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Calculation of maximum ground level concentration as a function of stack height

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Summary

A comparison has been made of four different methods of calculating the expected variation of maximum ground level concentration of pollutant downwind from a stack. These methods are closely related as they have all been derived from Gaussian plume models. The methods are intended to be applied for sources fairly near the surface (plume heights not exceeding about 150 metres) where the Pasquill/Gifford dispersion coefficient curves can be considered to apply. In the case of the third method considered (CONCAWE Model) the values of the Sutton dispersion coefficients were chosen with data from ground level sources specifically excluded. As an example of the application of the models the four methods have been used to predict the change in the maximum ground level concentration which would be expected to occur near the forming chimney of Pilkington's fibreglass factory at Pont-y-Felin near Pont-y-pool.

1. Introduction

The maximum concentration of a pollutant which will occur at ground level downwind from a stack can be estimated using one of several published formulae which have been derived for simple Gaussian plumes. These formulae enable one to calculate the values of 4 related parameters.

(a) C_{max} The maximum ground level concentration which will occur at any distance downwind from the stack as a function of wind speed (U) for a given stability class.

(b) x_{max} The distance downwind from the stack at which the maximum ground level concentration (C_{max}) occurs as a function of wind speed and stability.

(c) C_{crit} The maximum ground level concentration for a given stability class for all wind speeds (U) and all downwind distances (x).

(d) $h_{s_{crit}}$ The stack height which will ensure that a given value of C_{crit} is not exceeded. This values applies for any wind speed which occurs with a given stability category.

The various formulae are inter-related as they are all derived from a Gaussian plume model. However different assumptions have been made about the plume rise formulae which apply the values of σ_y and σ_z to be used and the way in which different sampling times and roughness lengths are allowed for. The formulae are intended to be applied over an approximately level plain of uniform roughness.

The concentrations will also depend on other factors such as site topography, the presence of a capping stable layer and entrainment by buildings but these factors are not dealt with in the simple models considered (except a capping stable layer which is included in the fourth model).

In this note four different models have been compared. The four models used are as follows:

Model A.	Simple Gaussian Plume.	Slade (1968)
Model B.	Gaussian Model.	Weil and Jepsen (1977)
Model C.	CONCAWE Model.	Brummage (1968)
Model D.	Atmospheric Dispersion Modelling Working Group Model. ADMWG/R2 (1979, unpublished draft paper)	

It is important to realise that the concentrations calculated by any method will be subject to some error. The Gaussian plume model in the form recommended by the Atmospheric Dispersion Modelling Working Group (Model D) was stated to be accurate to a factor of 3 in neutral conditions. Differences of this order between the different models may be expected.

When considering an individual model it is probable that all the estimates will either be systematically biased high or low when compared with actually measurements. Therefore individual calculated values of concentration must not be given too much weight. Probably the most useful way to consider the results is to use the model in a relative sense rather than an absolute sense. This means the proportionate change in maximum ground level concentration which occurs as a result of a change in stack height should be considered rather than the absolute change.

In calculating the maximum ground level concentration it is necessary to make some assumptions about the plume rise and the effective height of the plume. In order to make it possible to compare the different models under similar circumstances the same plume rise formula has been used for each model as far as possible. In Models A and B the effective height of the plume has been calculated according to Briggs (1969) equations for the final plume rise. In Model D the Briggs (1969) equations have been used for both the initial plume rise and the final plume rise. In Model C the CONCAWE plume rise formula is included. Results for Models A and B using Briggs (1970) plume rise equations are also included.

The four different models have been used to calculate the variation of the maximum ground level concentration of phenolics which would be expected if the height of the forming chimney at the Pont-y-Felin fibreglass factory were increased. The present height of the stack is 52 metres. The source strength of the stack is 5.0 kg/hr of phenolics (or 1.39 gm/sec phenolics). Calculations have been made for Pasquill stability categories D and C except for Model C where the calculations have only been made for category D.

The values of the horizontal and vertical dispersion coefficients (σ_y and σ_z) used in Models A and B - or more strictly the functions which depend on the choice of these parameters - have been derived from the Pasquill/Gifford curves (Hilsmeier and Gifford (1962)). This implies that the calculated concentrations are assumed to be representative of a sampling time of about 3 mins and a roughness length of about 3 cms which are approximately the values which applied during the experiments used to measure σ_y and σ_z originally (Hanna et al, (1977)). In Model C the values of σ_y and σ_z are defined in terms of Suttons dispersion constants. The values chosen are derived from a number of experimental measurements. However data from ground level sources are specifically disregarded and this includes the measurements used by Pasquill (See Brummage (1968), Table 2 and page 202). The results are stated to apply to elevated sources and to be representative of a 30 min sampling period, but there is no comment on a representative roughness length. In Model D the values of σ_y are taken from the Pasquill/Gifford curves. The values of σ_z are taken from the curves published by Smith (1972) using the equation fitted by Hosker (1974). These curves allow a choice to be made of the roughness length and for these calculations z_o equal to 50 cms has been used, a value which is thought to be representative of the fairly rough terrain surrounding the Pont-y-Felin factory.

