Met Office

Hadley Centre Technical Note 78

Conservative interpolation of fluxes between arbitrarily gridded ocean and atmosphere components of a GCM

Date December 2008
Writer Ian Culverwell

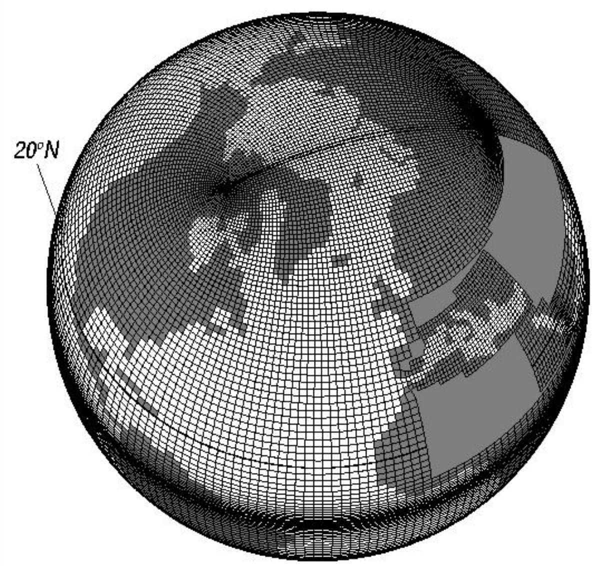
Conservative interpolation of fluxes between arbitrarily gridded ocean and atmosphere components of a GCM

Ian Culverwell
Met Office Hadley Centre
December 2008

Executive Summary

This report summarises the general theory of conservative coupling between the ocean and the atmosphere components of a climate General Circulation Model. It then applies this theory to the particular case of (prototype) HadGEM3-AO, the GCM which will form the basis of the next generation of Met Office Hadley Centre climate and seasonal-to-decadal prediction models. These GCMs comprise a regularly gridded atmospheric model coupled to an ocean model which has a tripolar grid configuration (as in the figure below). In the latter, the usual "meridionally convergent" grid in the Arctic is replaced by a quasi-uniform grid having two poles over the land masses of Canada and Siberia. This device removes the coordinate singularity at the north pole from the ocean, and eliminates the need for the extensive Fourier filtering that is known to have a deleterious effect on the integrity of regularly gridded ocean models. The price to be paid for this is that there is no longer any simple geometric relationship between the ocean grid and the overlying atmospheric grid. This has the consequences for the conservative coupling of fluxes between the two models – consequences which this report seeks to examine.

After some introductory remarks to put the problem in the HadGEM3-AO context, the basic theory is examined in a variety of simple problems at low and high resolution. First and second order interpolation schemes are compared with each other, and with schemes used to conservatively remap fluxes in the earlier (regularly gridded) GCM HadGEM1. Finally, some recommendations for conservative coupling of various fields in HadGEM3-AO are made.

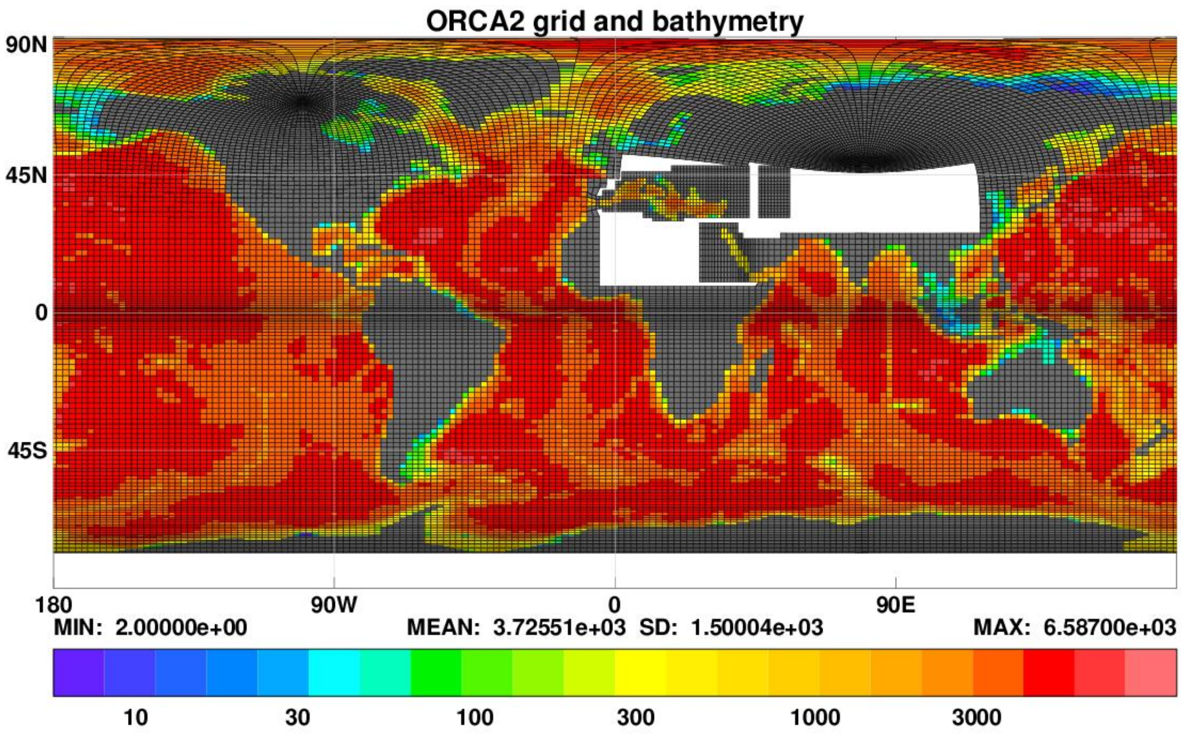


1. Introduction

The conservative interpolation of fluxes between atmosphere and ocean is essential for the integrity and stability of long-term coupled climate model integrations. The problem is more acute for coupled models whose ocean components are discretised on irregular grids, such as the tripolar grids used in some configurations of the NEMO [Madec, 2007] and MOM [Griffies et al, 2004] ocean models, or the "displaced pole" grids sometimes used in the CICE [Hunke et al, 2008] sea-ice model. On such grids, the longitude and latitude of each gridpoint are not separable functions of the *i* and *j* indices of the grid, but are general 2D functions $\lambda_{ij}=f(i,j)$ and $\phi_{ij}=g(i,j)$. (The coordinate lines are usually still orthogonal, however.) This means that identifying the gridcells of one grid that are located near those of the other is not straightforward. It also means that the shape and size of the gridcells can be very different to those of traditional grids, which has consequences for both qualitative and quantitative model analysis and development.

For example, Fig 1a shows the gridcells of the ORCA2 (nominal 2^o resolution) configuration of the NEMO ocean model. Note the many differences from a traditional uniform resolution grid: the global grid refinements in the tropics and the southern hemisphere; the two numerical poles in Canada and Russia; and the local grid refinement in the Mediterranean and Red Seas, and the consequent gaps over Central Asia and the Sahara. Fig 1b shows a close up of this grid around the mouth of the Mediterranean Sea, together with an example overlying atmosphere gridcell. (This is taken from the N48 (3.75^o x 2.5^o) resolution (early) HadGAM1 model, which is a forerunner of the atmospheric component of HadGEM1 [Johns et al, 2005], the Met Office's current climate modelling GCM, to which ORCA2 could be coupled in a low resolution GCM.) Finding which ocean model gridcells overlap this particular atmosphere model gridcell is more difficult than for regular cells, as it now requires a 2D search on the ocean gridcell corners, rather than two 1D searches. And calculating the resulting overlap integrals that are needed to perform the interpolation is much less straightforward than it would be for regular grids, where all such regions are rectangles in latitude-longitude space.

Fig 2a illustrates some other difficulties. It shows the arrangement near the north pole of the gridcells of the prototype HadGEM3-AO coupled model which is the archetypal model used in this study. The ocean component is the ORCA1 (1^o) configuration of NEMO, and the atmosphere component is the N96 (1.875^o x 1.25^o) configuration of HadGAM1. Traditional choices of the interpolation method between two grids depend on which one is considered to be high resolution and which is considered low. Clearly, for a tripolar grid in the high Arctic, the question is a rather open one. It will be discussed in this report. It is also clear from the expanded plot of the ORCA1 gridcells in Fig 2b that they span many degrees of longitude. Any traditional "small $\Delta\lambda$" approximation will likely suffer a large truncation error if applied to these cells. Likewise, any "regular grid" assumption is likely to go wrong near the pole, since the gridcell boundaries are far from being straight lines in latitude-longitude space. Finally, the passing of the ORCA1 grid directly over the north pole is yet another potential mathematical difficulty.[1]



[1] The quasi-regular resolution of the ocean grid over the Arctic Ocean is of course the main reason for using tripolar grids in ocean modelling. If the ocean model grid suffered the convergence of the meridians evident (Fig 2a) in the atmosphere model, then severe Fourier filtering, and possibly the invention of a polar land point, would be needed to handle the coordinate singularity at the north pole.

Fig 1a: Gridcells and bathymetry of the ORCA2 configuration of NEMO ocean model.

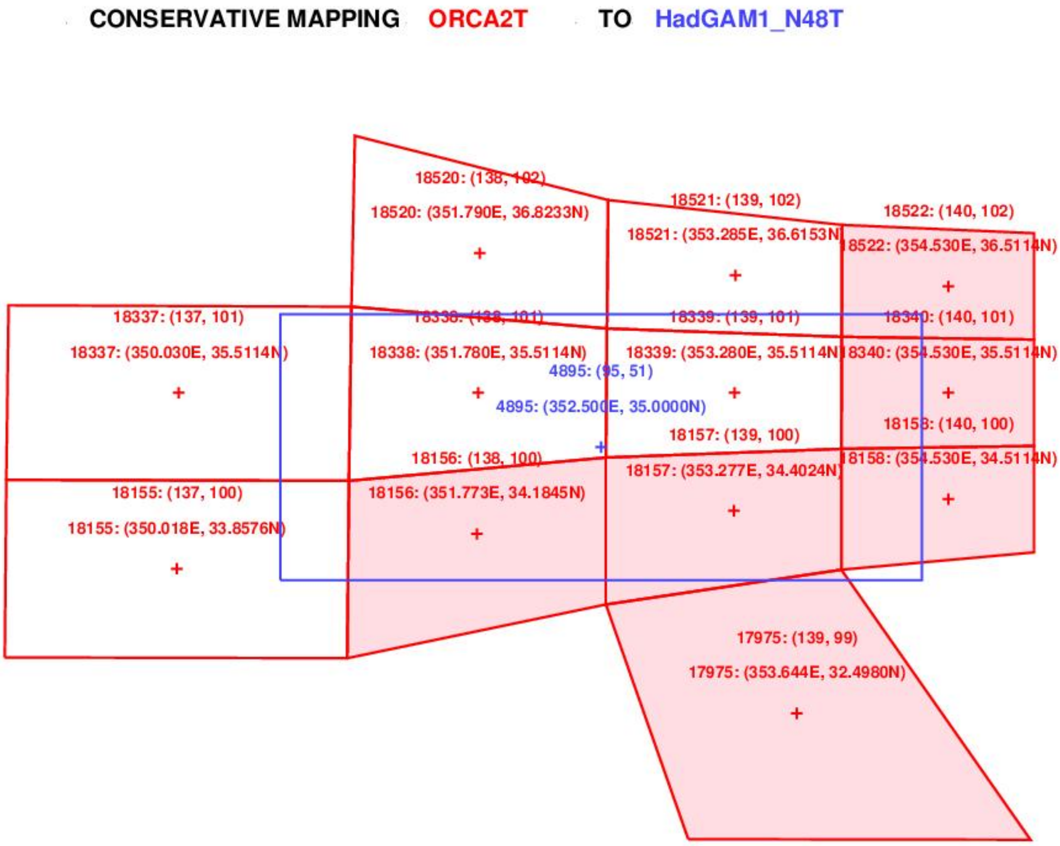


Fig 1b: Example of overlap between ORCA2 and N48 HadGAM1 grids at the mouth of the Mediterranean. (See Fig 1a.) Shaded cells are landpoints of the ocean model.

For all these reasons, the (conservative) interpolation of fields between general grids is considerably more involved than it is when the grids are regular. Fortunately, a tool already exists to interpolate between two general grids on the sphere: the SCRIP interpolation package [Jones, 1999]. This facility is being increasingly widely used as more and more coupled models employ irregular grids. Indeed, it lies at the heart of OASIS [Valcke, 2004], which is the coupler used in HadGEM3-AO. We have access to an offline version of SCRIP, which makes much easier the development and checking of its algorithms, performance, and so on.

The rest of this report falls into two main sections: an idealised discussion of conservative remapping between irregular grids, using various simple examples, and a practical assessment and evaluation of the method when applied to the prototype HadGEM3-AO GCM. Attention is restricted to the SCRIP interpolation methods. The account is intended as a pedagogical guide for modellers who are using SCRIP/OASIS to build coupled climate models, rather than a theoretical analysis of the whys and wherefores of conservative interpolation in GCMs. For this reason, much of the mathematical detail has been relegated to the Appendix.

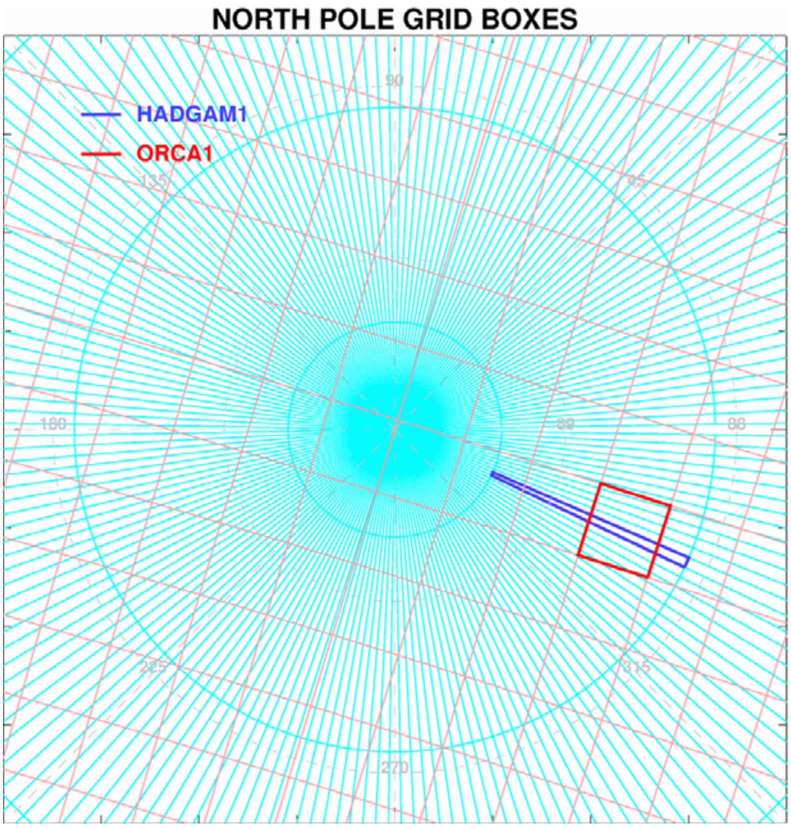


Fig 2a: Example of overlap between ORCA1 and N96 HadGAM1 grids near the north pole.

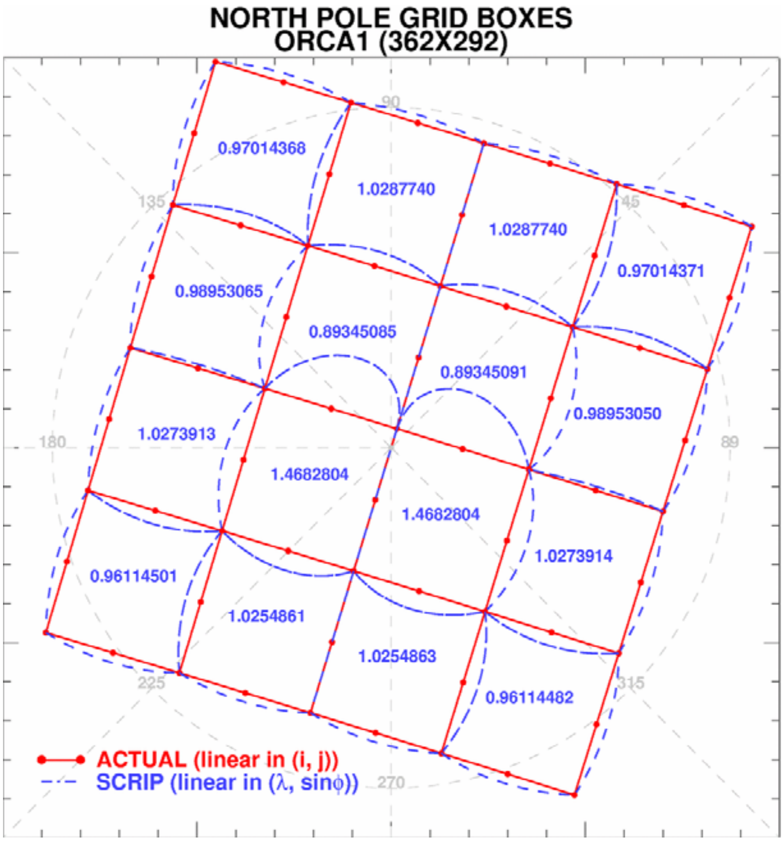


Fig 2b: ORCA1 gridcells very near the north pole, with actual boundaries (red) and those derived by assuming they are straight lines in (λ, sinϕ) space (blue). The numbers are the ratios of the gridcell areas.

2. Theory

Conservative interpolation depends on equating the integrated fluxes *f* and *F* on the source (*g*) and target (*G*) grids. Different assumptions about the form of *f* over the source cells lead to different orders of accuracy in the interpolated field *F*. Piecewise constant *f* gives a first order accurate interpolation; piecewise linear *f* gives second order accurate interpolation. In either case, the set of fluxes on the target grid $\boldsymbol{F}=\{F_I\}$ is expressed as a linear transformation of the fluxes on the source grid $\boldsymbol{f}=\{f_i\}$, thus:

$$\mathbf{F} = \mathbf{w}^1\mathbf{f} + \mathbf{w}^2\, \mathbf{df}/\mathbf{d}\varphi + \mathbf{w}^3\, \mathbf{df}/\mathbf{d}\lambda \qquad (1)$$

where (importantly) the **remapping matrices** $\boldsymbol{w}^1$, $\boldsymbol{w}^2$ and $\boldsymbol{w}^3$ depend only on the geometry of the two grids, not on the fluxes *f*, whose contribution is explicit in (1). SCRIP can (and does) therefore calculate and store these remapping coefficients once for each (source grid, target grid) pair. Interpolation is then effected by the simple matrix multiplication in (1).

For first order conservative remapping, otherwise known as area averaging, $\boldsymbol{w^2}$ and $\boldsymbol{w^3}$ are null.

Jones (1999) gives formulae for the remapping matrices, which are calculated by making a Taylor expansion of the source cell flux about its centroid. Details of the theory, and some deductions from it, are discussed in Appendix A.

There follow some simple examples of the principles and practice of conservative interpolation.

