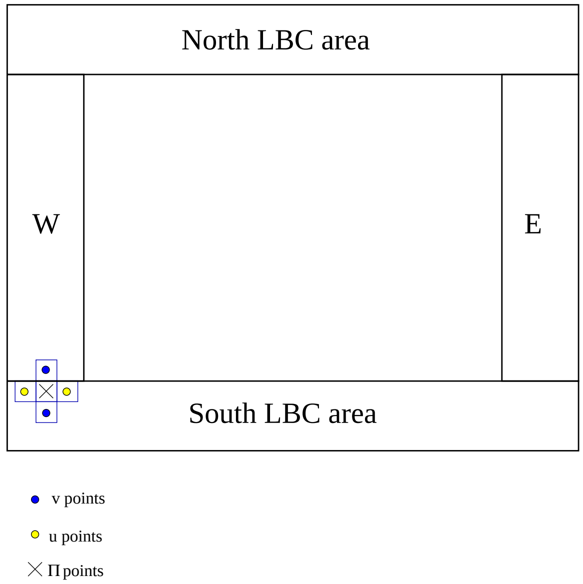


Figure 9: *Schematic to illustrate the segmented data storage of the LBC fields in UM and the difficulties of horizontal interpolation of the winds onto* Π *points near the edge of these segments.*

4 Results

In section 1, two situations were described where a PMSL bias occurred; firstly when imposing hydrostatic balance in the LBCs and secondly when interpolating LBC data from one vertical grid to another. This section describes tests of the new balanced formulation of the LBCs in both of these situations.

4.1 Comparison with hydrostatically balanced LBCs

The improved balance formulation of the LBCs is compared here with the use of hydrostatically balanced LBCs. Figure 10 shows the clear reduction in the PMSL bias in the idealised experiment. An improvement is also seen when using the improved formulation in the NAE (see Figure 11). In this case the PMSL bias is reduced from typical values of around -1.5hPa to around -0.5hPa.

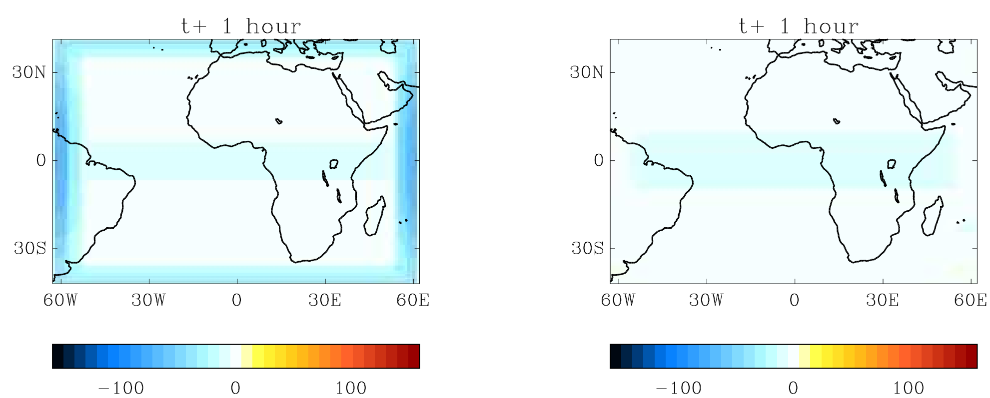


Figure 10: *PMSL difference [Pa] in idealised models (LAM - global) when using hydrostatic LBCs (left) and when using the new balanced LBCs (right).*

The maximum vertical velocities along the LBCs at the end of the first timestep in the idealised case study are found to reduce from around 0.018ms^{-1} to under 0.004ms^{-1} (not shown). In the NAE forecasts, the iteration count was found to reduce from 23 iterations when running with no balance imposed in the LBCs, to 20 when imposing hydrostatic balance and reducing to only 15 when running using the new balanced LBC formulation. This led to a runtime decrease of around 5% in the NAE compared with when imposing hydrostatic balance and almost 25% faster than when not imposing any balance. This increase in speed comes despite the fact that a new routine had been introduced to calculate the balanced Exner profiles.