2. Description of Models

2.1 Model A. Simple Gaussian Plume

The ground level concentration for an effective release height h_e with unrestricted growth of the plume above its centreline and with perfect reflection of the plume from the ground is

$$C = \frac{Q}{\pi \sigma_y \sigma_z U} \cdot \exp\left[-\left(\frac{y^2}{2\sigma_y^2} + \frac{h_e^2}{2\sigma_z^2}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where
C (gm/m^3) concentration
Q (gm/sec) source strength of pollutant
U (m/sec) wind speed
y (metres) crosswind direction
h_e (metres) effective plume height
σ_y (metres) horizontal diffusion coefficient
σ_z (metres) vertical diffusion coefficient

(See Slade (1968) equation 3.116 or Turner (1969) equation 3.2)

In the Simple Gaussian Plume model the ratio of σ_y to σ_z is assumed to be constant for a given stability category. The maximum ground level concentration below the centreline for a given wind speed and given stability category is

$$C_{max} = \frac{2Q}{\pi h_e^2 e U} \cdot \left(\frac{\sigma_z}{\sigma_y}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where $\frac{\sigma_y}{\sigma_z}$ is constant

and $h_e^2 = 2\sigma_z^2$

$e = 2{\cdot}7183$ (base of natural logarithms)

(See Slade (1968), equation 3.146)

In order to find the highest concentration (C_{crit}) which will occur with any wind speed for a given stability category it is necessary to differentiate this equation with respect to U. However the effective plume height is also a function of wind speed through a plume rise equation.

$$h_e = h_s + \Delta h \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

h_s (metres) stack height

Δh (metres) plume rise

For the present calculations the plume rise equations due to Briggs (1969) for neutral and unstable conditions are assumed to apply. The final plume rise is achieved at a distance $3x_*$ downwind from the stack.

From Briggs (1969), equation 4.35 on page 58, for 305 metres $> h_s \geq$ 17 metres

$$x_* = 5{\cdot}125\ Q_H^{2/5}\ h_s^{3/5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

x_* (metres) the distance at which atmospheric turbulence starts to dominate entrainment.

Q_H (MWatts) the rate of emission of heat from the stack.

From Briggs (1969), equation 4.32[1] on page 57 and text on page 58, for $Q_H <$ 20 MWatts

$$\Delta h_{final} = 3{\cdot}2844\ Q_H^{1/3}\ (3x_*)^{2/3}\ U^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Substituting for x_*

$$\Delta h_{final}.U = 20{\cdot}310\ Q_H^{3/5}\ h_s^{2/5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

It will be noted that in this equation the final plume rise is not only a function of the rate of heat emission from the stack but also the stack height.

In order to calculate C_{crit} it is convenient to rewrite the equation for C_{max} in the form

$$C_{max} = D.\frac{1}{h_e^2}.\frac{1}{U} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where $D = 0{\cdot}234\ Q \left(\frac{\sigma_z}{\sigma_y}\right)$

The equation for the final plume rise can also be rewritten in the form

$$\Delta h = B\,U^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where $B = 20{\cdot}310\ Q_H^{3/5}\ h_s^{2/5}$

Hence $h_e = h_s + B\,U^{-1}$ $\ldots\ldots\ldots\ldots\ldots\ldots (9)$

And $C_{max} = D.\,U.\,(B + h_s U)^{-2}$ $\ldots\ldots\ldots\ldots\ldots\ldots (10)$

Differentiating this with respect to U and setting $\frac{dC_{max}}{dU} = 0$ for the largest value of C_{max} yields the equation

$$C_{crit} = \frac{D}{4B\,h_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

And $$h_{s_{crit}} = \frac{D}{B.4\,C_{crit}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

Using Briggs (1969) plume rise formulae it has already been noted that B is a function of stack height h_s. Substituting in the last equation gives

$$h_{s_{crit}} = \left(\frac{D}{4\ .\ 20{\cdot}310\ \ Q_H^{3/5}\ \ C_{crit}}\right)^{5/7} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