2.1 1D toy problem

Consider the following example, where quadratically varying fluxes, $f(\lambda) = 144\lambda^2$, on a 3x1 atmosphere grid are mapped to a 4x1 ocean grid (assuming domain $0 \quad \lambda \quad 1$, ϕ=0):

Atmosphere grid

f_1 = 4	**f_2 = 36**	**f_3 = 100**

↓

Ocean grid

F_1 = ?	**F_2 = ?**	**F_3 = ?**	**F_4 = ?**

The remapping matrices $\boldsymbol{w^1}$ and $\boldsymbol{w^3}$ for this problem are, according to eqns A.3a-3c of Appendix A:

$$w^1 = \begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \\ 0 & 0 & 1 \end{pmatrix}; w^3 = \begin{pmatrix} -1/24 & 0 & 0 \\ 1/24 & -1/18 & 0 \\ 0 & 1/18 & -1/24 \\ 0 & 0 & 1/24 \end{pmatrix}$$

Note that:

1. For this 1D problem, $\mathbf{w^2}$ (the multiplier of $\partial f/\partial\phi$) is null.
2. The row sums of $\mathbf{w^1}$ are unity. This follows directly from equation (A.3a). It implies that a constant field would be mapped to a constant field (with the same value of the constant).
3. The column sums of $\mathbf{w^1}$ are all the same. This is a result of the uniform resolution of the source and target grids. The constant is (area source cell)/(area target cell) (=4/3 here).
4. The column sums of $\mathbf{w^3}$ are zero. In general this would also be (non-trivially) true of the column sums of $\mathbf{w^2}$, too. It is only true because the target grid has uniform resolution. It follows in that case from $_{AI}(\mathbf{r}-\mathbf{r}^0)$ dA = $\mathbf{0}$, ie the 2nd order gradient terms vanish upon integrating over an entire source cell.

5. By ignoring $\boldsymbol{w^3}$ (the multiplier of $\partial f/\partial\lambda$), and assuming the given quadratic f = (4, 36, 100), we get (using eqn 1) the 1st order remapped output F = (4, 25 1/3, 57 1/3, 100).
6. By including the contribution from $\boldsymbol{w^3}$, we get the 2nd order accurate remapped output F = (2, 19 1/3, 55 1/3, 110).[2]
7. In both cases we have $\int FdA$ = 140/3, which is, *of course*, equal to $\int fda$ (whether one thinks of f as being constant or varying linearly across each source gridcell).

These, and some other formal results about remapping matrices, are discussed in Appendix A.2.

The results of 1st and 2nd order conservative interpolation are plotted in Fig 3. Also shown is the result of "transa2o", which is the current Met Office Hadley Centre method for interpolating fluxes between regular grids in current GCMs. This uses a predictor-corrector method, in which an initial bilinear approximation to the flux on the target grid is adjusted to ensure conservation of fluxes. Different adjustments are made for positive fields like downward SW (which are scaled) and fields of either sign like P-E (which are shifted). See Appendix A.4 for further information on transa2o.

Such behaviour could in principle be emulated with SCRIP/OASIS[3], but in practice this is likely to be difficult. However, for this toy problem the (single) remapping matrix (using the "E-P" type correction) is easily shown to be (App A.4.1):

$$w^1 = \begin{pmatrix} 114/96 & -18/96 & 0 \\ 26/96 & 82/96 & -12/96 \\ -12/96 & 82/96 & 26/96 \\ 0 & -18/96 & 114/96 \end{pmatrix}.$$

(Note that properties 2 & 3 above are still true.) The result of applying this to f = (4, 36, 100) is F = (-2, 19 1/3, 57 1/3, 112). Once again, $\int FdA$ = 140/3, as required.

Fig 3 shows that 2nd order conservative and transa2o are equally good at respecting the gradients in the input data: much better than 1st order remapping, which smears them out through its area averaging. 2nd order conservative remapping is therefore the best method for "P-E"-type fields. For "SW"-type

---

[2] This uses the exact expression $\partial f/\partial\lambda$ = (48, 144, 240), although in practice the source gradients only need to be 1st order accurate to achieve 2nd order accuracy in the remapped field [Jones, 1999].

[3] At least for "additive" corrections; see App A.4.2.

fields, however, there is a problem: 2nd order interpolation can generate negative values where the source field abruptly falls to zero, because it uses an inaccurate estimate of the gradient (finite where it should be zero). In fact, it can overshoot as well as undershoot, leading in principle to, for example, ice fractions greater than 1 appearing on the other side of the coupler. Flux limiters are one possible means of avoiding this [Dukowicz and Kodis, 1987]. A more pragmatic approach may be to simply "top and tail" data that lie outside physically acceptable bounds, and live with the non-conservation – as long as it is small enough. More will be said about this in Section 3.

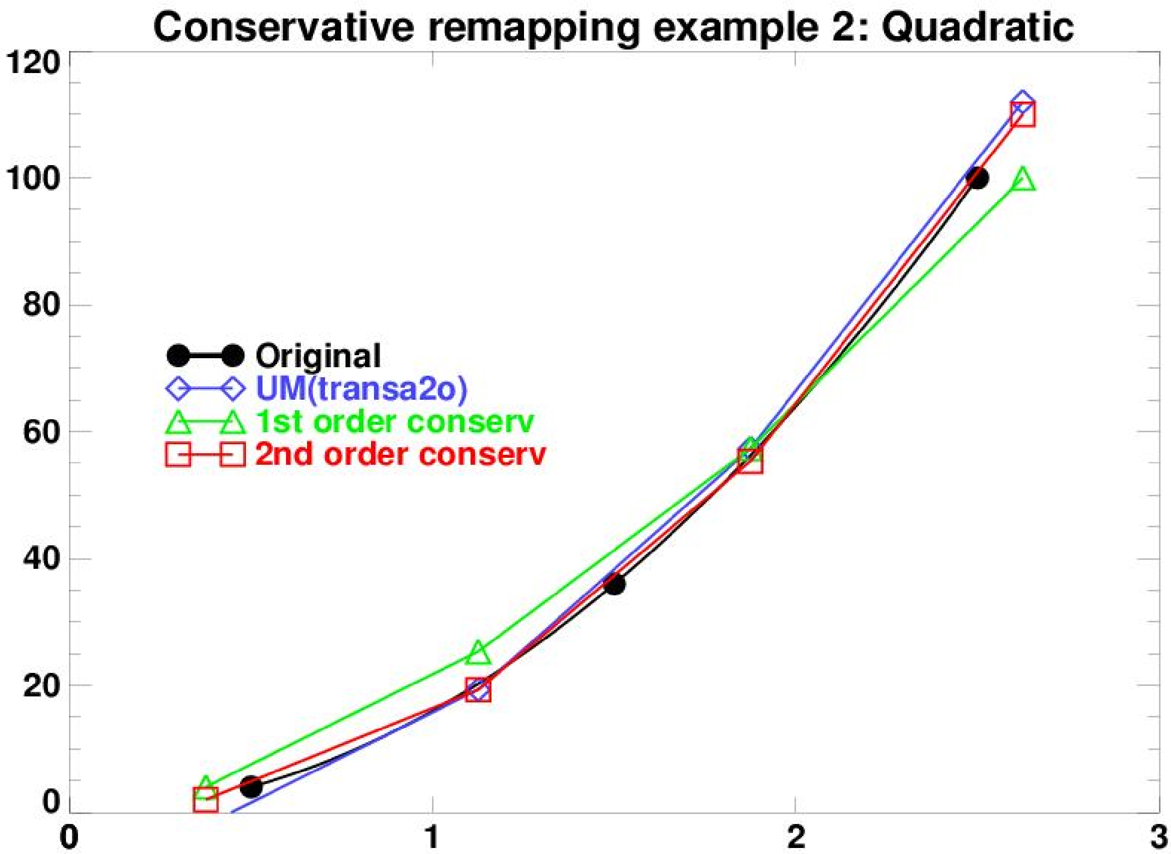


Fig 3: Results of quadratic remapping toy problem.

Another problem with 2nd order remapping occurs in the presence of masked data, and this will be discussed now.

2.2 1D toy problem (masked)

The masking problem is surprisingly thorny, and repays study even in a simple 1D problem. Therefore, consider again the 1D toy problem:

Atmosphere grid

f_1 = mdi	f_2 = 36	f_3 = 100

Ocean grid

F_1 = mdi	F_2 = mdi	F_3 = ?	F_4 = ?

This time, as indicated by the white colours, some of the atmosphere and ocean gridcells are masked. This pattern follows the usual coupled model rule that atmosphere gridcells are only masked out if *all* the underlying ocean gridcells are masked out (ie, are land points in that model).

One technicality: the **sea-fraction** α is defined as the fraction of each atmosphere gridcell which overlays unmasked (ie, sea) points in the ocean model. It is calculated by 1st order conservative interpolation of the ocean mask to the atmosphere grid. In this case, therefore, $\alpha = (0, ½, 1)$.

Atmosphere to ocean and ocean to atmosphere remapping need separate treatment.

2.2.1 3x1 atmos to 4x1 ocean

Under 1st order conservative remapping, $f_A = (0, 36, 100)$ is mapped to $F_O = (0, 24, 57\ 1/3, 100)$. We only conserve fluxes (as Fig 5 in Sec 2.2.2 suggests and Appendix A.3 confirms) if we multiply the source flux f_A by the sea-fraction and the target field by its mask:

$$\alpha f_A\, da = 1/3\ (0 \quad 4 + ½\cdot 36 + 1\cdot 100) = 118/3;$$

$$M_O F_O\, dA = 1/4\ (0 \quad 0 + 0\cdot 24 + 1\cdot 57\ 1/3 + 1\cdot 100) = 118/3.$$

This has an interesting consequence for 2nd order remapping. Conservation in this scheme relies on the fact that the 2nd order correction (the term in $(\mathbf{r}-\mathbf{r}^0).\nabla f^0$ in App A.1) vanishes upon integrating over a complete source cell. Some destination cells will see an excess over the 1st order value arising from this term, but others will see a deficit, so that overall no flux is gained or lost – it's still conservative. But of course if some of the overlying destination cells are masked, then this zero sum budget will not be respected. See Figure 4.

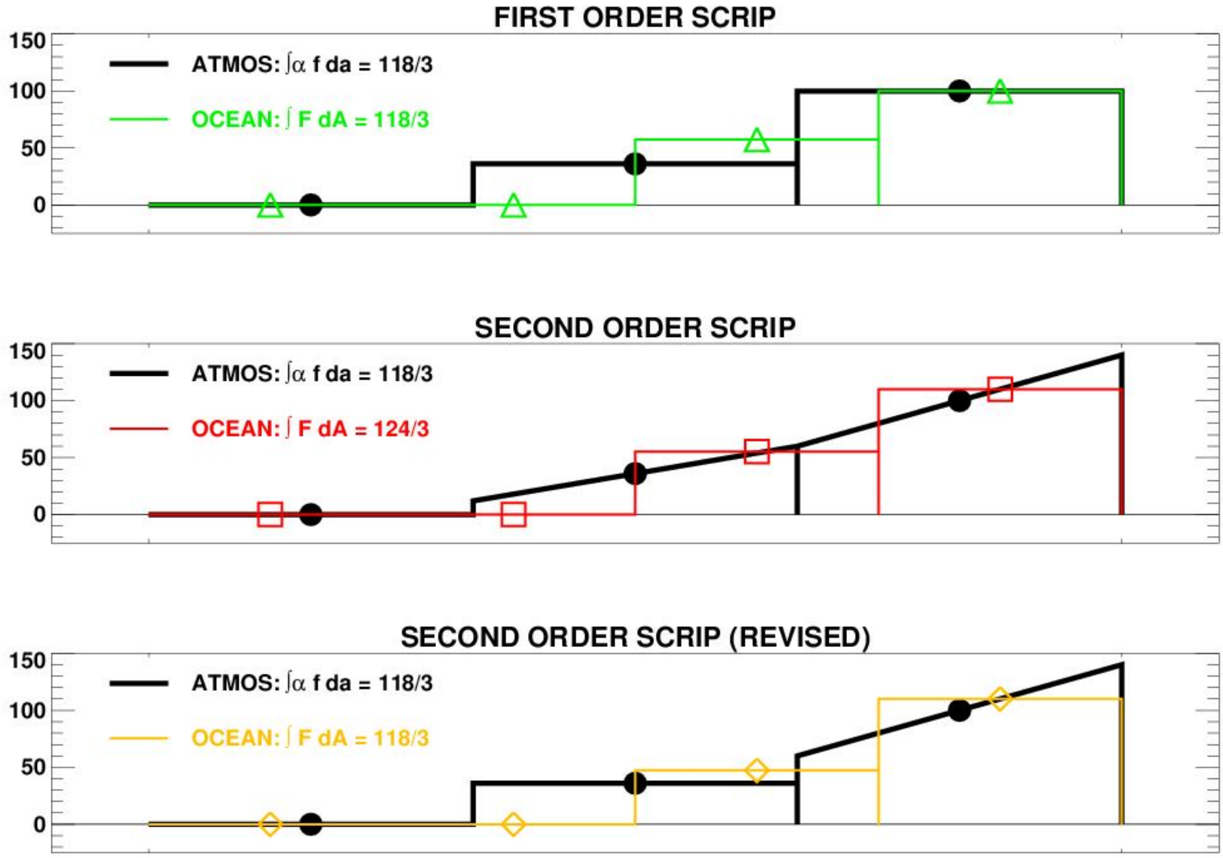


Fig 4: Impact of masking on 1st and 2nd order conservative interpolation. See text for details.

The top panel shows the results of 1st order interpolation, as just discussed, with the assumed piecewise constant source field. The middle panel shows the results of 2nd order interpolation, with the assumed piecewise linear source field. Note that the flux in the 4th ocean gridcell is greater than it is for 1st order interpolation (110 cf 100), because the average of the linearly increasing source flux over this destination cell is greater than the average over the whole of the source cell. (In other words, matrix element $w^3_{4,3} > 0$.) The same is true for the contribution from the 2nd atmosphere gridcell to the 3rd ocean gridcell (ie $w^3_{3,2} > 0$), although this is moderated by the smaller than average contribution from the 3rd atmosphere gridcell (ie $w^3_{3,3} < 0$). Note, however, that the total contribution from atmos gridcell 3 is the same as for 1st order interpolation, because the integral of the linear correction over the whole gridcell is zero (ie $w^3_{4,3} + w^3_{3,3} = 0$). But for atmosphere gridcell 2 this is no longer the case, because its (smaller than average) contribution to ocean gridcell 2 is *not* applied, because the latter is masked out. Therefore, the integrated flux on the ocean grid is *greater* than that on the atmosphere grid (124/3 cf 118/3).

An obvious solution is to revert to 1st order interpolation in the vicinity of coasts by nullifying the 2nd order weights, as illustrated in the bottom panel of Fig 4. To be specific, w^3 (and w^2 in general) must be set to zero for all links (ie non-zero remapping weights) starting on any partially masked source cell. In other words, the whole column must be set to zero if any of its non-zero elements apply to a masked destination cell. When this is done, conservation is restored, at the expense of some degradation in the accuracy of the solution near the coasts.

At the risk of labouring the point:

Source: $\alpha = \begin{pmatrix} 0 \\ 1/2 \\ 1 \end{pmatrix}; f = \begin{pmatrix} 0 \\ 36 \\ 100 \end{pmatrix}; \partial f / \partial x = \begin{pmatrix} 0 \\ 144 \\ 240 \end{pmatrix}; \Rightarrow \int \alpha \, f da = 118/3.$

1st order: $w^1 = \begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow F^{(1)} = w^1 \begin{pmatrix} 0 \\ 36 \\ 100 \end{pmatrix} = \begin{pmatrix} 0 \\ 24 \rightarrow 0 \\ 172/3 \\ 100 \end{pmatrix} \Rightarrow \int F^{(1)} dA = 118/3.$

2nd order:

$$w^3 = \begin{pmatrix} -1/24 & 0 & 0 \\ 1/24 & -1/18 & 0 \\ 0 & 1/18 & -1/24 \\ 0 & 0 & 1/24 \end{pmatrix} \Rightarrow F^{(2)} = w^3 \begin{pmatrix} 0 \\ 144 \\ 240 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \rightarrow 0 \\ -2 \\ 10 \end{pmatrix} \Rightarrow \int F^{(1)} + F^{(2)} dA = 124/3.$$

2nd order (revised, alterations in red):

$$w^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1/24 \\ 0 & 0 & 1/24 \end{pmatrix} \Rightarrow F^{(2)} = w^3 \begin{pmatrix} 0 \\ 144 \\ 240 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -10 \\ 10 \end{pmatrix} \Rightarrow \int F^{(1)} + F^{(2)} dA = 118/3.$$

It may be possible to bypass this difficulty if 2nd order interpolation were used to calculate sea-fractions etc – but this is not past practice, and it could have downstream implications for coastal tiling, ancillary file generation etc. In addition, we may wish to use 1st order interpolation for some fields (eg river runoff). It hardly seems sensible to have *two* sea-fractions, one for each interpolation scheme. For the moment, nullifying the 2nd order weights in the vicinity of coasts seems to be the simplest solution.

Unless otherwise stated, the 2nd order remapping weights used hereafter have been modified in this way.

2.2.2 4x1 ocean to 3x1 atmos

Coastal masking issues do not arise when mapping from ocean to atmosphere (because all masked atmosphere model points are fully covered by masked ocean points), but a complementary problem does: normalisation.

SCRIP allows two normalisation options for A_l in eqn 4:

- DESTAREA, in which A_l is the total area of destination cell l; and
- FRACAREA, in which A_l is the unmasked area of destination cell l.

(In fact, there's a third option, NONE, in which $A_l \equiv 1$, which we will ignore in this report.)
As stated above, the difference only matters when mapping from ocean to atmosphere, since when going from atmosphere to ocean the unmasked destination cells overlay completely unmasked atmosphere cells. But when remapping from ocean to atmosphere, DESTAREA conserves fluxes exactly at the cost of reducing the fields where the destination (ie atmosphere) cells are partially masked (ie, coastal points). FRACAREA gives more "realistic" fields near the coasts, but only conserves fluxes if account is taken of the sea-fraction α, as when remapping in the other direction. Figure 5 should clarify; if not, App A.3 provides some theoretical support.

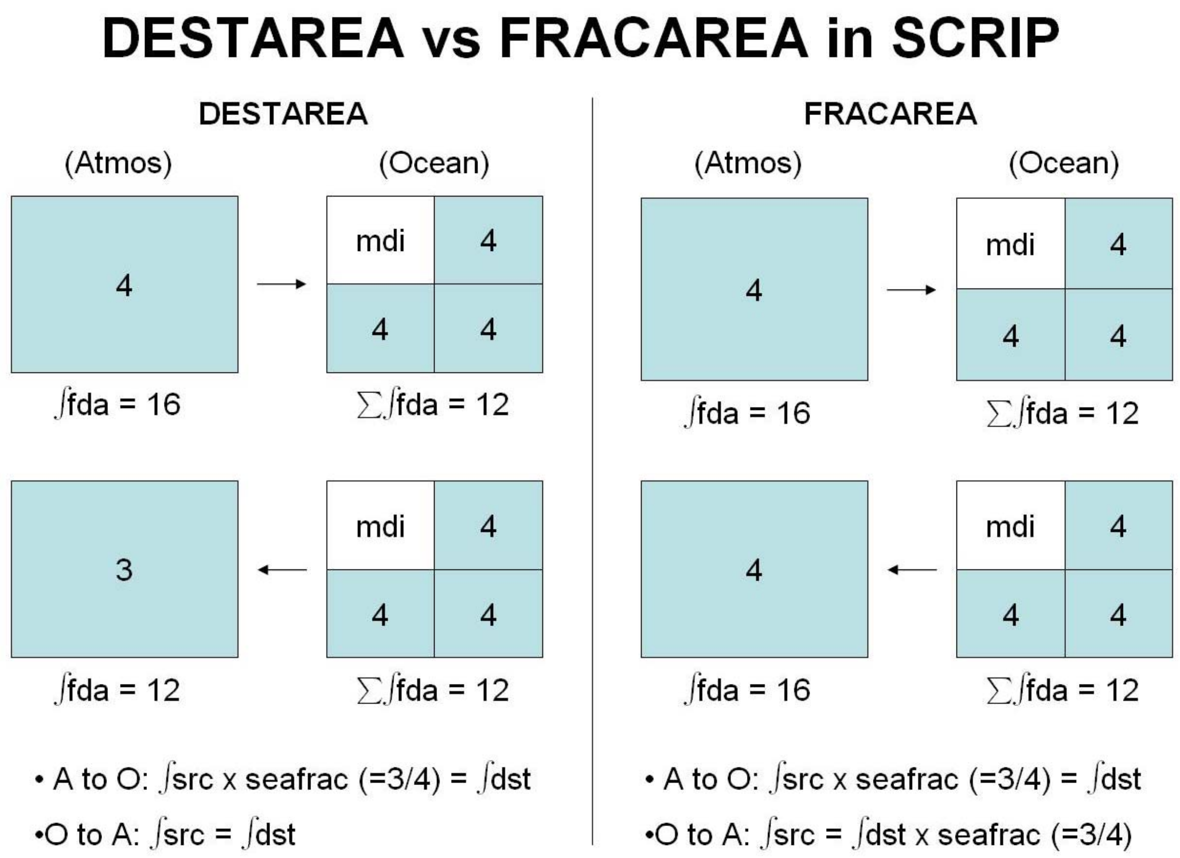


Fig 5: Comparison of DESTAREA and FRACAREA normalisations in SCRIP

FRACAREA, and its consequent dependence on use of the sea-fraction, is closer to what is done in the "transo2a" code used to remap fields (using area-averaging) from ocean to atmosphere in HadGEM1 and 2.