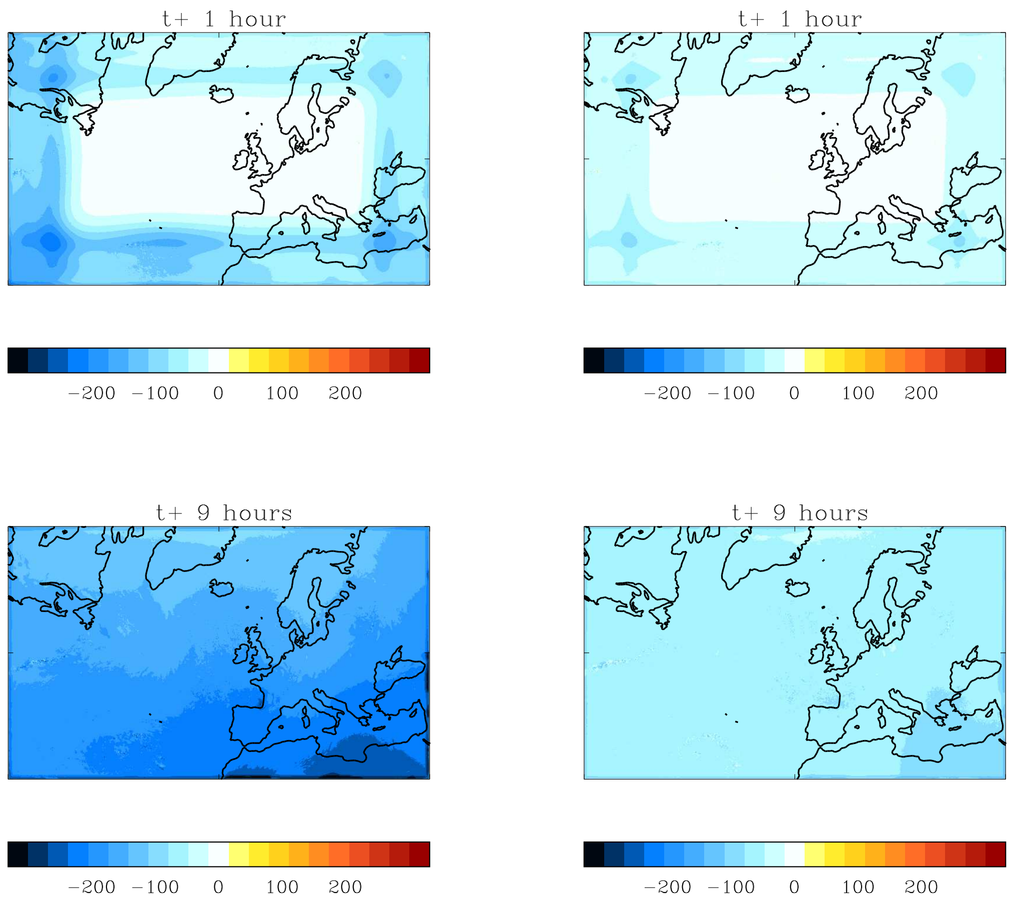


Figure 11: *PMSL difference [Pa] (NAE - global) when using hydrostatic LBCs (left panels) and when using the new balanced LBCs (right panels).*

4.2 Removal of PMSL bias induced by vertical interpolation

As mentioned in section 1, when running the UK4 model on a 56 level vertical grid (so-called "S2G3") driven by LBCs produced from a model on the standard 38 level grid ("G3") a PMSL bias of 8hPa was observed, even when hydrostatic balance was *not* imposed on the LBCs. i.e. this 8hPa PMSL bias was caused entirely by vertical imbalance generated by vertical interpolation from one model grid onto the other. A demonstration of how vertical interpolation from one discrete grid onto another can lead to dynamic imbalance is provided in Appendix B.