For Ponty-y-Felin $\quad Q_H = 7{\cdot}32$ MWatts

$Q = 1{\cdot}39$ gm/sec (phenolics)

For Category D from the Pasquill/Gifford curves

$$\frac{\sigma_y}{\sigma_z} = 1{\cdot}7 \quad \text{for } 0.5 \text{ km} < x < 1.0 \text{ km}$$

And for Category C

$$\frac{\sigma_y}{\sigma_z} = 2{\cdot}0 \quad \text{for } 0.5 \text{ km} < x < 1.0 \text{ km}$$

Hence for Pont-y-Felin, Category D, neutral stability

$$h_{s_{crit}} = \left(\frac{6{\cdot}0773\ .\ 10^{-4}}{C_{crit}}\right)^{5/7} \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

And for Pont-y-Felin, Category C, slightly unstable

$$h_{s_{crit}} = \left(\frac{7{\cdot}1336\ .\ 10^{-4}}{C_{crit}}\right)^{5/7} \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

In deriving these equations for $h_{s_{crit}}$ one of the important assumptions made is that the ratio of σ_y to σ_z is constant. This is certainly only approximately true for the Pasquill/Gifford curves for a limited range of distance downwind for category C and D. For Category A the relationship only applies rather poorly.

It should also be noted that equation (13) which gives $h_{s_{crit}}$ as a function of C_{crit} depends on the form of the plume rise equation used. If Briggs (1970) plume rise equation for neutral and unstable conditions had been used this yields an equation for the final plume rise which is independent of stack height.

$$\Delta h_{final} . u = 143\ \ Q_H^{3/5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

for $Q_H \geq 6.2$ MWatts.

Substituting in equation (12) with $B = 143\ Q_H^{3/5}$

gives

$$h_{s_{crit}} = \left(\frac{D}{4 \,.\, 143 \,.\, Q_H^{3/5}\ C_{crit}} \right) \quad \ldots\ldots\ldots\ (17)$$

And hence for Pont-y-Felin, Category D

$$h_{s_{crit}} = \frac{88{\cdot}59 * 10^{-6}}{C_{crit}} \quad \ldots\ldots\ldots\ (18)$$

And for Pont-y-Felin, Category C

$$h_{s_{crit}} = \frac{103{\cdot}98 \,.\, 10^{-6}}{C_{crit}} \quad \ldots\ldots\ldots\ (19)$$

Equations (16), (17), (18), (19) should be compared with equations (6), (13), (14), (15) respectively.

The importance of the differences between equations (13) and (17) is not simply in the difference in the absolute magnitude of the $h_{s_{crit}}$ values calculated but in the implications it has on the change of stack height which is required to achieve a desired proportionate reduction in C_{crit}. According to equation (13) a reduction of C_{crit} to 0.5 of its original value will require the stack height to be increased to $2^{5/7}$ times its original height (or 1.64 times the original height). On the other hand according to equation (17) a reduction of C_{crit} to 0.5 of its original value will require the stack height to be increased by a factor of 2. This difference has arisen simply from the choice of the form of the plume rise equation used.

2.2 Model B. Gaussian Model

The basic model used by Weil and Jepsen (1977) is the same as that already described by equation (1). However the variation of σ_y and σ_z is described by the functions

$$\left. \begin{aligned} \sigma_y &= a_1\, x^{b_1} \\ \sigma_z &= a_2\, x^{b_2} \end{aligned} \right\} \quad \ldots\ldots\ldots\ (20)$$

The values of the variables a_1 b_1 a_2 and b_2 for each stability class have been derived from the Pasquill/Gifford curves and are tabulated in Table A1 by Weil and Jepsen (1977). These functions adequately describe the variation of σ_y and σ_z for 0.5 km $< x <$ 20 km.

For a plume which is perfectly reflected at the ground but is unrestricted in spreading above its centreline the maximum ground level concentration for a given stability and wind speed is given by

$$C_{max} = \frac{Q\,\alpha^{\frac{\alpha}{2}}}{\pi\, u\, a_1 a_2} \cdot \frac{\exp\left(-\frac{\alpha}{2}\right)}{\left(\frac{h_e}{a_2}\right)^{\alpha}} \quad \ldots\ldots\ldots\ (21)$$

where $\alpha = 1 + \dfrac{b_1}{b_2}$

The distance downwind from the stack that this maximum occurs is given by

$$x_{max} = \left(\frac{h_e}{\sqrt{\alpha} \, . \, a_2} \right)^{1/b_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Equation (21) can be rewritten in the simplified form

$$C_{max} = Q \, . \, N \, . \, \frac{(h_e)^{-\alpha}}{U} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

C_{max}	(gm/m^3)	maximum ground level concentration
h_e	(metres)	effective plume height
U	(m/sec)	wind speed
Q	(gm/sec)	source strength of pollutant
N, α		numerical constants based on the Pasquill/Gifford curves for σ_y and σ_z .