2.3 2D toy problem: 3x4 atmosphere to and from 4x6 ocean

To see how these ideas come together in a slightly more realistic scenario, we consider remapping the simple function $\cos(\lambda+\phi)$ between the following two global grids:

Atmosphere grid

10	11	12
7	8	9
4	5	6
1	2	3

↕

Ocean grid

21	22	23	24
17	18	19	20
13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

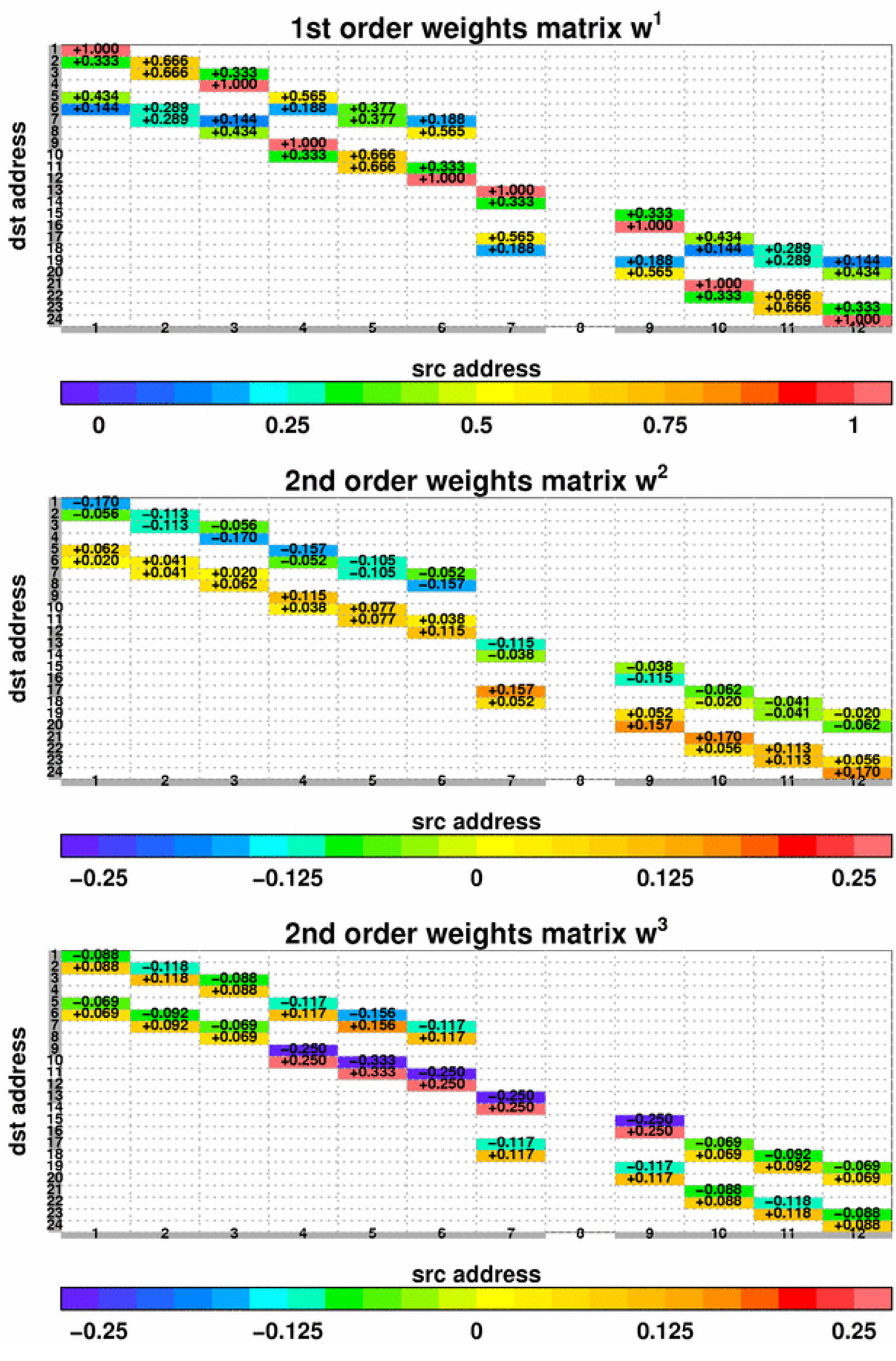
White gridcells denote missing data regions (“continents”).

Again, atmosphere-to-ocean and ocean-to-atmosphere need separate treatment.

2.3.1 3x4 atmosphere to 4x6 ocean

Figure 6 displays the remapping matrices $\mathbf{w^1}$-$\mathbf{w^3}$ for this problem. (Source and destination addresses are labelled above.)

These results tally with eqns A.3a-A.3c of App A. For example, $\mathbf{w^1} \geq 0$, and is large when the destination cell is largely covered by the given source cell (eg $w^1_{11} = 1$). And the super/sub diagonal elements of $\mathbf{w^2}$ are (from eqn A.3b) the differences of ϕ, from its mean value, over the southern/northern parts of the source cell, and are therefore negative/positive so that (if $\partial f/\partial\phi > 0$) the “meridional” second order correction $w^2\ \partial f/\partial\phi$ is negative/positive. Similarly for w^3 in the zonal direction.



1st order weights matrix w^1
dst address
src address
0
0.25
0.5
0.75
1
2nd order weights matrix w^2
dst address
src address
−0.25
−0.125
0
0.125
0.25
2nd order weights matrix w^3
dst address
src address
−0.25
−0.125
0
0.125
0.25

Fig 6: Conservative remapping matrices w^1-w^3 for 3x4 atmosphere to 4x6 ocean problem.

Figure 7 shows the new values of w^2 and w^3 which result from making the coastal masking adjustment described in Sec 2.2.1. (w^1 is of course unaffected.)

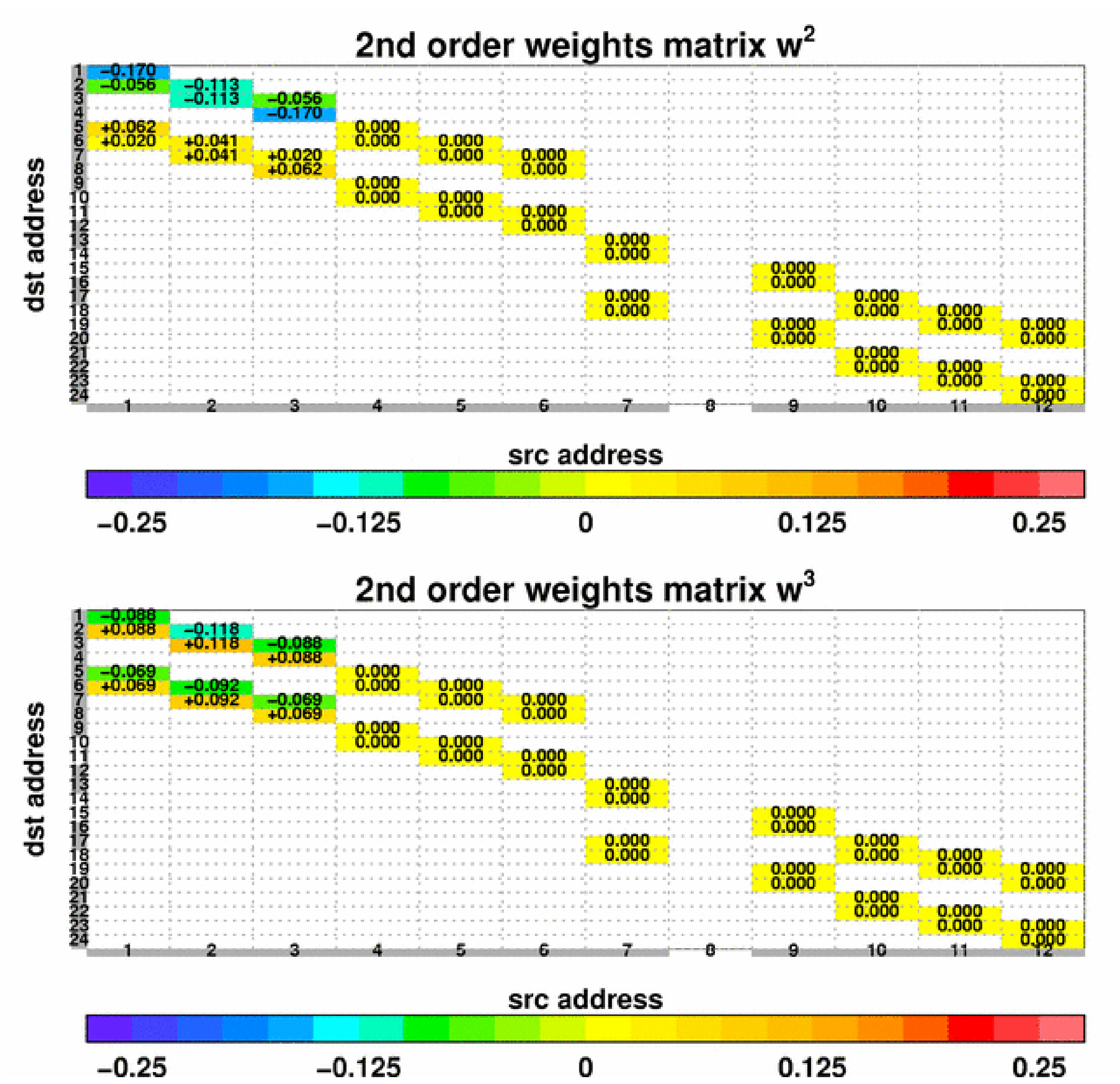
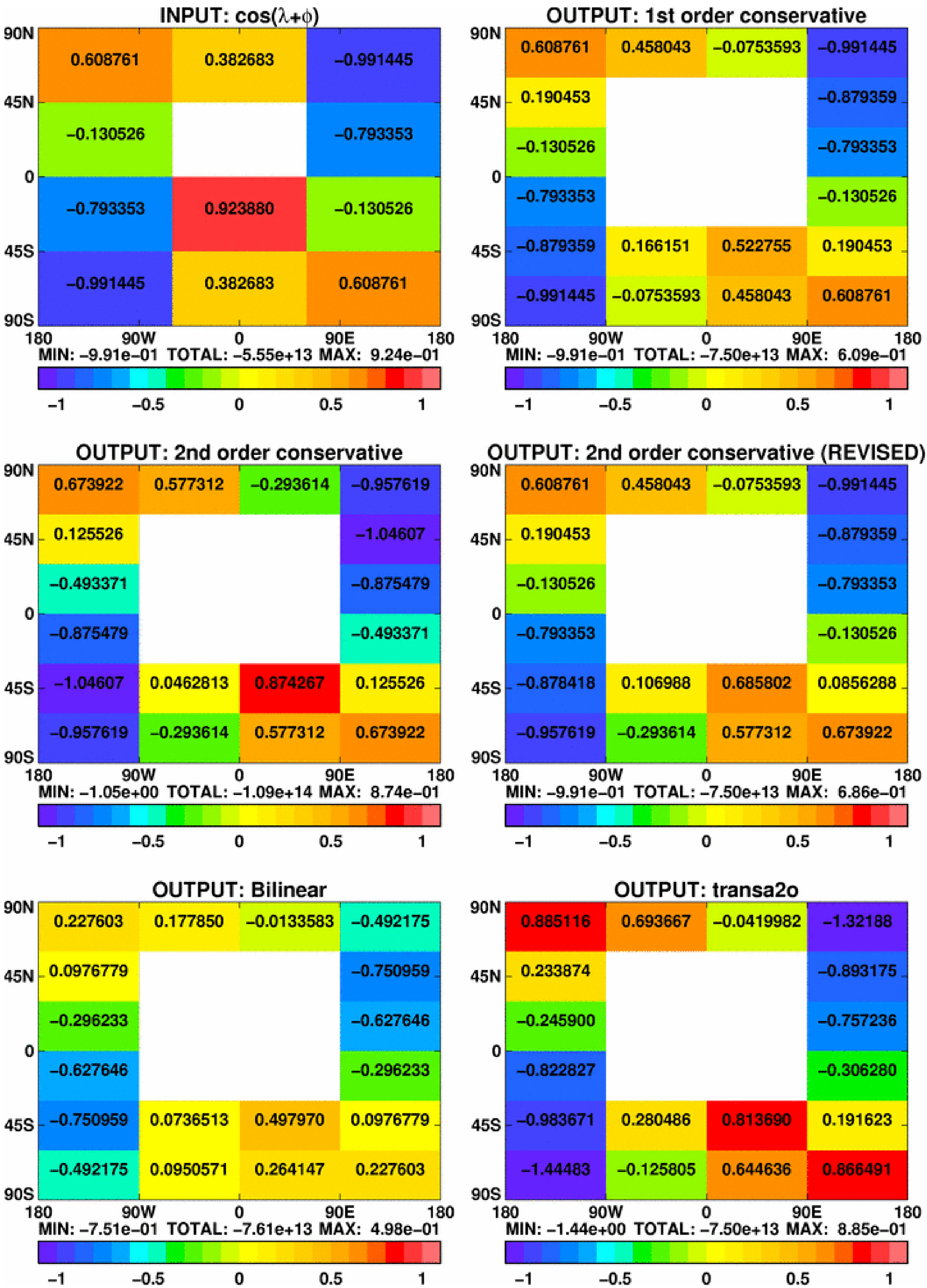


Fig 7: Revised conservative remapping matrices w^2-w^3 for 3x4 atmosphere to 4x6 ocean problem.

The impact of the coastal adjustment is striking: all of the 2nd order weights from source cells 4-12 are nullified. This is because these cells are precisely those that (partially) overlap the masked ocean gridcells. Only those ocean cells that overlap the fully uncovered bottom row of atmosphere cells (ie ocean cells 1-8) retain any 2nd accuracy. In other words, $\mathbf{w^2}$ and $\mathbf{w^3}$ are zero in any columns corresponding to an atmosphere gridcell having a sea-fraction less than unity.

For higher resolution grids, the impact of the coastal masking is less widespread.

What happens when these (and other) interpolation schemes are used to remap the test function $\cos(\lambda+\phi)$ from atmosphere to ocean? The results are shown in Figure 8.



INPUT: cos(λ+ϕ)
0.608761
0.382683
-0.991445
-0.130526
-0.793353
-0.793353
0.923880
-0.130526
-0.991445
0.382683
0.608761
MIN: -9.91e-01 TOTAL: -5.55e+13 MAX: 9.24e-01
OUTPUT: 1st order conservative
0.608761
0.458043
-0.0753593
-0.991445
0.190453
-0.879359
-0.130526
-0.793353
-0.793353
-0.130526
-0.879359
0.166151
0.522755
0.190453
-0.991445
-0.0753593
0.458043
0.608761
MIN: -9.91e-01 TOTAL: -7.50e+13 MAX: 6.09e-01
OUTPUT: 2nd order conservative
0.673922
0.577312
-0.293614
-0.957619
0.125526
-1.04607
-0.493371
-0.875479
-0.875479
-0.493371
-1.04607
0.0462813
0.874267
0.125526
-0.957619
-0.293614
0.577312
0.673922
MIN: -1.05e+00 TOTAL: -1.09e+14 MAX: 8.74e-01
OUTPUT: 2nd order conservative (REVISED)
0.608761
0.458043
-0.0753593
-0.991445
0.190453
-0.879359
-0.130526
-0.793353
-0.793353
-0.130526
-0.878418
0.106988
0.685802
0.0856288
-0.957619
-0.293614
0.577312
0.673922
MIN: -9.91e-01 TOTAL: -7.50e+13 MAX: 6.86e-01
OUTPUT: Bilinear
0.227603
0.177850
-0.0133583
-0.492175
0.0976779
-0.750959
-0.296233
-0.627646
-0.627646
-0.296233
-0.750959
0.0736513
0.497970
0.0976779
-0.492175
0.0950571
0.264147
0.227603
MIN: -7.51e-01 TOTAL: -7.61e+13 MAX: 4.98e-01
OUTPUT: transa2o
0.885116
0.693667
-0.0419982
-1.32188
0.233874
-0.893175
-0.245900
-0.757236
-0.822827
-0.306280
-0.983671
0.280486
0.813690
0.191623
-1.44483
-0.125805
0.644636
0.866491
MIN: -1.44e+00 TOTAL: -7.50e+13 MAX: 8.85e-01
90N
45N
0
45S
90S
180
90W
0
90E
180
-1
-0.5
0
0.5
1

Fig 8: Results of remapping cos($\lambda+\phi$) from 3x4 atmosphere grid to 4x6 ocean grid.

Table 1 lists some statistics of the fields.

Table 1: Statistics of the fields in Fig 8

Grid/method	Min	Total	Max
Atmos	-0.991445	-5.55400e+13	0.923880
Atmos (*seafrac)†	-0.991445	-7.49872e+13	0.608761
Ocean (1st cons)	-0.991445	-7.49872e+13	0.608761
Ocean (2nd cons)	-1.046075	-10.87660e+13	0.874267
Ocean (2nd cons´)	-0.991445	-7.49872e+13	0.685802
Ocean (bilinear)	-0.750960	-7.61081e+13	0.497970
Ocean (transa2o)	-1.444833	-7.49872e+13	0.885116

Table 1: Statistics of the fields shown in Fig 8. Undesirable properties are in red.
† Not shown in Fig 8.

Several points stand out:

- 1st order conservative conserves flux exactly (after multiplying input by sea-fraction).
- 2nd order suffers from overshoots, and it doesn't conserve unless coastal masking is applied (labelled 2nd cons'). It then reduces to 1st order everywhere but the bottom two ocean rows (Fig 7). In this case, this is enough to wipe out the overshoots, but this wouldn't be true in general – they're different issues.
- Bilinear doesn't respect gradients in the source data very well. This is because the correlation scale of the input field is not much larger than the input gridcell size, so bilinear interpolation mixes up very different input values. It doesn't conserve, of course.
- The transa2o algorithm, which starts from the bilinear approximation, does conserve. Because it's using the "P-E" correction, it has generated some overshoots – which are rather larger than those of 2nd order SCRIP.