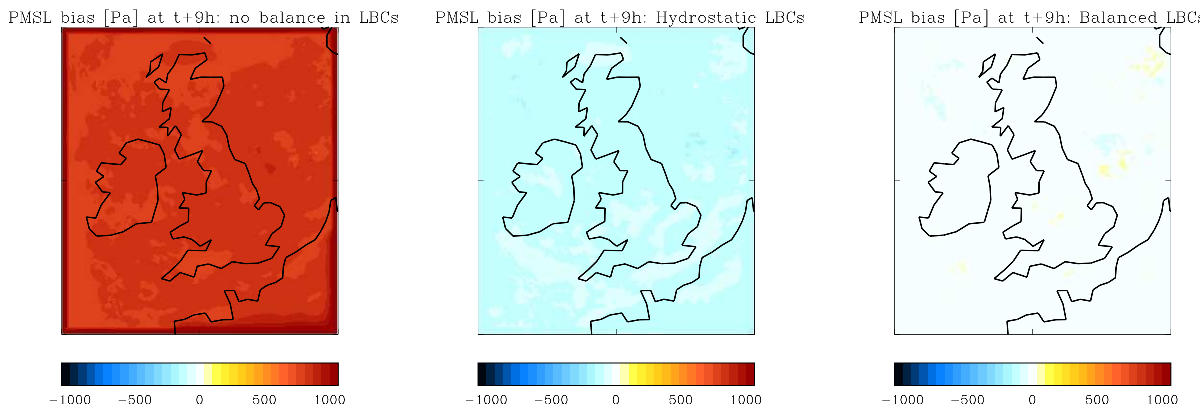


Figure 12: *PMSL difference [Pa] (PMSL in S2G3 UK4 run - PMSL in G3 UK4 run) when using no balance in LBCs (left), hydrostatic LBCs (centre) and when using the new balanced LBCs (right).*

This model setup was repeated in order to reproduce this result. After this 8hPa PMSL bias had been verified, the UK4 was rerun but this time imposing hydrostatic balance in the LBCs and finally a forecast was made with the use of the new balanced LBCs. The results are shown in Figure 12. When running with no balance in the LBCs, the imbalance caused by vertical interpolation leads to a PMSL bias of almost exactly +8.0hPa. Running with hydrostatic LBCs changes this bias to -1.3hPa. Finally, forecasts made with the new balanced LBCs have a bias of only +0.05hPa.

5 Trials and operational implementation

The code changes described in this report underwent three periods of testing. Firstly, a trial suite was run for 6 weeks comparing NAE forecasts with and without the balanced LBCs. One forecast was run per day in this suite meaning that a total of 43 forecasts were run throughout the period. Secondly, the changes were tested along with the w-based CAPE closure timescale changes to the convection scheme in 8 days of trials prior to parallel suite 14. Finally, the changes were tested in parallel suite 14 for 5 weeks running a full operational forecast cycle. The results in all these trials indicated that the PMSL bias, which had been a problem in the NAE up until that point, had been significantly reduced (in Figure 13 compare verification from the operational forecasts (blue) with those using improved LBCs (red)). Similarly, the iteration count reduction agreed with that stated in section 4.1.

The new formulation appeared to be computationally stable with no increase in the number of model failures. The change to other model variables was assessed to be neutral. Since the UK Index does not include PMSL, the impact of the balanced LBCs on it was also neutral (less than 0.25% change).

The changes became operational in the NAE forecasts from the 12z run on March 6th, 2007. Later they were also operationally implemented in the Crisis Area Mesoscale Models following similar problems there. With the vertical resolution of the operational forecasts due to increase in October 2007, it seems likely that it will be implemented in the new 70 level UK4 in PS17 which will run on a different vertical levels set from that of the 70 level NAE and global model.