The values of N and α for each stability class are given in the table below

Pasquill Stability Category	α	N
A	1.401	0.0101
B	1.791	0.0512
C	1.967	0.1096
D	2.420	0.523
E	2.571	0.656
F	2.978	1.950

Equation (22) can also be rewritten in the simplified form

$$x_{max} = (h_e)^{1/b_2} \, M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

x_{max}	(metres)	distance of maximum ground level concentration from stack.
b_2, M		numerical constants based on the Pasquill/Gifford curves for σ_y and σ_z .

Pasquill Stability Category	$\left(\frac{1}{b_2}\right)$	M
A	0.4717	53.92
B	0.9091	11.69
C	1.099	7.802
D	1.613	1.777
E	1.786	1.944
F	2.222	0.8302

In order to calculate C_{crit} we must substitute an expression for the effective plume height in equation (23). Using Briggs (1969) plume rise formula again

$$\Delta h = B\, u^{-1} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

where $$B = 20{\cdot}310\ Q_H^{3/5}\ h_s^{2/5}$$

$$C_{max} = Q.N.\ U^{\alpha-1}.\left(B + h_s U\right)^{-\alpha} \quad \ldots\ldots \quad (25)$$

Differentiating this with respect to U and setting $\frac{dC_{max}}{dU}=0$ for the largest value of C_{max} yields the equations

$$C_{crit} = \frac{QN}{B}.\ \alpha^{-\alpha}.\left(\frac{\alpha-1}{h_s}\right)^{\alpha-1} \quad \ldots\ldots \quad (26)$$

and $$h_{s_{crit}} = (\alpha-1).\left(\frac{QN}{B}.\frac{\alpha^{-\alpha}}{C_{crit}}\right)^{\left(\frac{1}{\alpha-1}\right)} \quad \ldots\ldots \quad (27)$$

Substiuting for B

$$h_{s_{crit}} = (\alpha-1)^{\left(\frac{\alpha-1}{\alpha-0{\cdot}6}\right)}.\left(\frac{Q.N.\ \alpha^{-\alpha}}{20{\cdot}310\ Q_H^{3/5}\ C_{crit}}\right)^{\left(\frac{1}{\alpha-0{\cdot}6}\right)} \quad \ldots \quad (28)$$

For Category D

$$\alpha = 2{\cdot}420$$
$$N = 0{\cdot}523$$

$$h_{s_{crit}} = 1{\cdot}315\left(\frac{Q\ .\ 3{\cdot}03\ .\ 10^{-3}}{Q_H^{3/5}\ C_{crit}}\right)^{0{\cdot}5495} \quad \ldots\ldots \quad (29)$$

For Category C

$$\alpha = 1{\cdot}967$$
$$N = 0{\cdot}1096$$

$$h_{s_{crit}} = 0{\cdot}9765\left(\frac{Q\ .\ 1{\cdot}43\ .\ 10^{-3}}{Q_H^{3/5}\ C_{crit}}\right)^{0{\cdot}7315} \quad \ldots\ldots \quad (30)$$

<u>For Pont-y-Felin</u>

$Q = 1{\cdot}39$ gm/sec (phenolics)

$Q_H = 7{\cdot}32$ MWatts

Category D $$h_{s_{crit}} = 1{\cdot}315\left(\frac{1{\cdot}277\ .\ 10^{-3}}{C_{crit}}\right)^{0{\cdot}5495} \quad \ldots\ldots \quad (31)$$

Category C $$h_{s_{crit}} = 0.9765 \left(\frac{6.0039 \cdot 10^{-4}}{C_{crit}} \right)^{0.7315} \quad \ldots\ldots\ldots (32)$$

If the Briggs (1970) plume rise formula for neutral and unstable conditions is used

$$\Delta h_{final} \cdot U = 143 \; Q_H^{3/5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

for $Q_H \geqslant 6.2$ MWatts

Substituting in equation (27) with $B = 143 \; Q_H^{3/5}$ gives

$$h_{s_{crit}} = (\alpha - 1) \left(\frac{Q . N . \alpha^{-\alpha}}{143 \; Q_H^{3/5} \; C_{crit}} \right)^{\left(\frac{1}{\alpha - 1}\right)} \quad \ldots\ldots (33)$$