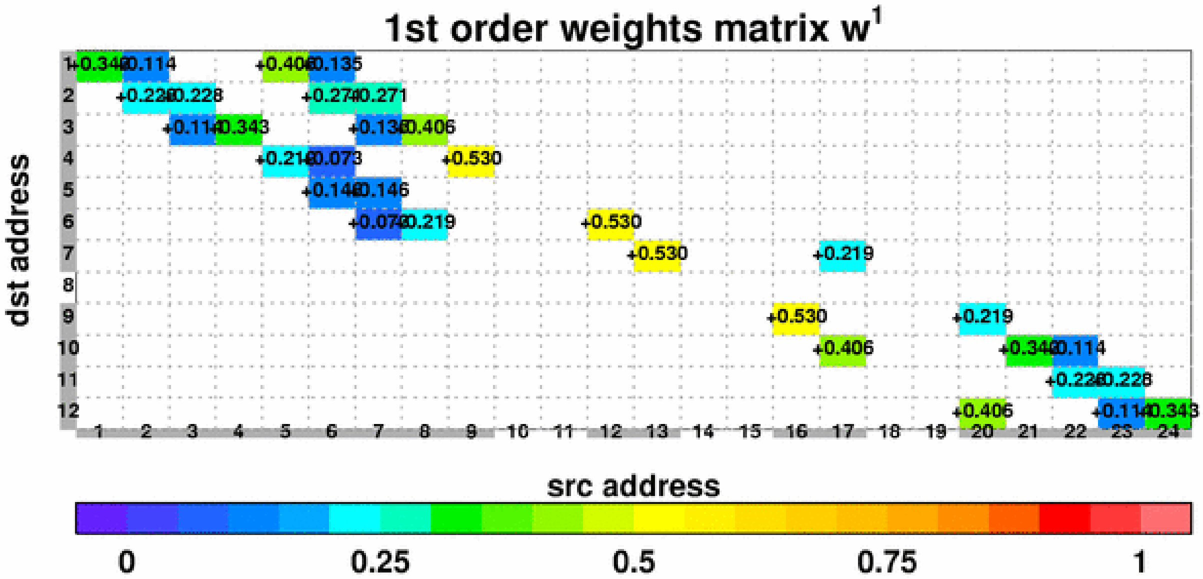
We will see that the same features appear when dealing with high resolution coupled model fields.

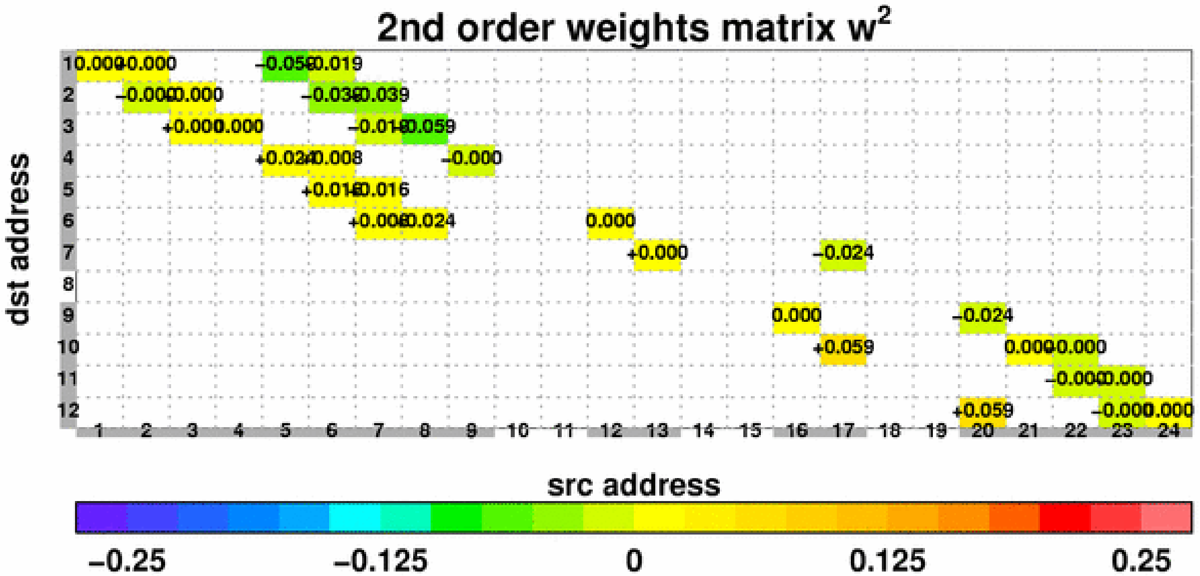
2.3.2: 4x6 ocean to 3x4 atmosphere

We now consider the reverse mapping, from ocean to atmosphere.

The "DESTAREA" remapping matrices for this problem are shown in Figure 9. Note that:

- The 1st order weights are generally smaller than they are for atmosphere to ocean. This is because less of each target gridcell is covered by any single source gridcell – in fact, they're never fully covered, which is why the maximum of $\mathbf{w^1}$ is less than 1.
- There can be no effect of coastal masking since there are no matrix elements connecting the (one) masked output cell to any source cells (because of the relative arrangement of atmosphere and ocean gridcells in coupled models).
- The 2nd order weights $\mathbf{w^2}$ and $\mathbf{w^3}$ have the same patterns as before, but are smaller.


1st order weights matrix w1
dst address
src address
0
0.25
0.5
0.75
1


2nd order weights matrix w2
dst address
src address
−0.25
−0.125
0
0.125
0.25

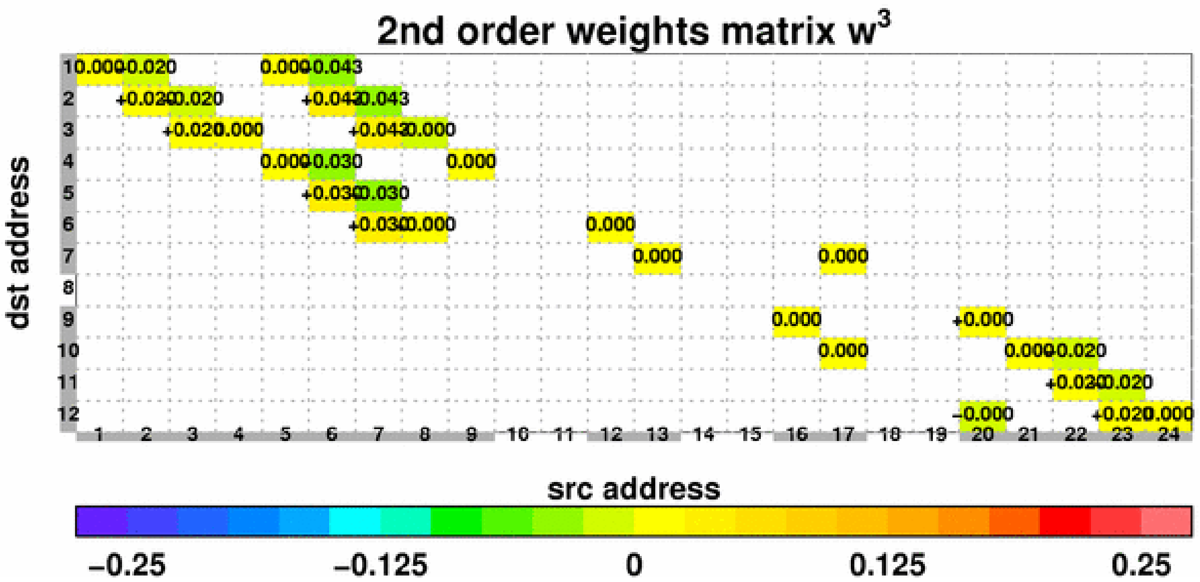

2nd order weights matrix w3
dst address
src address
−0.25
−0.125
0
0.125
0.25

Fig 9: DESTAREA conservative remapping matrices w^1-w^3 for 4x6 ocean to 3x4 atmosphere problem.

Although there is no impact of coastal masking when mapping in this direction, recall that the FRACAREA vs DESTAREA distinction is now important. Hence, Figure 10 shows the “FRACAREA” remapping matrices.

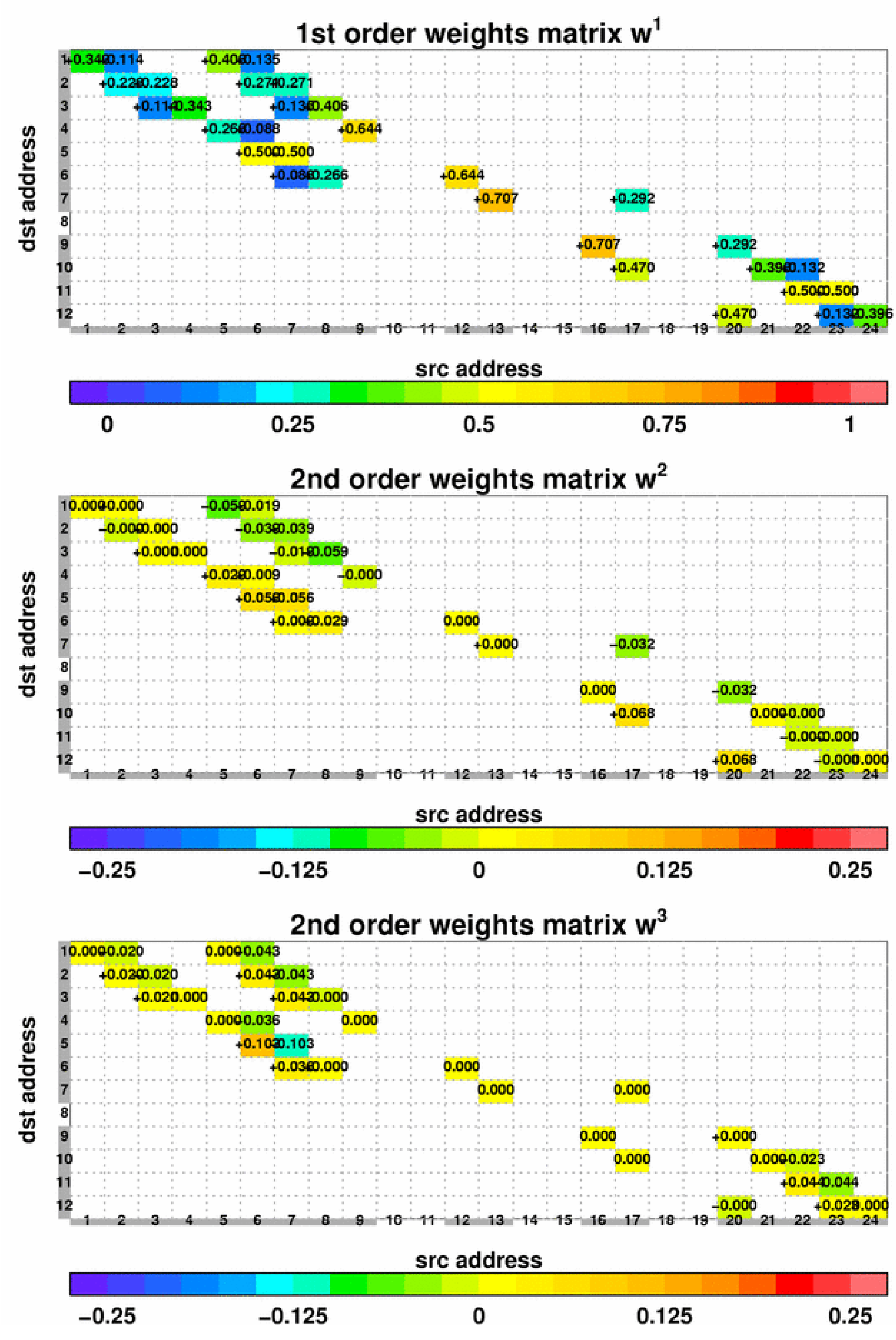


Fig 10: FRACAREA conservative remapping matrices w^1-w^3 for 4x6 ocean to 3x4 atmosphere problem.

This shows that the only effect of using FRACAREA rather than DESTAREA is a rescaling of all matrix elements w^{1-3}_{Ii} associated with a particular destination cell (ie all elements in a given row) by the same amount, namely the reciprocal of the sea-fraction in that cell. This follows from the definition in Sec 2.2.2.

The results of remapping cos($\lambda+\phi$) from 4x6 ocean grid to the 3x4 atmosphere grid are shown in Figure 11, and some statistics are listed in Table 2.

Table 2: Statistics of the fields in Fig 11

Grid/method	Min	Total	Max
Ocean	-1.000000	-1.10440e+14	1.000000
Atmos (bilinear)	-0.839402	-1.12940e+14	0.500000
Atmos (1st cons DEST)	-0.761215	-1.10440e+14	0.406211
Atmos (2nd cons DEST)	-0.842137	-1.10440e+14	0.475907
Atmos (1st cons FRAC)	-0.905265	-1.18790e+14	0.500000
Atmos (1st cons FRAC) *seafrac†	-0.761215	-1.10440e+14	0.406211
Atmos (2nd cons FRAC)	-0.905266	-1.12660e+14	0.660071
Atmos (2nd cons FRAC) * seafrac†	-0.842137	-1.10440e+14	0.475907

Table 2: Statistics of the fields shown in Fig 11. Undesirable properties are in red. † Not shown in Fig 11.

Points of interest:

- Traditionally, the problem of mapping ocean to atmosphere has been considered one of high to low resolution (as it is with regular grids). It has therefore been treated by area-averaging, ie 1st order remapping, or even just bilinear interpolation.
- No overshooting problems arise in this case – probably because the averaging implicit in the remapping to a coarser grid is washing out excesses derived from extrapolating source data.
- FRACAREA uses the same "unmasked normalisation" as we are used to (eg compare the small (<0.2) fluxes assigned to atmos gridcell 5 by the DESTAREA schemes, compared to the 0.5 or greater generated by the FRACAREA scheme). It is therefore probably to be preferred. However, Table 2 (and Fig 5) makes clear, it is only conservative if the resulting atmosphere field is multiplied by the sea-fraction.
- The two (1st and 2nd order) FRACAREA output fields have different total fluxes through them, unlike 1st and 2nd order DESTAREA. This is because the piecewise linear 2nd order corrections to the source flux only integrate

to zero over the entire source cell. In this case of FRACAREA, the parts of the source cell under separate destination cells are normalised in different ways, and so the target grid sees a *weighted* sum of the overlap integrals over the source grid, which therefore does not equal zero. When these weights are undone, by multiplying by the sea-fraction, the 2nd order contributions integrate to zero, and the 1st and 2nd order schemes give the same total flux (which equals that of the input ocean field, and of the 1st and 2nd order DESTAREA schemes).

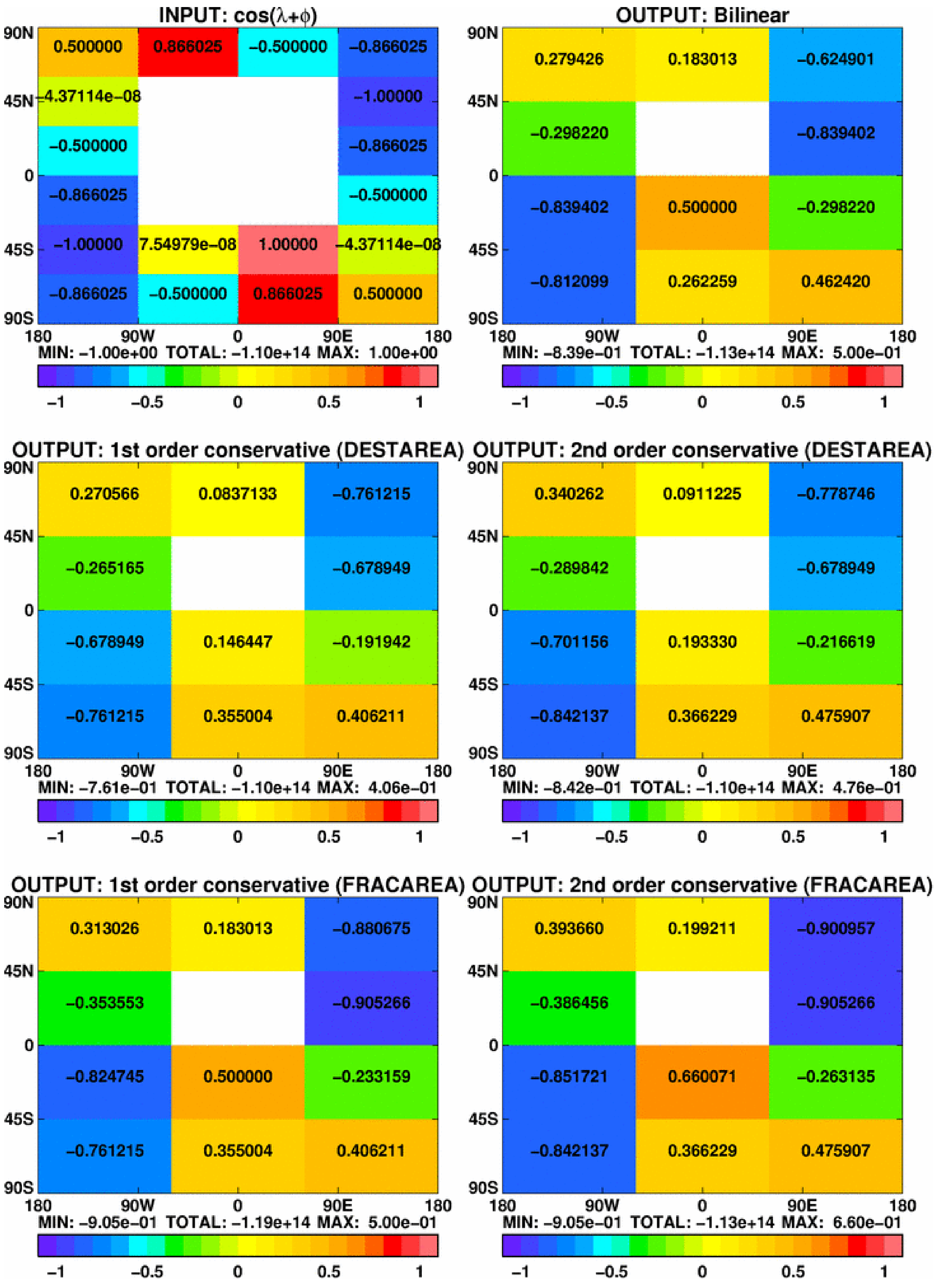

INPUT: cos(λ+φ)
0.500000
0.866025
-0.500000
-0.866025
-4.37114e-08
-1.00000
-0.500000
-0.866025
-0.866025
-0.500000
-1.00000
7.54979e-08
1.00000
-4.37114e-08
-0.866025
-0.500000
0.866025
0.500000
MIN: -1.00e+00 TOTAL: -1.10e+14 MAX: 1.00e+00
OUTPUT: Bilinear
0.279426
0.183013
-0.624901
-0.298220
-0.839402
-0.839402
0.500000
-0.298220
-0.812099
0.262259
0.462420
MIN: -8.39e-01 TOTAL: -1.13e+14 MAX: 5.00e-01
OUTPUT: 1st order conservative (DESTAREA)
0.270566
0.0837133
-0.761215
-0.265165
-0.678949
-0.678949
0.146447
-0.191942
-0.761215
0.355004
0.406211
MIN: -7.61e-01 TOTAL: -1.10e+14 MAX: 4.06e-01
OUTPUT: 2nd order conservative (DESTAREA)
0.340262
0.0911225
-0.778746
-0.289842
-0.678949
-0.701156
0.193330
-0.216619
-0.842137
0.366229
0.475907
MIN: -8.42e-01 TOTAL: -1.10e+14 MAX: 4.76e-01
OUTPUT: 1st order conservative (FRACAREA)
0.313026
0.183013
-0.880675
-0.353553
-0.905266
-0.824745
0.500000
-0.233159
-0.761215
0.355004
0.406211
MIN: -9.05e-01 TOTAL: -1.19e+14 MAX: 5.00e-01
OUTPUT: 2nd order conservative (FRACAREA)
0.393660
0.199211
-0.900957
-0.386456
-0.905266
-0.851721
0.660071
-0.263135
-0.842137
0.366229
0.475907
MIN: -9.05e-01 TOTAL: -1.13e+14 MAX: 6.60e-01
90N
45N
0
45S
90S
180
90W
0
90E
180
-1
-0.5
0
0.5
1

Fig 11: Results of remapping $\cos(\lambda+\phi)$ from 4x6 ocean grid to 3x4 atmosphere grid.