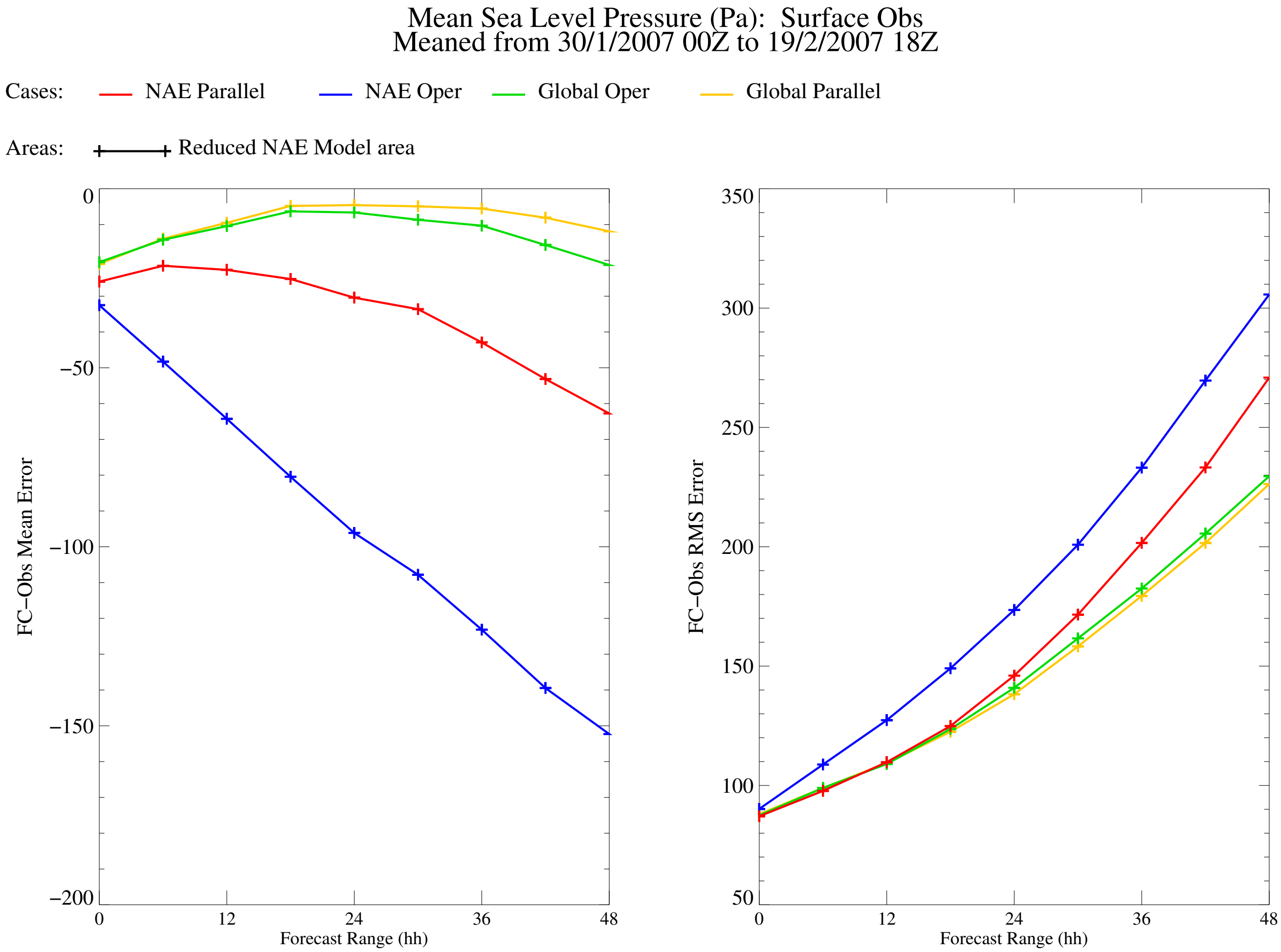


Figure 13: *PS14 verification: PMSL mean error (left panel) and RMS error (right panel) [Pa]. The operational NAE (blue) quickly develops a negative PMSL bias, where as the trial NAE (red) forecast is much closer to that of the global model.*

6 Conclusion

Problems caused by the linear interpolation in height of Exner pressure, potential temperature and density to create LBCs on a different vertical grid from that of the driving model have been identified with the use of an idealised setup of the UM and solved by imposing vertically balanced profiles of Exner pressure and density in the LBCs. The solution provided has successfully removed anomalously high dynamics solver iteration counts in the LAMs, with a subsequent reduction in operational runtime of the NAE of 5%. Similarly, a PMSL bias (experienced with certain combinations of vertical levels in the driving and driven model) has been significantly reduced in the LAM forecasts resulting in improved guidance in high resolution, short-range forecasts. It has been shown that imposing hydrostatic balance in the LBCs is insufficient as accelerations resulting from the vertical coriolis force still lead to spurious vertical accelerations large enough to create a reduction in sea level pressure. To prevent these vertical accelerations, all the terms in the vertical momentum must be balanced. These changes have been successfully implemented operationally in the NAE and the CAMMs and have become a standard option after UM6.5.

The use of balanced lateral boundary conditions allows a free choice of vertical grid in each model. This freedom should provide benefits not only for the operational weather forecasts but also for regional climate modellers who may wish to use a higher resolution vertical grid in their limited area simulations.

It is suggested that the results from this work may also be applicable to other non-hydrostatic NWP models when LBCs are provided by another model on a different vertical grid and/or with a different dynamical formulation.

Appendix A: Scale analysis of the vertical momentum equation

It was shown in section 2.3 that when hydrostatic balance was imposed on an atmosphere with strong horizontal winds, significant vertical accelerations still occurred. This scale analysis is intended to show that the magnitude of these accelerations is consistent with those expected from the vertical component of the coriolis and metric terms.