For Category D

$$h_{s_{crit}} = 1.420 \left(\frac{Q \;\; 4.31 \cdot 10^{-4}}{Q_H^{3/5} \; C_{crit}} \right)^{0.7042} \quad \ldots\ldots (34)$$

For Category C

$$h_{s_{crit}} = 0.967 \left(\frac{Q \;\; 2.03 \cdot 10^{-4}}{Q_H^{3/5} \; C_{crit}} \right)^{1.034} \quad \ldots\ldots (35)$$

For Pont-y-Felin

$Q = 1.39$ gm/sec (phenolics)
$Q_H = 7.32$ MWatts

Category D

$$h_{s_{crit}} = 1.420 \left(\frac{1.81 \cdot 10^{-4}}{C_{crit}} \right)^{0.7042} \quad \ldots\ldots\ldots (36)$$

Category C

$$h_{s_{crit}} = 0.967 \left(\frac{0.853 \cdot 10^{-4}}{C_{crit}} \right)^{1.034} \quad \ldots\ldots\ldots (37)$$

Equation (33), (34), (35), (36), (37) which have been derived using Briggs (1970) plume rise formula should be compared with equations (28), (29), (30), (31), (32) respectively which were derived using the Briggs (1969) plume rise formula.

In neutral conditions, category D, according to equation (31) a reduction of C_{crit} to 0.5 of its original value will require the stack height to be increased to $2^{0.5495}$ times its original height (or 1.46 times the original height). On the other hand according to equation (36) a reduction of C_{crit} to 0.5 of its original value will require the stack height to be increased to $2^{0.7042}$ of its original height (or 1.63 times the original height). The corresponding figures for category C are $2^{0.7315}$ (or 1.66) using equation (32) and $2^{1.034}$ (or 2.05) using equation (37).

2.3 Model C. CONCAWE Model

The basic model used by Brummage (1968) in the derivation of the CONCAWE formulae is the same as that already described by equation (1). The variations of σ_y and σ_z are described by the functions

$$\left.\begin{aligned} \sigma_y &= \frac{1}{\sqrt{2}} . C_y . x^{\left(\frac{2-n}{2}\right)} \\ \sigma_z &= \frac{1}{\sqrt{2}} . C_z . x^{\left(\frac{2-n}{2}\right)} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (38)$$

C_y	(metres)$^{n/2}$	Suttons horizontal dispersion constant
C_z	(metres)$^{n/2}$	Suttons vertical dispersion constant
n		Dispersion constant used by Sutton.

This leads to the equation for the maximum ground level concentration

$$C_{max} = \frac{2Q}{\pi u e h_e^2} . \frac{C_z}{C_y} \quad \ldots\ldots\ldots\ldots\ldots (39)$$

C_{max}	(gm/m^3)	maximum ground level concentration
Q	(gm/sec)	rate of production of pollutant
h_e	(metres)	effective stack height
e	= 2.7183	

Note that some of the units and some of the notation used in this equation has been altered from Brummage (1968), equation (9), in order to keep the units and symbols consistent throughout these notes. This equation is directly analagous to equation (2) for the Simple Gaussian Model except that the term $\frac{C_z}{C_y}$ has replaced $\frac{\sigma_z}{\sigma_y}$.

The values of the ratio $\frac{C_z}{C_y}$ to be used for each stability category were derived by Brummage (1968) from a number of experimental measurements for elevated sources. Measurements which related to ground level sources were specifically excluded. For category D for a 30 minute sampling period the value of $\frac{C_z}{C_y}$ recommended by Brummage (see page 204) is 0.7 (for a distance $x = 1000$ metres downwind from the stack). This should be compared with the ratio of $\frac{\sigma_z}{\sigma_y}$ used in the Simple Gaussian Model of $1/1.7$ or 0.59 which was derived from the Pasquill/Gifford curves for ground level sources for a sampling time of about 3 minutes.