2.4 2D real model grids

We now extend the study of test examples to high resolution atmosphere and ocean grids used in existing GCMs. We use two test functions:

- The "speedbump": $f = \max(0, \sin(2\lambda+2\phi))$. This allows various interpolation schemes' preservation of positivity to be examined.
- The "delta function": $f_{jj} = \delta_{i-i0\ j-j0}$. This is a severe test of the interpolation schemes' handling of gradients in the source.

2.4.1 HadGAM1 to HadGOM1

2.4.1.a Speedbump

We map the speedbump from N96 HadGAM1 to HadGOM1 (the ocean model component of HadGEM1-AO). This ocean grid is regular in latitude and longitude, which means it is susceptible to interpolation by transa2o, so we can compare its results with those of SCRIP.

Fig 12 compares the results of remapping by transa2o, and 1st and 2nd order SCRIP, and of their differences with respect to the exact solution. In all three cases the integrated flux is the same as that of the original field. Overall, transa2o and 2nd order SCRIP are closer to the exact solution (standard deviation of difference ~2e-3) than 1st order SCRIP (sd ~5e-3). 2nd order SCRIP does, however, suffer undershoot near the edge of the speedbump, as can be seen in the Fig 12. If we simply set those negative values to zero, to maintain positivity, the resulting field fails to conserve fluxes by less than 0.01%.

Second order SCRIP overshoots at the high end of the filed range too: its maximum value is 1.0004. First order SCRIP is completely bounded by 0 and 1. Interestingly – and this isn't clear from the picture – transa2o also overshoots at the upper end: its maximum value is 1.001. This doesn't seem to cause problems in practice (eg HadGEM1 or HadGEM2), but if *f* were the sea-ice fraction, it could have severe consequences.

Second order SCRIP is slightly better than transa2o near the coasts.

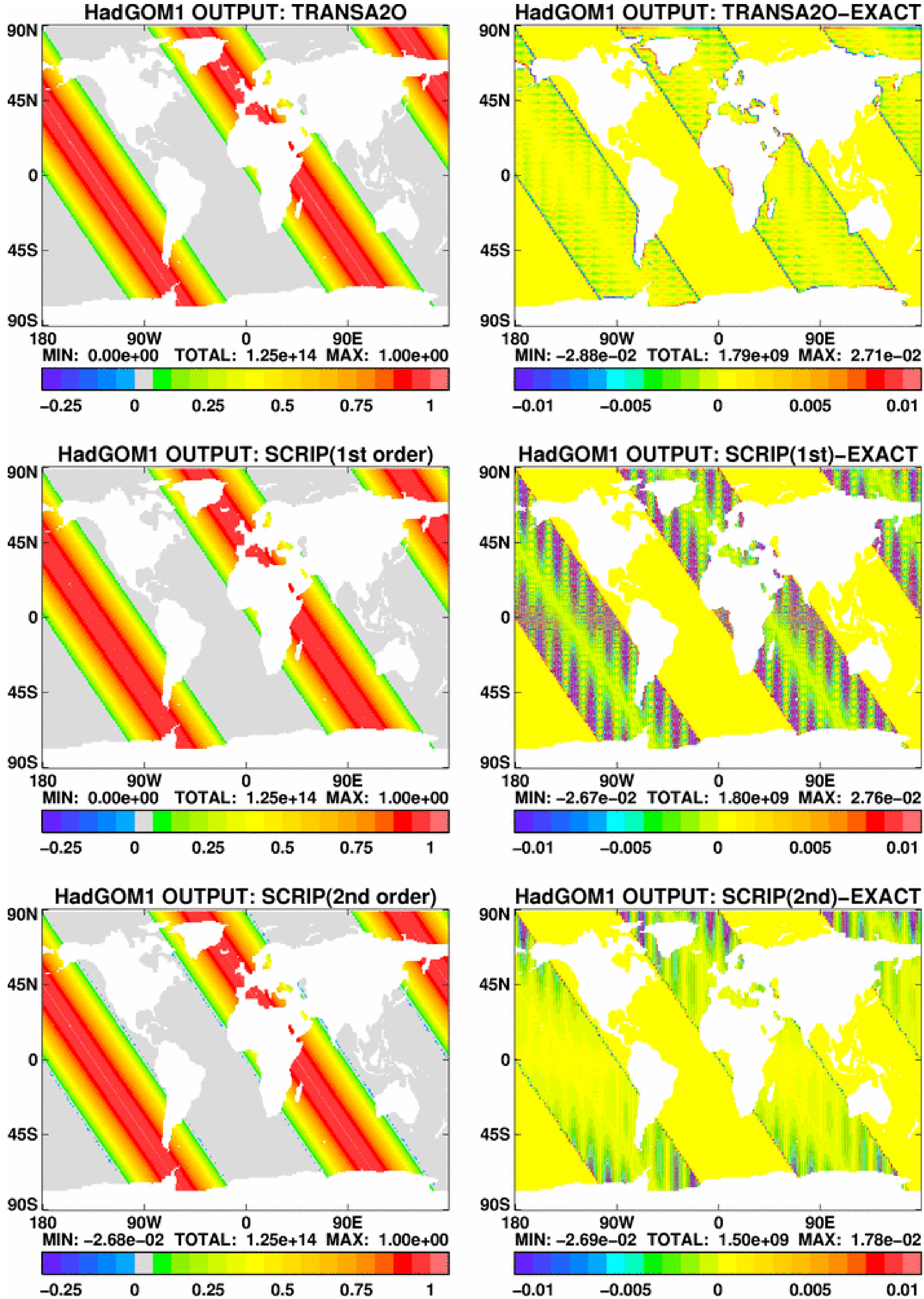

HadGOM1 OUTPUT: TRANSA2O
MIN: 0.00e+00 TOTAL: 1.25e+14 MAX: 1.00e+00
HadGOM1 OUTPUT: TRANSA2O-EXACT
MIN: -2.88e-02 TOTAL: 1.79e+09 MAX: 2.71e-02
HadGOM1 OUTPUT: SCRIP(1st order)
MIN: 0.00e+00 TOTAL: 1.25e+14 MAX: 1.00e+00
HadGOM1 OUTPUT: SCRIP(1st)-EXACT
MIN: -2.67e-02 TOTAL: 1.80e+09 MAX: 2.76e-02
HadGOM1 OUTPUT: SCRIP(2nd order)
MIN: -2.68e-02 TOTAL: 1.25e+14 MAX: 1.00e+00
HadGOM1 OUTPUT: SCRIP(2nd)-EXACT
MIN: -2.69e-02 TOTAL: 1.50e+09 MAX: 1.78e-02
90N
45N
0
45S
90S
180
90W
0
90E
-0.25
0
0.25
0.5
0.75
1
-0.01
-0.005
0
0.005
0.01

Fig 12: Results of remapping max(0,sin($2\lambda+2\phi$)) from HadGAM1 atmosphere grid to HadGOM1 ocean grid. The results of remapping by transa2o, and 1st and 2nd order SCRIP, and of their differences wrt the exact solution are shown.

2.4.2 HadGAM1 to HadORCA1

2.4.2.a Speedbump

We map the speedbump from N96 HadGAM1 to HadORCA1 (the ocean model component of HadGEM3-AO). This ocean grid is not regular in latitude and longitude, which means that transa2o is not available for interpolation. For comparison, therefore, we show the results of bilinear interpolation (the "predictor" step of transa2o) which is available from SCRIP. We again compare the results of all three schemes with the analytical solution on the irregular grid.

The results are shown in Fig 13. Qualitatively they are similar to the above: 2nd order SCRIP is better than 1st order, but again it undershoots and overshoots. (Bilinear itself doesn't overshoot: it's the corrections applied to ensure conservation that cause that.)

Fortuitously, bilinear interpolation is very nearly conservative in this case (difference in integrated flux < 0.0006%): this is probably due to the smooth nature of the input field.

Overall, bilinear is probably best here.

2.4.2.b Delta function

A more severe test of the interpolation schemes is provided by a delta function input, in which the source is zero everywhere except for one gridcell, where it is unity. In this case the non-zero source cell is the highlighted blue HadGAM1 gridcell in Fig 2a. The results of interpolating this to the red HadORCA1 grid by bilinear, 1st and 2nd order conservative schemes are shown in Fig 14a.

Bilinear interpolation clearly does the best job here, because the centre of the target cell is close to that of the only non-zero source cell. Both conservative interpolation methods are compromised (highly diluted) by their integration over the many overlapping source cells where the field is zero, as is clear from Fig 2a.

This is rather different to the situation that obtains in the tropics and Southern hemisphere, where both grids are regular. In that case, the ocean gridcells are smaller than the atmosphere ones, often falling entirely within them, which means that conservative interpolation schemes (1st or 2nd order) reflect the "blockiness" of the input delta function, while the bilinear scheme smears out the source cell discontinuity, because it can be one of the four nearest neighbours of many ocean cells. This is apparent from Fig 14b, which shows the results of interpolating a delta function centred on the equator. Conservative interpolation is clearly better here, and, if it is important to maintain positivity, 1st order is to be preferred.

In the high Arctic, however, atmosphere-to-ocean remapping is rather more like the interpolation from *ocean-to-atmosphere* in the regular regions of the grid. (Compare the bottom panel of Fig 14a with Fig 16b, which shows the result of remapping a tropically situated delta function in this sense.) At very high latitudes, then, it appears that the *atmosphere* model can be considered to have the fine mesh.

Overall, bilinear is best here.

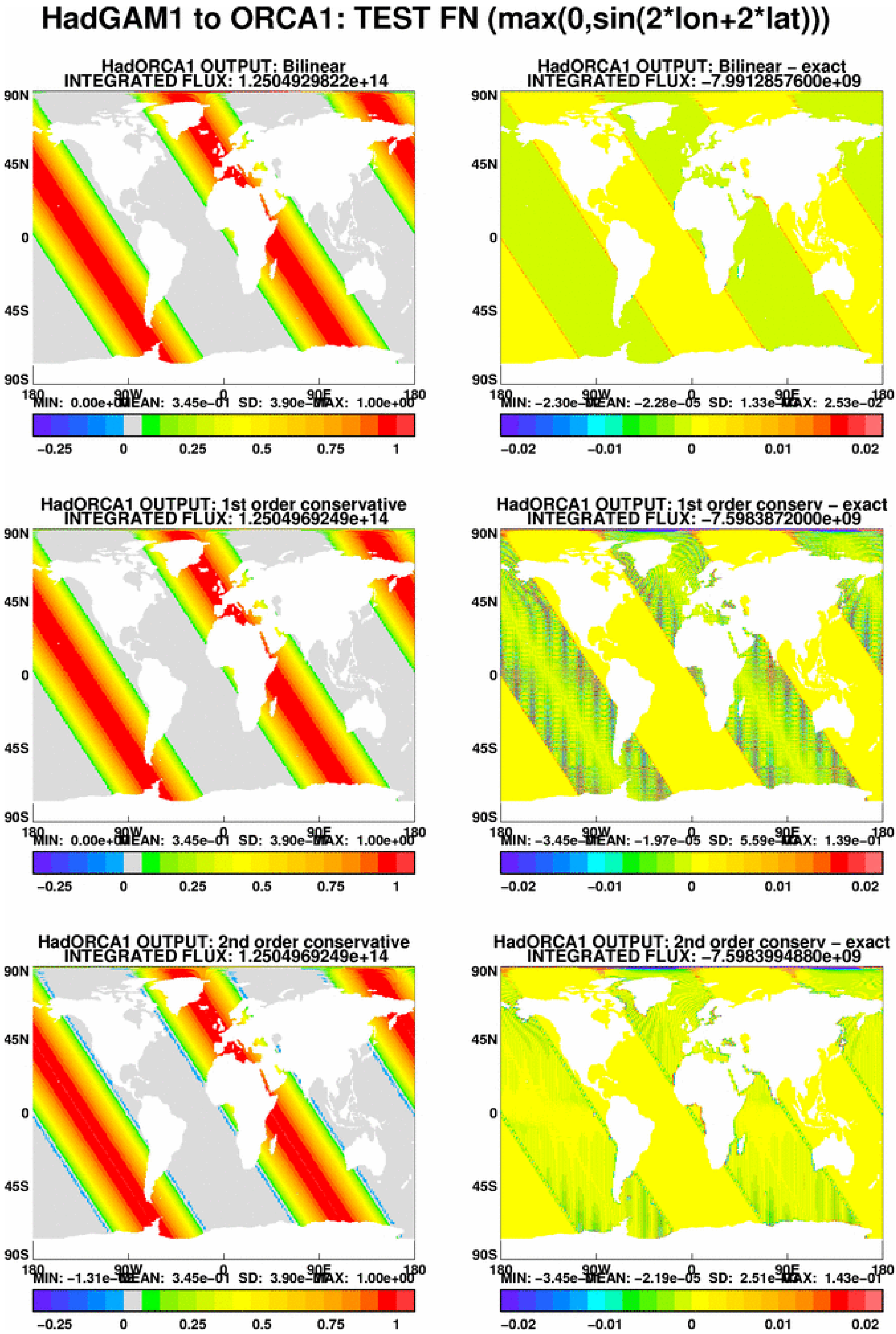

HadGAM1 to ORCA1: TEST FN (max(0,sin(2*lon+2*lat)))
HadORCA1 OUTPUT: Bilinear
INTEGRATED FLUX: 1.2504929822e+14
HadORCA1 OUTPUT: Bilinear – exact
INTEGRATED FLUX: –7.9912857600e+09
HadORCA1 OUTPUT: 1st order conservative
INTEGRATED FLUX: 1.2504969249e+14
HadORCA1 OUTPUT: 1st order conserv – exact
INTEGRATED FLUX: –7.5983872000e+09
HadORCA1 OUTPUT: 2nd order conservative
INTEGRATED FLUX: 1.2504969249e+14
HadORCA1 OUTPUT: 2nd order conserv – exact
INTEGRATED FLUX: –7.5983994880e+09
90N
45N
0
45S
90S
180
90W
90E
–0.25
0.25
0.5
0.75
1
–0.02
–0.01
0.01
0.02

Fig 13: Results of remapping max(0,sin(2λ+2ϕ)) from HadGAM1 atmosphere grid to HadORCA1 ocean grid. The results of remapping by bilinear, and 1st and 2nd order SCRIP, and of their differences wrt the exact solution, are shown.

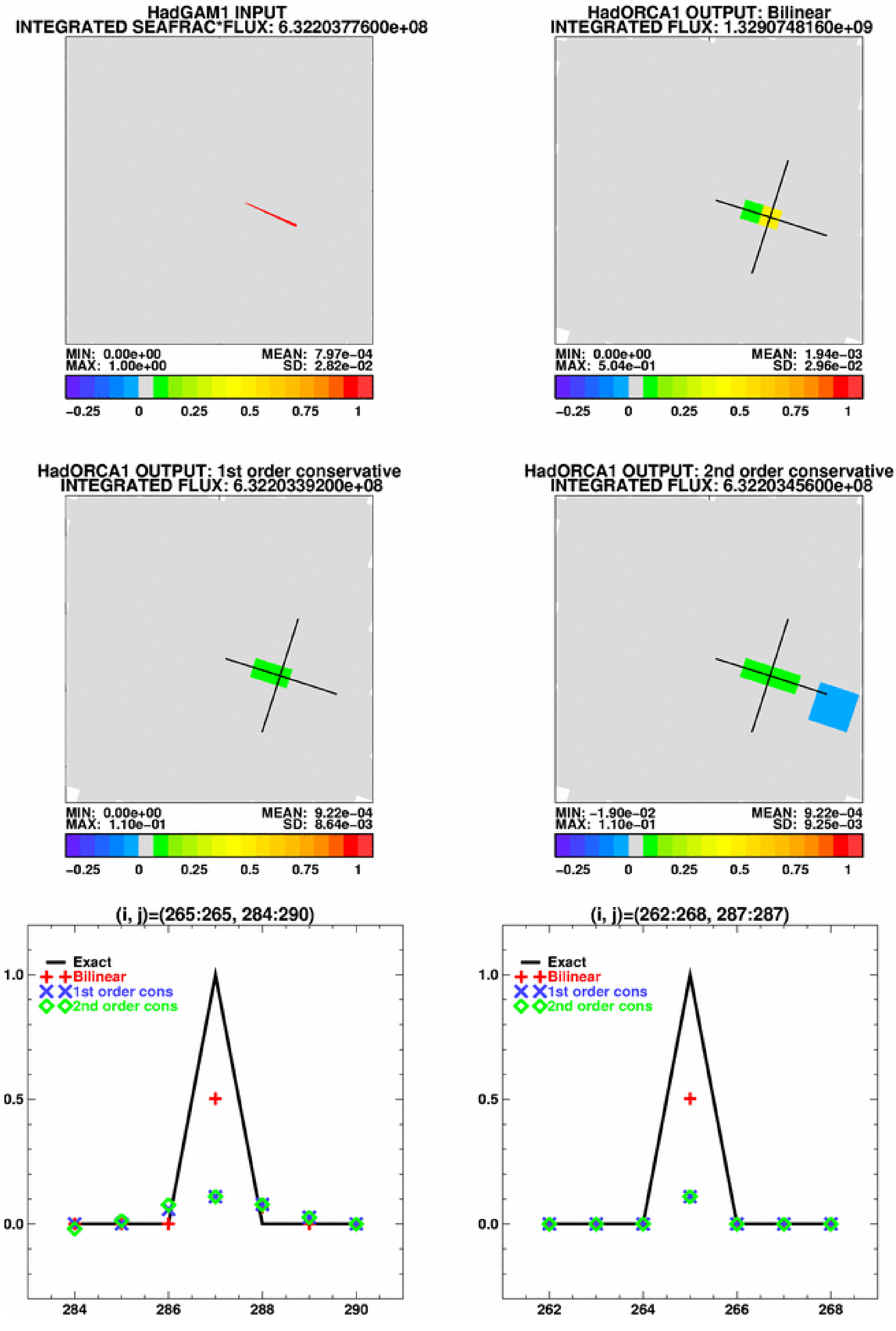
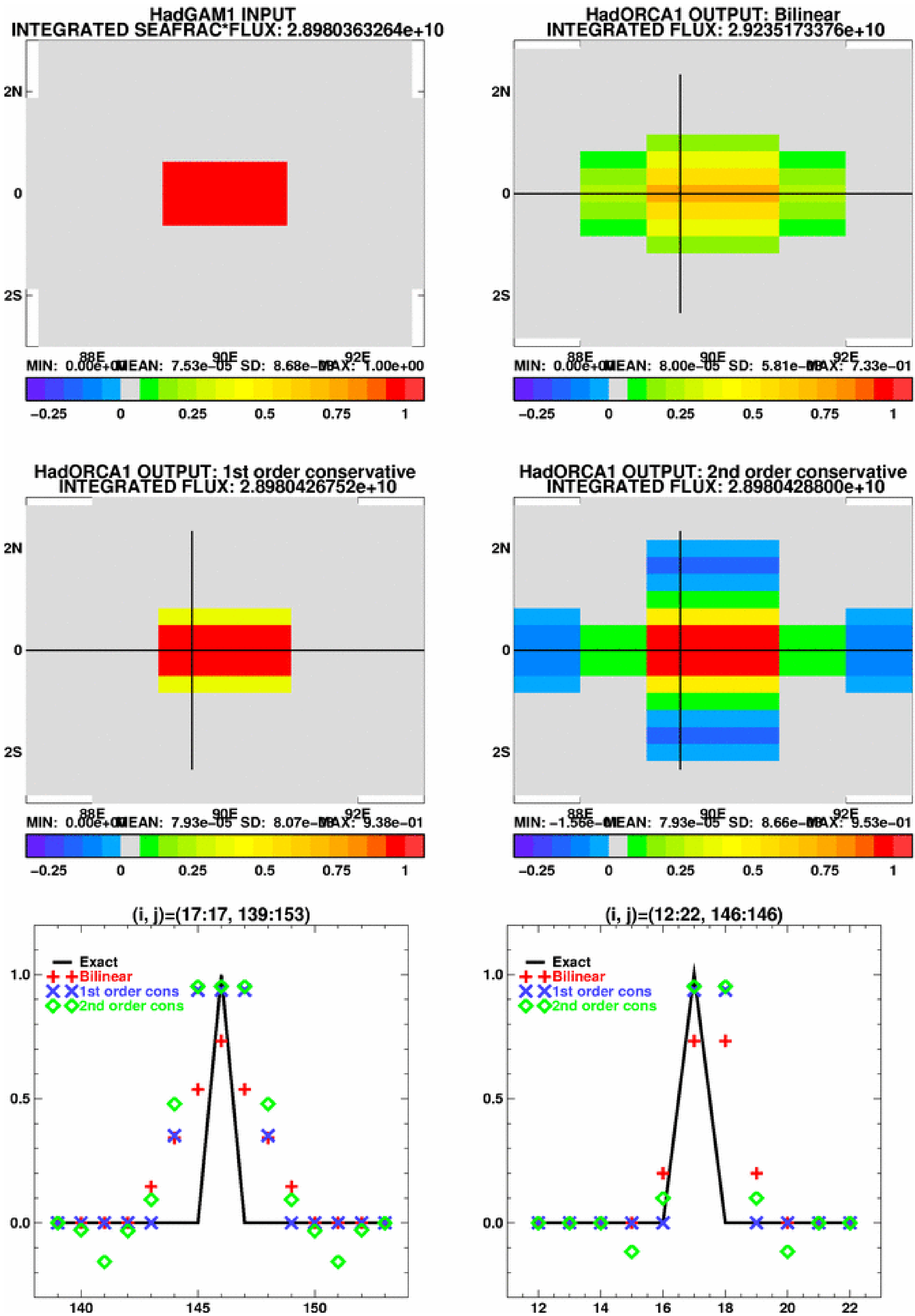