For clarity the vertical momentum equation is repeated here,

$$\underbrace{\frac{Dw}{Dt}}_{termA} = \underbrace{\frac{(u^2+v^2)}{r}}_{termB} + \underbrace{2\Omega ucos\phi}_{termC} - \underbrace{g}_{termD} - \underbrace{c_{pd}\theta_v\frac{\partial\Pi}{\partial r}}_{termE} + \underbrace{S^w}_{termF}$$

U=100ms^{-1}, R=10^{7}m, 2Ω=10^{-4}rads^{-1}, g=10ms^{-2}, c=10^{3}Jkg^{-1}K, θ=10^{2}K, $\frac{\partial\Pi}{\partial r}=\frac{1}{10^4}$

Term B = $\frac{U^2}{R}=\frac{100^2}{10^7}$ = 10^{-3}ms^{-2}
Term C = 2ΩU = 10^{-4}.100 = 10^{-2} ms^{-2}
Term D = 10 ms^{-2}
Term E = c$\theta\frac{\partial\Pi}{\partial r}$ = 10^3.10^2.10^{-4} = 10 ms^{-2}

The vertical accelerations in the model (term A) which cause the problems are of order 10^{-5}ms^{-2}, which is smaller than terms B and C. Therefore, terms B and C cannot be ignored when imposing balance to remove these accelerations.

i.e. hydrostatic balance *is not precise enough* **to remove the vertical accelerations. In an atmosphere with fast moving jets of air, all the terms in the vertical momentum equation must be balanced to reduce the spurious vertical accelerations within acceptable levels.**

Appendix B: The destruction of balance by vertical interpolation

The destruction of a dynamically balanced state by vertical interpolation from one discrete grid onto another can be shown (Wood, 2007) by considering a hydrostatically balanced atmosphere,

$$c_p\theta\frac{D\Pi}{dz} + g = 0.$$

If this profile is sampled on a discrete set of grid points, k=1,2,3,...N, and then interpolated onto a grid using linear interpolation weights, w_1 and $w_2 = 1 - w_1$, then,

$$c_p\left(w_1\theta_k + w_2\theta_{k+1}\right)\left(w_1\left(\frac{D\Pi}{Dz}\right)_k + w_2\left(\frac{D\Pi}{Dz}\right)_{k+1}\right) + g = 0$$

which can be re-arranged to become,

$$\left(w_1^2+w_2^2\right)g + w_1w_2c_p\left(\theta_k\left(\frac{D\Pi}{Dz}\right)_{k+1} + \theta_{k+1}\left(\frac{D\Pi}{Dz}\right)_k\right) = g$$

as Dz $\rightarrow$ 0,

$$\left(w_1^2+w_2^2\right)^2 g = g$$

i.e. $g = g$, since w_2=1-w_1 ; the new state after interpolation is still in exact hydrostatic balance. Whereas for a finite Dz this will not be the case.

References

Bechtold, P., Krueger, S., Lewellen, W., van Meijgaard, E., Moeng, C., Randall, D., van Ulden, A., and Wang, S. (1996). Modeling a Stratocumulus-Topped PBL: Intercomparison among Different One-Dimensional Codes and with Large Eddy Simulation. *Bull. Amer. Meteor. Soc.*, **77**, 2033–2042.

Davies, T., Cullen, M., Malcolm, A., Mawson, M., Staniforth, A., White, A., and Wood, N. (2005). A new dynamical core for the Met Office's global and regional modelling of the atmosphere. *Quart. J. Roy. Meteor. Soc.*, **131**, 1759–1782.

Staniforth, A., White, A., Wood, N., Thuburn, J., Zerroukat, M., Cordero, E., and Davies, T. (2003). Joy of UM5.5 - Model Formulation. Unified Model Documentation Paper 15, Met Office Forecasting Research.

Tardif, R. (2004). On the impact of vertical resolution in the numerical forecasting of fog. *11th Conference on Aviation, Range and Aerospace Meteorology*, **Extended Abstract**.

Walters, D., Earnshaw, P., Milton, S., Rawlins, R., Keil, M., Davies, T., and Heming, J. (2007). Enhancing vertical and horizontal resolution of the Met Office global NWP (Unified) Model. *Quart. J. Roy. Meteor. Soc.*, **In press**.

Wood, N. (2007). Personal Communication.