In order to derive an equation for $h_{s\,crit}$ it is again necessary to use an appropriate plume rise equation. The CONCAWE plume rise formula recommended by Brummage for small industrial stacks such as those which exist at an oil refinery is

$$\Delta h = \frac{88.0 \quad Q_H^{1/2}}{u^{3/4}} \quad \ldots\ldots\ldots\ldots\ldots (40)$$

Δh (metres) plume rise

Q_H (MWatts) thermal output of stack

U (m/sec) wind speed

In this equation Q_H has been expressed as MWatts (in place of cals/sec used by Brummage, equation (19)). This formula is said to be applicable for a stack volume output of between 15 and 100 Nm^3/sec which is roughly equivalent to a heat emission Q_H between 2 and 25 MWatts. Replacing the effective plume height in equation (39) with $h_e = \Delta h + h_s$ and substituting for Δh from equation (40) gives a formula for C_{max} as a function of U.

$$C_{max} = \frac{2Q}{\pi U e \left(h_s + \frac{88.0 \; Q_H^{1/2}}{U^{3/4}} \right)^2} \cdot \frac{C_z}{C_y} \quad \ldots\ldots\ldots (41)$$

In order to find the largest value of C_{max} differentiate equation (41) with respect to U and set $\frac{dC_{max}}{dU} = 0$. This gives the equation

$$C_{crit} = 2.268 \; Q \left[\frac{1}{Q_v . \Delta T . h_s} \right]^{2/3} \quad \ldots\ldots\ldots\ldots (42)$$

and

$$h_{s_{crit}} = \frac{3.415}{Q_v \, \Delta T} \left[\frac{Q}{C_{crit}} \right]^{3/2} \quad \ldots\ldots\ldots\ldots (43)$$

Q_v (Nm^3/hr) Volume rate of stack

ΔT (degK) (Flue temperature-Ambient temperature)

In deriving these equations Brummage has replaced the thermal output of the stack Q_H by an expression involving Q_v and ΔT and the approximate specific heat of the flue gases.

Equations (42) and (43) are applicable to elevated sources in neutral stability for a 30 minute sampling period.

At Pont-y-Felin

Q = 1.39 gm/sec

$Q_v = 2.43 \, . \, 10^5 \; Nm^3/hr$

ΔT = 90 degK

Hence

$$h_{s_{crit}} = 1.561 \, . \, 10^{-7} \left(\frac{1.39}{C_{crit}} \right)^{3/2} \quad \ldots\ldots\ldots\ldots (44)$$

If the value of C_{crit} is reduced to 0.5 times its original value then $h_{s_{crit}}$ must be increased to $2^{3/2}$ (= 2.83) times its original height.

2.4 Model D. Atmospheric Dispersion Modelling Working Group model

The model described in this section differs in some respects from the three previous models. The basic equation used to describe the Gaussian Plume allows the effects of an elevated inversion or capping stable layer to be incorporated. σ_y and σ_z are specified as continuous functions for all downwind distances. An allowance can be made for the effect of different sampling times by modifying the σ_y values used. An allowance can also be made for the effect of different values of surface roughness by modifying the σ_z values used. Finally the rise of the plume is more completely described by incorporating equations to describe both the transitional and final plume rise.

Because of the increased complexity which arise because of the inclusion of these effects it is not possible to derive a simple equation which directly relates stack height and critical concentration. For a given stack height, stability and wind speed it is necessary to obtain C_{max} by solving the equations for ground level concentration at a large number of downwind distances and select C_{max} by inspection. Subsequently for a given stack height and stability, the value of C_{crit} can be obtained by inspecting the values of C_{max} derived for different wind speeds. The effect that a change of stack height will have on C_{crit} can only be ascertained by repeating all the calculations with the new value of stack height.

A computer programme (called MUCK4) which incorporates the features outlined above has been written to facilitate these calculations.

The equation for the ground level concentrations which allows for multiple reflection of the plume from the ground and from the capping stable layer is

$$C = \frac{Q}{2\pi U \sigma_y \sigma_z} \cdot \exp\left\{\frac{-y^2}{2\sigma_y^2}\right\} \cdot F \quad \ldots\ldots\ldots\ldots (45)$$

where $F = 2\left\{\exp\left\{-\frac{1}{2}\left(\frac{h}{\sigma_z}\right)^2\right\} + \exp\left\{-\frac{1}{2}\left(\frac{2H+h}{\sigma_z}\right)^2\right\} + \exp\left\{-\frac{1}{2}\left(\frac{2H-h}{\sigma_z}\right)^2\right\}\right\}$

h	(metres)	height of the plume above ground
H	(metres)	height of capping stable layer

The values of σ_y as a function of downwind distance x are obtained by using the equations due to McMullen (1975) which describe the Pasquill/Gifford curves. The σ_y value can be corrected for sampling time following the method due to Hanna et al (1977)

$$\frac{\sigma_{y_a}}{\sigma_{y_b}} = \left(\frac{t_a}{t_b}\right)^{0.2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

t_b	(mins)	Sampling time for Pasquill/Gifford curves (t_b = 3 mins)
σ_{y_b}	(metres)	Pasquill/Gifford value of σ_y
t_a	(mins)	New sampling time
σ_{y_a}	(metres)	New value of σ_y for 3 mins $< t_a <$ 1 hour

The values of σ_z are obtained by using the equations due to Hosker (1974) which describe the curves due to Smith (1972). These incorporate a correction for different roughness lengths (z_0).