HadGAM1 INPUT
INTEGRATED SEAFRAC*FLUX: 6.3220377600e+08
MIN: 0.00e+00
MAX: 1.00e+00
MEAN: 7.97e-04
SD: 2.82e-02
-0.25 0 0.25 0.5 0.75 1
HadORCA1 OUTPUT: Bilinear
INTEGRATED FLUX: 1.3290748160e+09
MIN: 0.00e+00
MAX: 5.04e-01
MEAN: 1.94e-03
SD: 2.96e-02
-0.25 0 0.25 0.5 0.75 1
HadORCA1 OUTPUT: 1st order conservative
INTEGRATED FLUX: 6.3220339200e+08
MIN: 0.00e+00
MAX: 1.10e-01
MEAN: 9.22e-04
SD: 8.64e-03
-0.25 0 0.25 0.5 0.75 1
HadORCA1 OUTPUT: 2nd order conservative
INTEGRATED FLUX: 6.3220345600e+08
MIN: -1.90e-02
MAX: 1.10e-01
MEAN: 9.22e-04
SD: 9.25e-03
-0.25 0 0.25 0.5 0.75 1
(i, j)=(265:265, 284:290)
Exact
Bilinear
1st order cons
2nd order cons
1.0
0.5
0.0
284 286 288 290
(i, j)=(262:268, 287:287)
Exact
Bilinear
1st order cons
2nd order cons
1.0
0.5
0.0
262 264 266 268

Fig 14a: Maps and cross-sections of the results of interpolating a delta function source centred on the dark blue HadGAM1 gridcell of Fig 2a. (The “exact” solution in the cross-sections is a nominal value.)



HadGAM1 INPUT
INTEGRATED SEAFRAC*FLUX: 2.8980363264e+10
HadORCA1 OUTPUT: Bilinear
INTEGRATED FLUX: 2.9235173376e+10
HadORCA1 OUTPUT: 1st order conservative
INTEGRATED FLUX: 2.8980426752e+10
HadORCA1 OUTPUT: 2nd order conservative
INTEGRATED FLUX: 2.8980428800e+10
(i, j)=(17:17, 139:153)
(i, j)=(12:22, 146:146)
Exact
Bilinear
1st order cons
2nd order cons

Fig 14b: Maps and cross-sections of the results of interpolating a delta function source in a HadGAM1 gridcell at (90E, 0N). (The "exact" solution in the cross-sections is a nominal value.)

2.4.3 HadORCA1 to HadGAM1

2.4.3.a Speedbump

The results of interpolating the speedbump from the HadGEM3 ocean to atmosphere grids are shown in Fig 15[4]. Apart from being smaller in magnitude overall, the errors in the right hand column tell the same story as when going from HadGAM1 to HadORCA1 (Fig 13): bilinear has a small, systematic error, 1st order conservative has a large error, 2nd order conservative has a smaller one, but does not maintain positivity.

Overall, bilinear is best here.

2.4.3.b Delta function

The delta function test is again more instructive. The results of interpolating a delta function source concentrated in the highlighted red ORCA1 gridcell of Fig 2a, are shown in Fig 16a. In the N-S direction, we have the familiar fine-to-coarse remapping, for which bilinear is marginally better. In the E-W direction, however, it's a coarse-to-fine remapping, and bilinear is much too diffusive. This is like the situation when remapping in the reverse direction, from HadGAM1 to HadORCA1, where both grids are regular. (Compare the bottom panel of Fig 16a with Fig 14b, which shows the results of mapping a delta function on the equator from atmosphere to ocean.) In other words, in the E-W direction in the high Arctic, the ocean grid is the coarse one.

Overall, 1st order conservative is probably best here.

Note that for a delta function source in the tropics (see Fig 16b), where the ocean grid is much finer than the atmosphere, 1st and 2nd order conservative remapping are similar, and much worse than bilinear interpolation.

These somewhat inconclusive findings have prompted the study of further idealised tests, which, to avoid trying the reader's patience further, will not be discussed here. The overall conclusion is that they do not strongly favour one interpolation scheme over another. We are therefore led to a more pragmatic study of interpolation, using real fields passed between real, high resolution GCM atmosphere and ocean grids.

---

[4] The derivatives $\partial f/\partial\lambda$ and $\partial f/\partial\varphi$ that are needed to use the 2nd order conservative interpolation scheme (see eqn 1) are calculated by writing $\partial f/\partial i$ and $\partial f/\partial j$ (which are known) in terms of $\partial f/\partial\lambda$ and $\partial f/\partial\lambda$ through a matrix with elements like $\partial\lambda/\partial i$ (which are known), and then inverting. In other words we follow the rules for a covariant transformation between (i,j) and (λ,φ) coordinates.

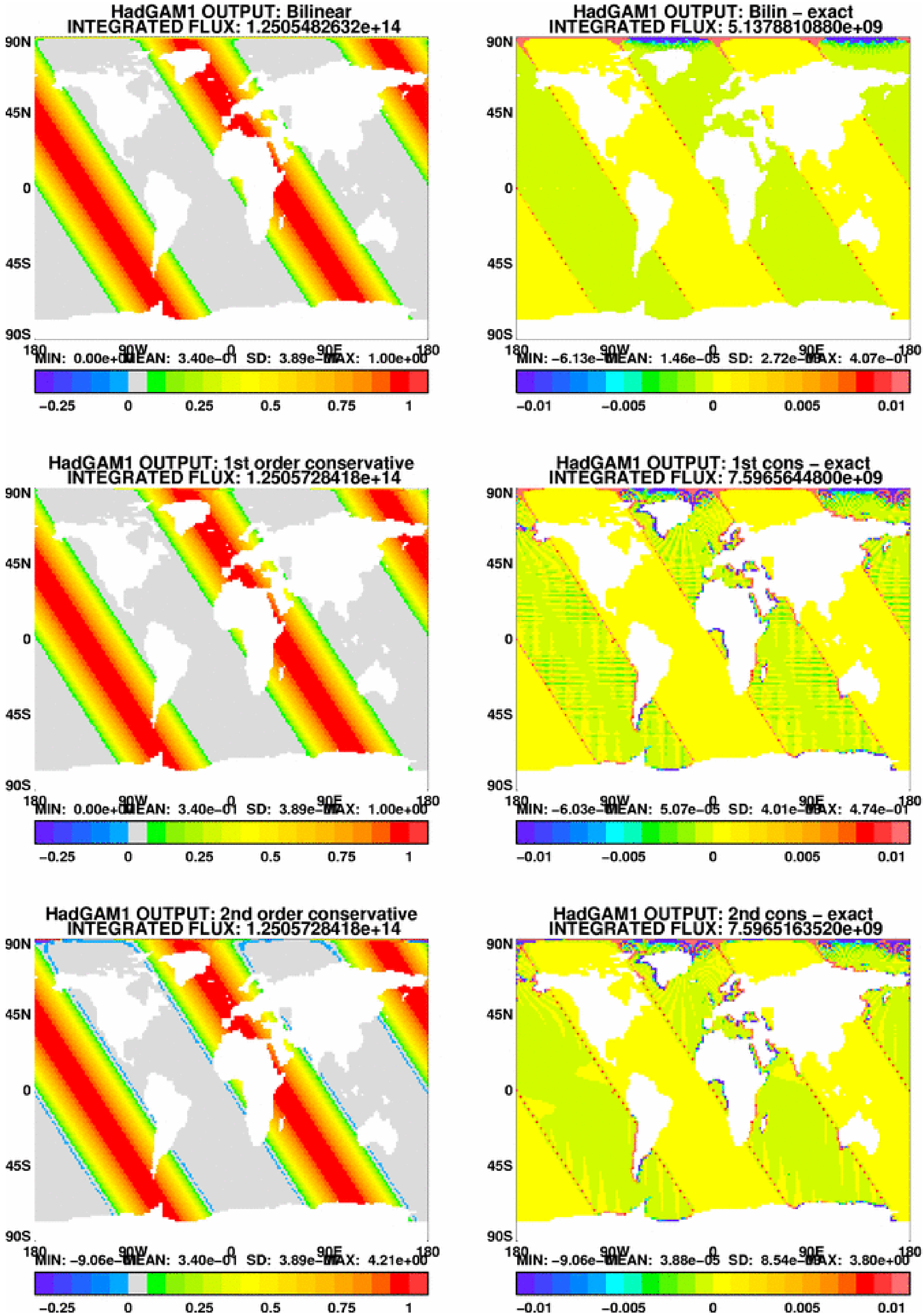

HadGAM1 OUTPUT: Bilinear
INTEGRATED FLUX: 1.2505482632e+14
90N
45N
0
45S
90S
180
90W
0
90E
180
MEAN: 3.40e-01
−0.25
0
0.25
0.5
0.75
1
HadGAM1 OUTPUT: Bilin – exact
INTEGRATED FLUX: 5.1378810880e+09
MEAN: 1.46e-05
−0.01
−0.005
0
0.005
0.01
HadGAM1 OUTPUT: 1st order conservative
INTEGRATED FLUX: 1.2505728418e+14
MEAN: 3.40e-01
HadGAM1 OUTPUT: 1st cons – exact
INTEGRATED FLUX: 7.5965644800e+09
MEAN: 5.07e-05
HadGAM1 OUTPUT: 2nd order conservative
INTEGRATED FLUX: 1.2505728418e+14
MEAN: 3.40e-01
HadGAM1 OUTPUT: 2nd cons – exact
INTEGRATED FLUX: 7.5965163520e+09
MEAN: 3.88e-05

Fig 15: Results of remapping max(0,sin(2λ+2ϕ)) from HadORCA1 ocean grid to HadGAM1 atmosphere grid. The results of remapping by bilinear, and 1st and 2nd order SCRIP, and of their differences wrt the exact solution are shown.

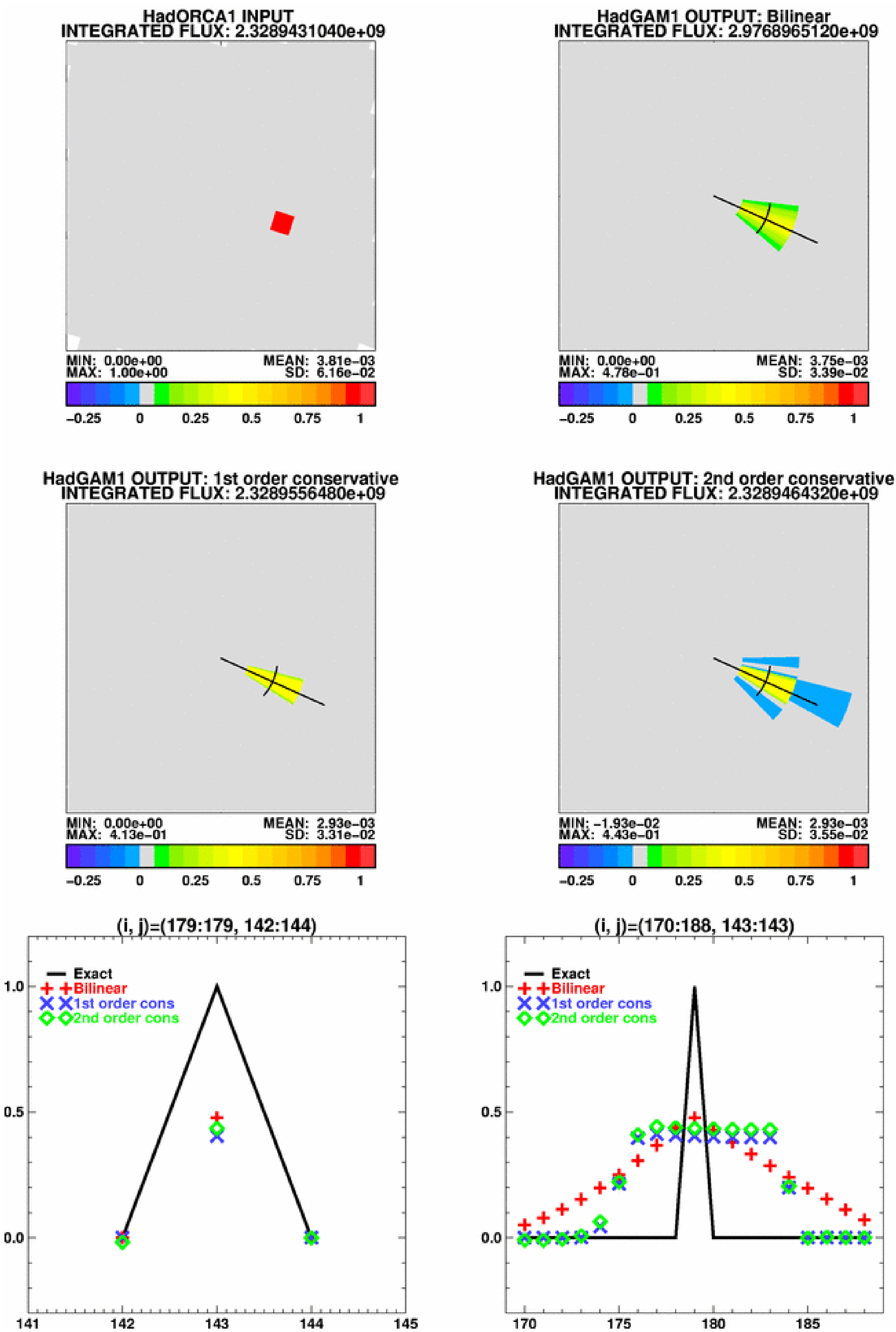

HadORCA1 INPUT
INTEGRATED FLUX: 2.3289431040e+09
MIN: 0.00e+00 MEAN: 3.81e-03
MAX: 1.00e+00 SD: 6.16e-02
HadGAM1 OUTPUT: Bilinear
INTEGRATED FLUX: 2.9768965120e+09
MIN: 0.00e+00 MEAN: 3.75e-03
MAX: 4.78e-01 SD: 3.39e-02
HadGAM1 OUTPUT: 1st order conservative
INTEGRATED FLUX: 2.3289556480e+09
MIN: 0.00e+00 MEAN: 2.93e-03
MAX: 4.13e-01 SD: 3.31e-02
HadGAM1 OUTPUT: 2nd order conservative
INTEGRATED FLUX: 2.3289464320e+09
MIN: -1.93e-02 MEAN: 2.93e-03
MAX: 4.43e-01 SD: 3.55e-02
-0.25 0 0.25 0.5 0.75 1
(i, j)=(179:179, 142:144)
(i, j)=(170:188, 143:143)
Exact
Bilinear
1st order cons
2nd order cons
1.0
0.5
0.0
141 142 143 144 145
170 175 180 185

Fig 16a: Maps and cross-sections of the results of interpolating a delta function source centred on the dark red HadORCA1 gridcell of Fig 2a. (The “exact” solution in the cross-sections is a nominal value.)