The plume rise equations used are those due to Briggs (1969). For the final plume rise equation (6) is used. For the initial plume rise when $x < 3x_*$ the plume rise is given by

$$\Delta h = 3{\cdot}2844 \; Q_H^{1/3} \; x^{2/3} \; U^{-1} \qquad \ldots\ldots\ldots\ldots\ldots \quad (47)$$

for 305 metres $> h_s \geqslant$ 17 metres

and $Q_H < 20$ MWatts

3. Results of calculations for Pont-y-Felin

Equations have been obtained which relate the variations of the maximum ground level concentration C_{crit} and the stack height $h_{s\,crit}$ for three of the models considered. For model D no such simple equation can be derived.

Model	Plume Rise Equation	Stability Category	Equation
Model A Simple Gaussian	Briggs (1969) for Final Plume Rise	D C	(14) (15)
	Briggs (1970) for Final Plume Rise	D C	(18) (19)
Model B Gaussian Plume	Briggs (1969) for Final Plume Rise	D C	(31) (32)
	Briggs (1970) for Final Plume Rise	D C	(36) (37)
Model C CONCAWE	CONCAWE	D	(44)
Model D ADMWG Model	Briggs (1969) for Initial and Final Plume Rise	D C	-

The results obtained by applying equations (14), (15), (31), (32) and (44) are plotted as curves in figure 1. For Model D the results of the individual calculations of C_{crit} for $h_{s\,crit}$ = 52 and 100 metres for Category D and C have been plotted. The results from equations (18),(19),(36) and (37) are plotted in figure 2.

When considering these results the following points must be noted

(1) The equations for Model A are only applicable when $\frac{\sigma_y}{\sigma_z}$ is approximately constant. This is only true for Category C and D for a fairly limited range of downwind distance. The values of $\frac{\sigma_y}{\sigma_z}$ were chosen to apply between 0.5 km and 1.0 km from the stack. For other distances and other categories the agreement between this model and the other models is expected to be poorer.

(2) The calculated maximum ground level concentration for all the different models for a stack height of 52 metres shows a variation between 1.09 and 3.06 $\mu g/m^3$ for category D. This is approximately a factor of 3.

(3) The category C and D curves for Model A are quite similar. This arises from the fairly small difference between the values of $\frac{\sigma_y}{\sigma_z}$ used. (ie 2.0 for category D and 1.7 for category C.)

(4) The category C curve for Model B lies quite close to the category C curve for Model A. This arises because the value of α (= 1.967) is very close to α = 2 at which (assuming N = 0.117) equation (28) will degenerate into equation (13). Similarly equation (33) will simplify into equation (17).

(5) The new stack height which would be required to reduce to maximum ground level concentration (C_{crit}) to half the value calculated for the existing stack height of 52 metres shows a very large range of variation. For category D the new stack height varies between 76 and 145 metres.

(6) The results from Model C in particular warrant some attention as the predicted increase in $h_{s_{crit}}$ required for a given proportionate decrease in C_{crit} is much larger when compared with the other models. Model C is the only model that is said to apply to elevated sources. The sampling time is 30 minutes as against 3 minutes for the other models so that the assumed values of σ_y would be expected to be larger. In fact the assumed values of $\frac{\sigma_z}{\sigma_y}$ for category D is 0.7 while the ratio of σ_z/σ_y for the Simple Gaussian model for category D is only 0.59. Hence the effective σ_y values for Model C appear to be smaller rather than larger.

However the important difference between Model C and the other models may arise from the different plume rise formula used. Moore (1968) in a comment on the paper in which Brummage (1968) set out the CONCAWE method makes the point that the formulae for calculating $h_{s_{crit}}$ "... depend very critically on the values of the indices used in the diffusion and plume rise equations ...".