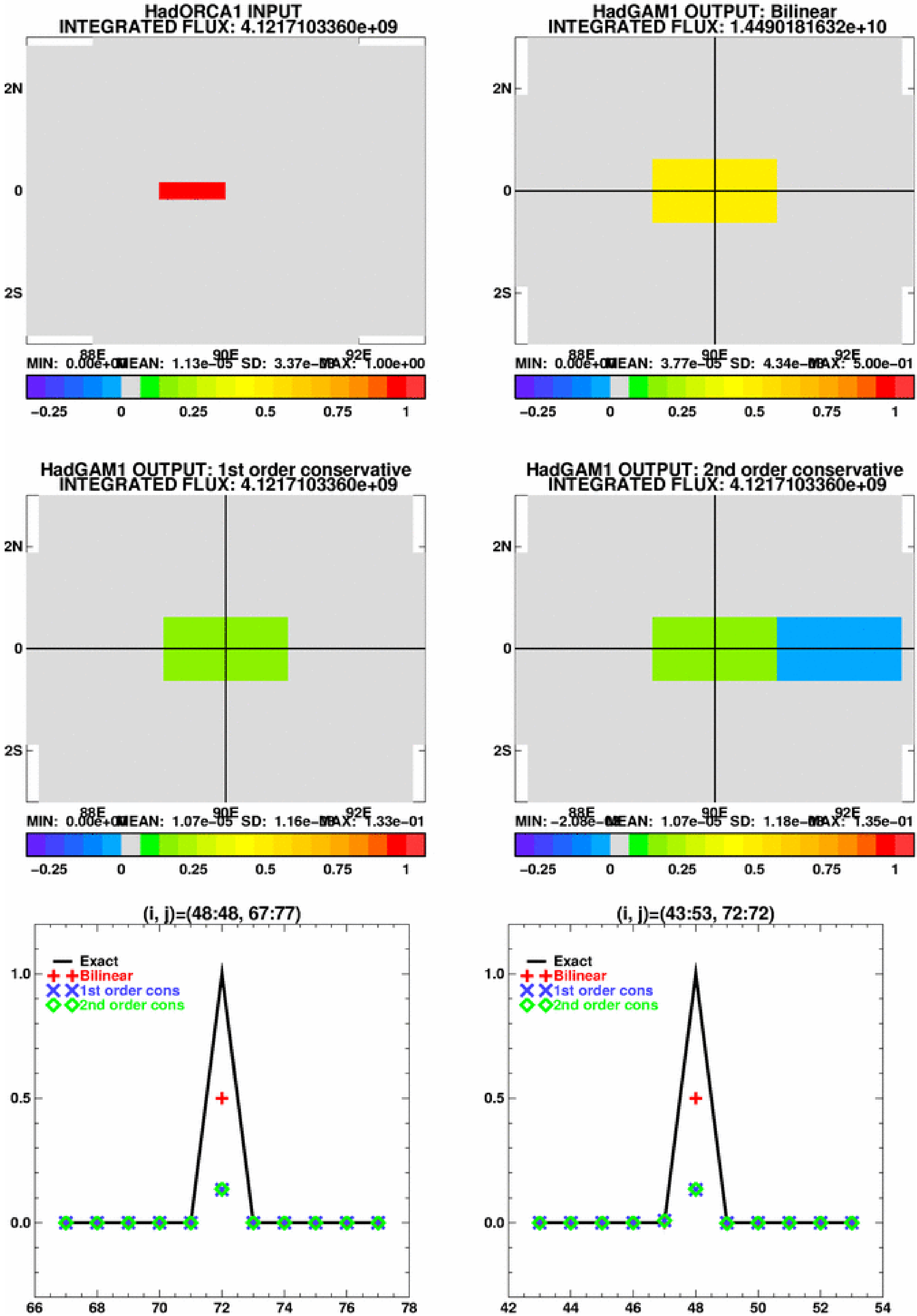

HadORCA1 INPUT
INTEGRATED FLUX: 4.1217103360e+09
MIN: 0.00e+00 MEAN: 1.13e-05 SD: 3.37e-03 MAX: 1.00e+00
HadGAM1 OUTPUT: Bilinear
INTEGRATED FLUX: 1.4490181632e+10
MIN: 0.00e+00 MEAN: 3.77e-05 SD: 4.34e-03 MAX: 5.00e-01
HadGAM1 OUTPUT: 1st order conservative
INTEGRATED FLUX: 4.1217103360e+09
MIN: 0.00e+00 MEAN: 1.07e-05 SD: 1.16e-03 MAX: 1.33e-01
HadGAM1 OUTPUT: 2nd order conservative
INTEGRATED FLUX: 4.1217103360e+09
MIN: -2.08e-02 MEAN: 1.07e-05 SD: 1.18e-03 MAX: 1.35e-01
2N
0
2S
88E
90E
92E
-0.25
0
0.25
0.5
0.75
1
(i, j)=(48:48, 67:77)
(i, j)=(43:53, 72:72)
Exact
Bilinear
1st order cons
2nd order cons
1.0
0.5
0.0
66
68
70
72
74
76
78
42
44
46
48
50
52
54

Fig 16b: Maps and cross-sections of the results of interpolating a delta function source in a HadORCA1 gridcell at (90E, 0N). (The "exact" solution in the cross-sections is a nominal value.)

3. Practical example: HadGEM3-AO

Finally – and we acknowledge it's been a long time coming – we apply all the preceding ideas to a practical problem: that of interpolating fields between the N96 HadGAM1 atmosphere model and the HadORCA1 ocean model, which lie at the heart of the HadGEM3-AO coupled GCM. The sea-ice component of this model is the ORCA1 configuration of the CICE model, so, apart from some technicalities concerning the mapping of velocities between NEMO's C-grid and CICE's B-grid (which won't be discussed here), the ocean-to-sea ice remapping is straightforward.

3.1 HadGAM1 to ORCA1

Fig 17 shows the results of interpolating an example positive field (surface SW down) from N96 HadGAM1 to the ORCA1 grid. Since the output fields look very similar, only the differences are plotted. These, and the statistics in Table 3, show that:

1. Bilinear interpolation fails to conserve the flux exactly.
2. Bilinear is generally closer to 2nd order SCRIP than it is to 1st order.
3. 2nd order interpolation undershoots the minimum of the input field, giving, in this case, negative SW down at the surface. Zeroing these values out violates conservation (obviously), but only very slightly (1 part in 10^6).
4. Only 1st order conservative remapping conserves integrated fluxes and stays within the range of the input field.

Table 3: Statistics of Fig 17

Grid	Min	Total	Max
HadGAM1	1.9401133759e-03	3.2685585886e+16	1.4028228760e+02
HadORCA1 (Bilinear)	2.9598724842e-01	3.2687091272e+16	1.3597830200e+02
HadORCA1 (1st order)	7.2713509202e-02	3.2685585886e+16	1.3570767212e+02
HadORCA1 (2nd order)	-3.2710268497e+00	3.2685585886e+16	1.3570767212e+02
HadORCA1 (2nd order, truncated) †	0.0000000000e+00	3.2685605213e+16	1.3570767212e+02

Table 3: Statistics of the fields shown in Fig 17. HadGAM1 total understood to be calculated after multiplying by seafrac. Undesirable properties are shown in red.
†Not shown in Fig 17.

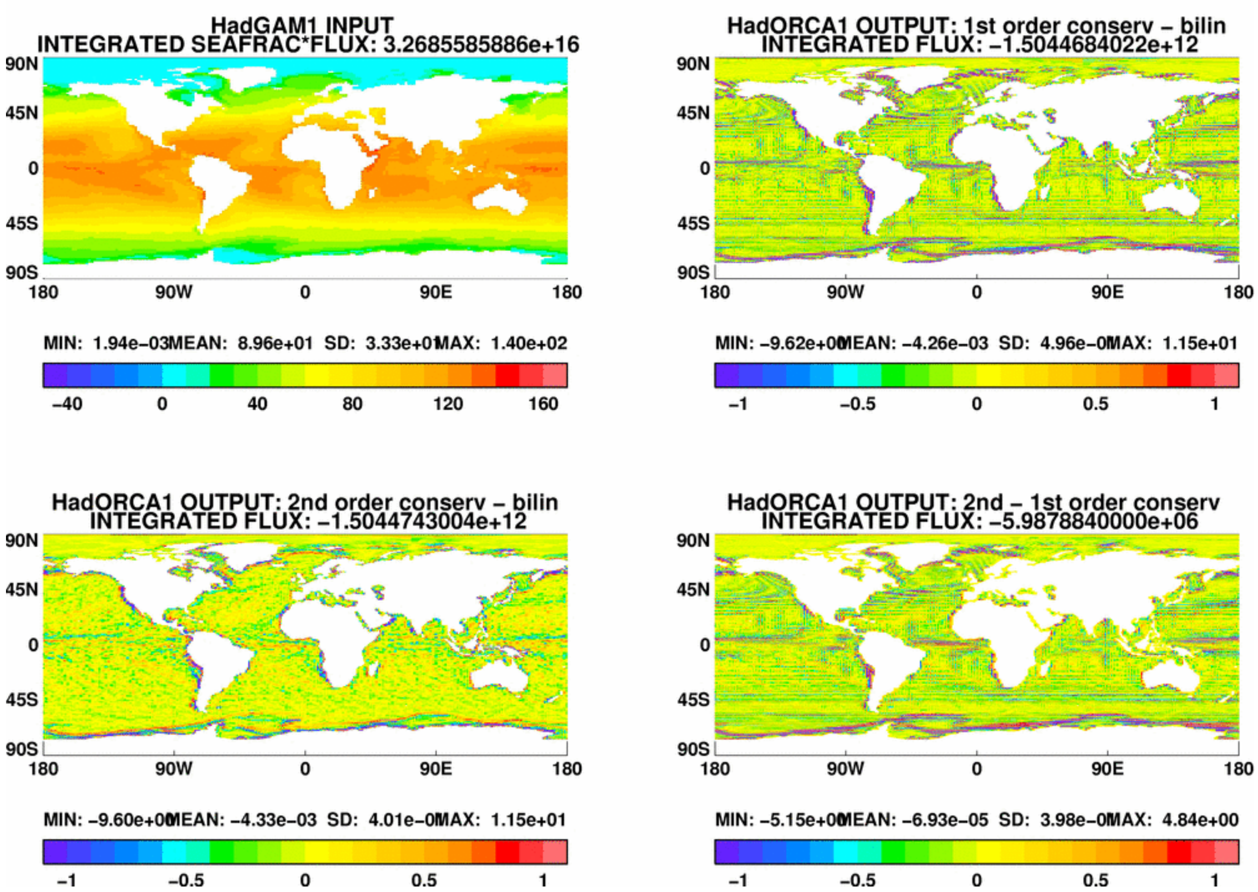


Fig 17: Results of remapping downwelling SW radiation from HadGAM1 atmosphere grid to HadORCA1 ocean grid. Differences in the results of remapping by bilinear, and 1st and 2nd order SCRIP are shown.

The small impact of "topping and tailing" the 2nd order SCRIP results suggests that this might be a suitable way of proceeding until flux limiters or "transa2o" emulators are incorporated in SCRIP. To this end, we have interpolated all the fields that will be passed from the atmosphere to the ocean in HadGEM3-AO, and examined the impact of trimming the output in general. The results are given in Fig 18, which compares the minimum, maximum and integrated total of the variously interpolated output fields with those of the input, for a set of fields. On this (stretched) plot, the input has a minimum of -1, a total of 0 and a maximum of 1.

For most fields, the fractional error resulting from topping and tailing the 2nd order SCRIP results is less than 10^{-3}, and often less than 10^{-4}. This therefore seems a safe method to use when interpolating fields conservatively from HadGAM1 to HadORCA1. (It is always much closer than bilinear.)

The only field for which this is not true is the river runoff field. However, the extremely localised nature of this field means that we would probably prefer to use 1st order interpolation on it anyway. The same is arguably true of topmelt. (The plotted statistics are actually for the sum of the topmelts over all (5) categories. We may expect the category topmelts to have finer scale structure, and to therefore be more susceptible to interpolation errors. And if category topmelts are remapped with 1st order SCRIP, category botmelts (penultimate field of plot) probably should be too, for consistency.

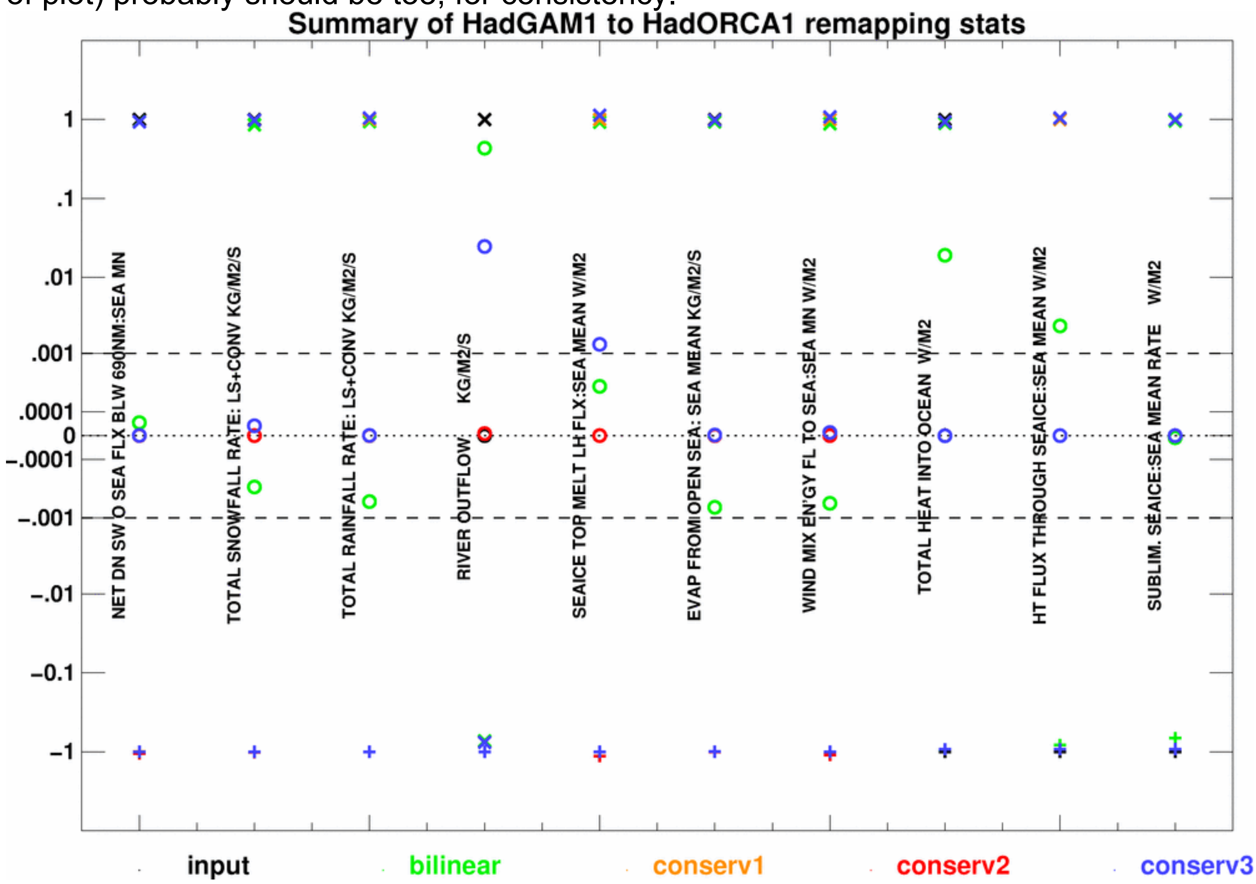


Fig 18: Summary of results of remapping required atmosphere fields from HadGAM1 to HadORCA1. The results of remapping by bilinear, and 1st and 2nd order conservative interpolation are shown. “conserv3” means “conserv2” with negative values set to zero.

3.2 ORCA1 to HadGAM1

Recall that in mapping from ocean to atmosphere, the distinction between DESTAREA and FRACAREA is important, and that we prefer FRACAREA as it extrapolates “without dilution” near the coasts.

Recall from Fig 2a, and the discussion in Sec 2.4.3.b, that HadORCA1-to-HadGAM1 remapping is not necessarily the simple fine-to-coarse interpolation that can be handled by area-averaging (as it is in transo2a).

One of the fields that needs mapping from ocean to atmosphere is the category-by-category ice fields. Although strictly these do not need to be mapped conservatively, there would be no harm in doing so if it could be done with sufficient accuracy.

The results of remapping category 1 ice areas from ORCA1 to HadGAM1 are shown in Fig 19 and detailed in Table 4. In this case, the undershoot effected by 2^{nd} order SCRIP needs amending. Simply setting negative values to zero doesn't drastically violate conservation (the totals still agree to within 1 part in 10^4), which doesn't need to hold exactly for these fields anyway. Interestingly, in this case the two SCRIP values are nearer each other than either is to the bilinear result, which suggests that the "fine-to-coarse grid" argument for area averaging still holds to some extent.

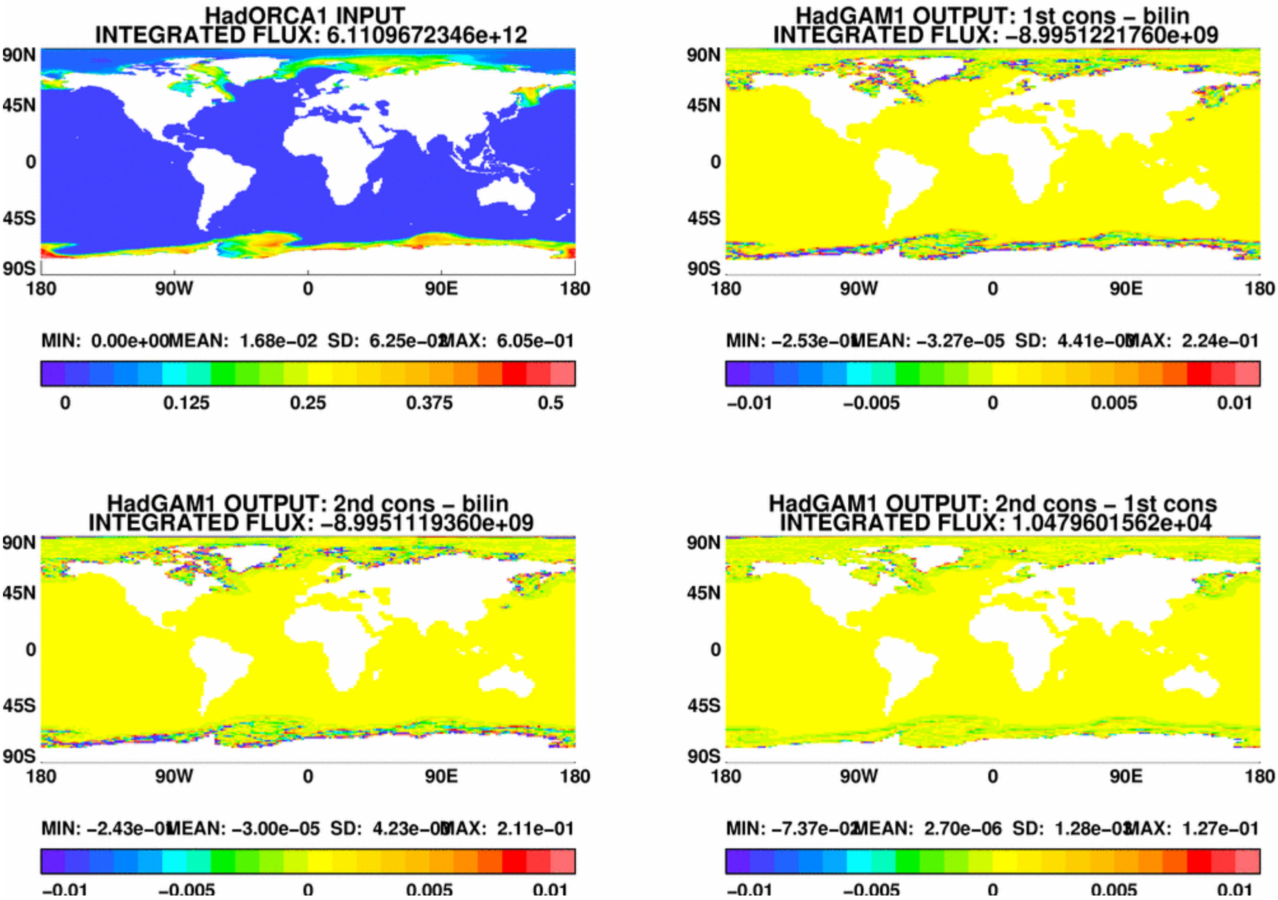


Fig 19: Results of remapping category 1 ice area from the HadORCA1 ocean grid to the HadGAM1 atmosphere grid. The results of remapping by bilinear, and 1^{st} and 2^{nd} order SCRIP are shown.

Table 4: Statistics of Fig 19

Grid	Min	Total	Max
HadORCA1	0.0000000000e+00	6.1109672346e+12	6.0528779030e-01
HadGAM1	0.0000000000e+00	6.1199619195e+12	5.9619092941e-

(Bilinear)			01
HadGAM1 (1st order)	0.0000000000e+00	6.1109672346e+12	5.9291255474e-01
HadGAM1 (2nd order)	-1.2267022394e-03	6.1109672346e+12	6.0049217939e-01
HadGAM1 (2nd order, truncated) †	0.0000000000e+00	6.1112382915e+12	6.0049217939e-01

Table 4: Statistics of the fields shown in Fig 19. HadGAM1 totals understood to be calculated after multiplication by sea-fraction. Undesirable properties are shown in red. †Not shown in Fig 19.