(7) For Model D values of C_{crit} have only been calculated for two values of $h_{s_{crit}}$. These indicate a fairly rapid decrease of C_{crit} as $h_{s_{crit}}$ is increased. This is in agreement with the use of σ_z values from Smith's (1972) curves for a roughness length z_o of 50 cms. The σ_z values at about 1 kilometre downwind from the stack are approximately 1.5 times the corresponding values from the Pasquill/Gifford curves for category D and about 1.2 times for category C. For low stack heights the larger values of σ_z are more efficient in bringing the pollution to the ground but as the stack height increases the increasing dilution associated with the larger values of σ_z leads to a more rapid decrease of C_{crit} .

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FIGURE 1

COMPARISON OF MODELS TO CALCULATE MAXIMUM GROUND LEVEL CONCENTRATION

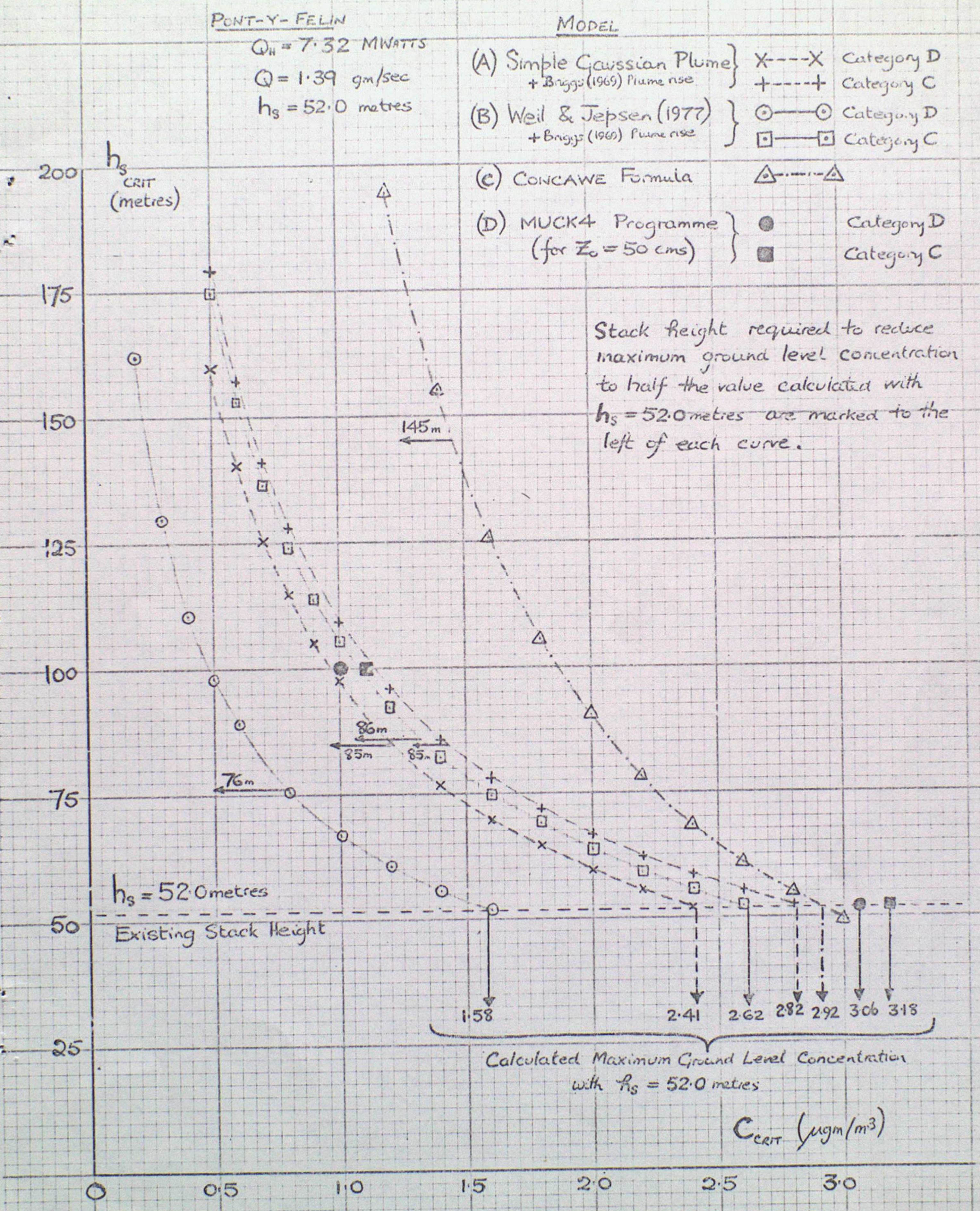

FIGURE 2

COMPARISON OF MODELS TO CALCULATE MAXIMUM GROUND LEVEL CONCENTRATION