As before, we have made a similar analysis for all the fields that need to be transferred from ocean to atmosphere in HadGEM3-AO. The results are shown in Fig 20, which again makes it clear that truncating the NEMO fields has little overall impact on the conservation (which is of arguable importance in going from ocean to atmosphere). 1st and 2nd order conservative interpolation are equally acceptable, and tie in with what we currently do in transo2a.

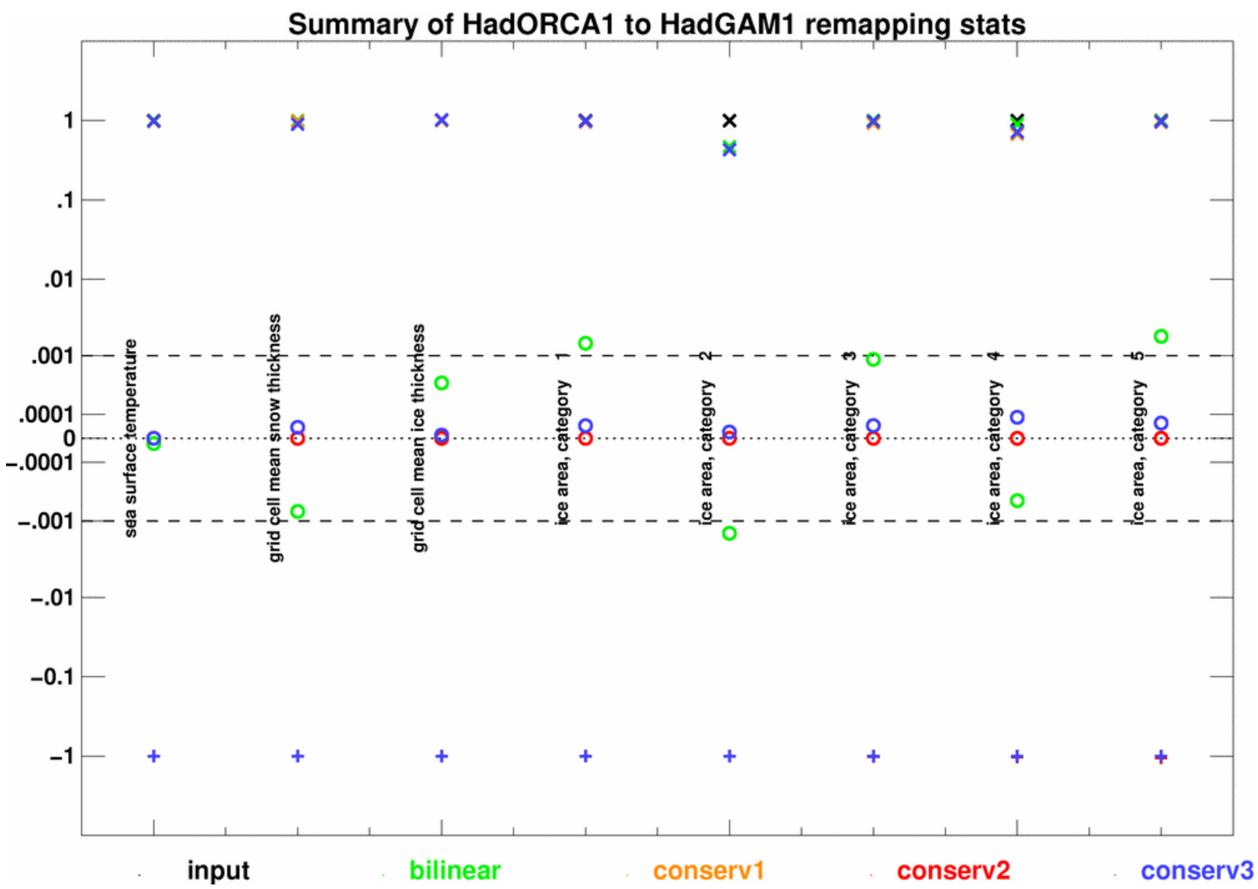


Fig 20: Summary of results of remapping required atmosphere fields from HadORCA1 to HadGAM1. The results of remapping by bilinear, and 1st and 2nd

order conservative are shown. “conserv3” means “conserv2” with negative values set to zero.

3.3 Recommendations

The preceding results lead us to the construction of Table 5, which lists some recommended methods for interpolating fields between atmosphere and ocean in a HadGEM3-AO-type coupled model.

Table 5: Recommended interpolation methods for HadGEM3-AO

Atmosphere to ocean	Ocean to atmosphere
Wind stress: Bilinear	Currents: Bilinear
RUNOFF, TOPMELT and BOTMELT: 1st order conservative (DESTAREA)	
Other scalars: 2nd order conservative (DESTAREA), truncated if necessary.	Scalars: 2nd order conservative (FRACAREA), truncated if necessary.

The remapping of vector fields has not been discussed in this report. This can be done via SCRIP, but the required rotations involve considerably more effort. Since vectors are interpolated bilinearly in our current coupling codes (transa2o and transo2a), we simply suggest doing the same for HadGEM3-AO, and postpone any further discussion of this point.

3.4 Coupled model results

As an example of these ideas in practice, Fig 21 shows the global atmosphere and ocean heat budgets of a typical proto-HadGEM3-AO coupled model. It can be seen that the heat fluxes across the coupler are conserved to an accuracy of about 10^{-3} W/m^2. A steady heat flux imbalance of this magnitude would lead to a mean ocean temperature trend of 0.3mK per century – far smaller than would be expected to arise in a climate change simulation. In other words, heat fluxes are being conserved well enough.

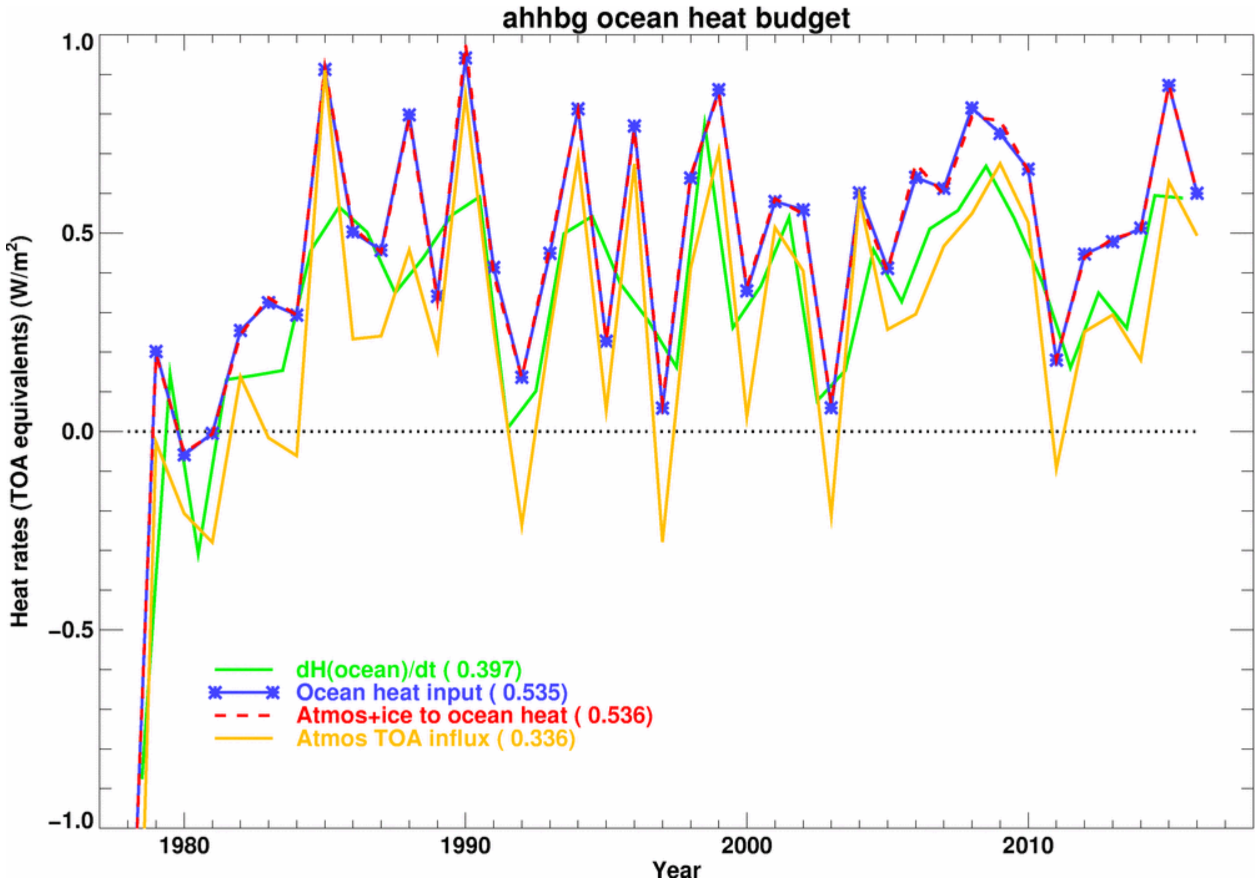


Fig 21: Global mean heat budgets for the first 40 years of a coupled HadGEM3-AO run. The difference between the red and blue curves is the non-conservation of heat across the coupler.

4. Summary and suggestions for further work

This report has explored the theory and practice of conservative interpolation of fluxes between the atmosphere and ocean components of a General Circulation Model. No very strong assumptions about the regularity of the two grids have been made. This means that the results can be applied to HadGEM3-AO, the next generation Hadley Centre GCM, whose ocean component (NEMO) is modelled on a tripolar grid.

Important features of the general theory, such as the place of normalisation and masking, and the differences between 1st and 2nd order conservative remapping, have been explained by appeal to simple test problems at low and high resolution. The interpolation of actual model fields has then been examined, and recommendations for the interpolation of all the fields in HadGEM3-AO have been made.

The practical implementation of these ideas is straightforward. The interpolation tool SCRIP takes source and target grids, and generates remapping matrices which can then be used to carry out the interpolation between the two. Files containing these matrices can be used directly by OASIS, the coupler employed in HadGEM3-AO, which means that we can be confident that the fluxes in the GCM are being interpolated as expected. Calculations of heat and freshwater budgets from coupled model integrations confirm that fluxes are indeed being conserved across the coupler.

Further work that could be done in this area, suggested by this study, includes:

- Incorporation of the transa2o-type predictor-corrector methods within SCRIP;
- Possible use of flux limiters to make 2nd order interpolation positivity-maintaining;
- Further analysis of the interpolation of vector fields (including wind stress);
- More accurate treatment of gridcell areas in SCRIP.

These matters may be the subject of future reports. In the present one, we have shown that the SCRIP remapping code, and the use of its remapping files in OASIS, allows us to interpolate fluxes conservatively between the N96 HadGAM1 atmosphere and HadORCA1 ocean components of the HadGEM3-AO coupled GCM.

5. Acknowledgements

Useful discussions with Helene Banks and Alison McLaren, particularly on the subject of masking/normalisation and the workings of transa2o/transo2a, are gratefully acknowledged. Transa2o is the work of many people, chiefly Jonathan Gregory, Tim Johns and Chris Durman, to whom thanks are extended. We are also grateful to Alistair Sellar for making SCRIP available, and for help with running it. This work was supported by the Joint DECC and MoD Integrated Climate Programme - (DECC) GA01101, (MoD) CBC/2B/0417_Annex C5.

6. References

Dukowicz, J K and J W Kodis, 1987: Accurate conservative remapping (rezoning) for arbitrary Lagrangian-Eulerian computations. *SIAM J Sci Stat Comput*, **8**, 305-321.

Griffies, S. M., M. J. Harrison, R. C. Pacanowski, and A. Rosati, 2004: A Technical Guide to MOM4, NOAA/Geophysical Fluid Dynamics Laboratory, Princeton, USA, 337 pp.

Hunke E C and W H Lipscombe, 2008: CICE: the Los Alamos Sea Ice Model Documentation and Software User's Manual, Version 4.0, LA-CC-06-012.

Johns T, C Durman, H Banks, M Roberts, A McLaren, J Ridley, C Senior, K Williams, A Jones, A Keen, G Rickard, S Cusack, M Joshi, M Ringer, B Dong, H Spencer, R Hill, J Gregory, A Pardaens, J Lowe, A Bodas-Salcedo, S Stark, Y Searl, 2005: HadGEM1 – Model description and analysis of preliminary experiments for the IPCC Fourth
Assessment Report, Hadley Centre Technical Note No. 55.

Jones, P W, 1999: First and Second-Order Conservative Remapping Schemes for Grids in Spherical Coordinates, *Mon Wea Rev*, **127**, 2204-2210.

Madec, G, 2007: "NEMO ocean engine". Note du Pole de modélisation, Institut Pierre-Simon Laplace (IPSL), France, No 27 ISSN No 1288-1619.

Valcke S (ed), 2004: OASIS3 User Guide, PRISM Support Initiative, Report No 3.

Appendix A: Some technical details of remapping

This appendix records some more abstract, general remarks about remapping. The aim is to show the place of normalisation, masking and composition of maps in a general formalism of linear transformations between vector spaces.

A.1 General formalism

In general then, we consider remapping a flux *f* from a source grid *g*, having gridcell areas a_i, *(i=1,n*) to a target grid *G*, having gridcell areas A_I, (*I=1,N*) where it has the value *F*.

Let us adopt at the outset the principle that data are masked but grids are not. (SCRIP allows one to incorporate masking of grids during the calculation of the remapping weights. This can lead to confusion.) The question of masking can therefore be postponed to Section A.3.

We therefore seek a mapping from $\{f_i\}$ to $\{F_I\}$ which conserves fluxes, so that $\int_R F_I\, dS =$
$\int_R f_i\, ds$ over any region *R*. By taking this to be gridcell *I* of the target grid, we obtain

$$F_I = A_I^{-1} \sum_i \int_{A_I \cap a_i} f_i ds \qquad (A.1)$$

A first order Taylor expansion of *f* about the centroid of the cell, $f = f^0 + (\mathbf{r}-\mathbf{r}^0).\mathrm{grad} f^0$, as described by Jones (1999) results in

$$F_I = A_I^{-1} \sum_i \int_{A_I \cap a_i} f_i^0 + (\partial f / \partial \varphi)_i^0 (\varphi - \varphi^0) + (\partial f / \partial \lambda)_i^0 \cos \varphi^0 (\lambda - \lambda^0) ds$$

$$= \sum_i w_{Ii}^1 f_i^0 + w_{Ii}^2 (\partial f / \partial \varphi)_i^0 + w_{Ii}^3 (\partial f / \partial \lambda)_i^0 \qquad (A.2)$$

where

$$w_{Ii}^1 = A_I^{-1} \int_{A_I \cap a_i} ds = A_I^{-1} \oint_{\partial (A_I \cap a_i)} -\sin \varphi d\lambda \qquad (A.3a)$$

$$w_{Ii}^2 = A_I^{-1} \int_{A_I \cap a_i} (\varphi - \varphi^0) ds = A_I^{-1} \oint_{\partial (A_I \cap a_i)} -(\cos \varphi + \varphi \sin \varphi) d\lambda$$
$$- w_{Ii}^1 a_i^{-1} \oint_{\partial a_i} -(\cos \varphi + \varphi \sin \varphi) d\lambda \qquad (A.3b)$$

$$w_{Ii}^{3} = A_I^{-1}\int_{A_I \cap a_i} \cos\varphi(\lambda - \lambda^0)ds = A_I^{-1}\oint_{\partial(A_I \cap a_i)} -\lambda/2\,(\sin\varphi\cos\varphi + \varphi)d\lambda$$
$$- w_{Ii}^{1} a_i^{-1}\oint_{\partial a_i} -\lambda/2\,(\sin\varphi\cos\varphi + \varphi)d\lambda \qquad (A.3c)$$

Eqn (A.2) can be written in matrix form as

$$\mathbf{F} = \mathbf{w}^1\mathbf{f} + \mathbf{w}^2\,\mathbf{df}/\mathbf{d}\boldsymbol{\varphi} + \mathbf{w}^3\,\mathbf{df}/\mathbf{d}\boldsymbol{\lambda} \qquad (A.4)$$

which emphasises the linear nature of the remapping procedure.

For most pairs of gridcells $A_I \cap a_i = \varnothing$, and therefore the corresponding element of the matrices $w^{1\text{-}3}_{Ii}$ is zero. In other words, the matrices $\mathbf{w}^{1\text{-}3}$ are sparse. This is essential for the practical implementation of this formulation, as $\mathbf{w}^{1\text{-}3}$ are of order N x n, which is 10^5 x 10^5 even for modest resolution GCM component models. This makes storing the full matrices impractical. SCRIP exploits the sparseness of the remapping matrices by simply storing $\{i(L), I(L), w^{1\text{-}3}{}_{I(L)i(L)}\}$ where the *link L* runs over the *non-zero* weights. The number of non-zero weights is O(max(N, n)), and storing the links therefore uses O(1/min(N, n)) as much space as storing the full matrix.

The area integrals in eqns A.3 above are converted to line integrals by means of Gauss' Theorem. This idea is due to Dukowicz and Kodis [1987], and makes practical the evaluation of the matrix elements. It does, however, introduce an error when implemented in SCRIP, because the line integrals along the segments of the overlap region $A_I \cap a_i$ are calculated using the end points only, by means of the trapezoidal rule. Thus, for example, the line element for the matrix element w^1, eqn A.3a, is calculated in SCRIP as

$$\int_A^B -\sin\varphi d\lambda = -1/2(\sin\varphi_A + \sin\varphi_B)(\lambda_B - \lambda_A) \qquad (A.5)$$

This would be correct if the gridcell boundary were linear in (λ, sinφ) space. This is the case for "regular" grids, in which the gridcell boundaries are defined by λ=constant or φ=constant, but unfortunately not in general, eg for the curvilinear grids used in NEMO and CICE. For the ORCA1 (1°, tripolar) configurations of such models, the largest error in the estimate of gridcell error incurred by the "linear" approximation in SCRIP occurs very near the North Pole, where the gridcell boundaries are most different to latitude circles, and is around 50%, as shown on Fig 2b. Other measures of gridcell area, which do not suffer these failings are possible, but for now we simply accept the infelicity implied by eqn (A.5) (and its analogues for w^2 and w^3). One consequence of this is that we must use the areas calculated from eqn A.5 when checking the conservation of fluxes using remapping matrices calculated by SCRIP.

A.2 Some consequences

A.2.1 Row sums

$\sum_i w^1_{Ii} = A_I^{-1}\sum_i\int_{AIi}ds = A_I^{-1}\, A_I = 1$, as noted in the 1D toy problem example in Sec 2.1. Here, A_{Ii} is shorthand for $A_I \quad a_i$.

A.2.2 Column sums

$\sum_I w^1_{Ii} = \sum_I A_I^{-1}\int_{AIi}ds$. If the target grid is of uniform resolution, $A_I \equiv A_o$ say, this equals
$A_o^{-1}\sum_I \int_{AIi}ds = A_o^{-1}\, a_i$. If in addition the source grid is of uniform resolution, $a_i \equiv a_o$ say, then $\sum_I w^1_{Ii} = A_o^{-1}\, a_o$ = constant, as noted in Sec 2.1 (where the constant equals 4/3).

Similarly, for a target grid of uniform resolution, the column sum of w^2, $\sum_I w^2_{Ii} = A_o^{-1}\sum_I \int_{AIi}(\varphi-\varphi^0)ds = 0$ by definition of the latitude of the centroid of the cell, φ^0. Likewise $\sum_I w^3_{Ii} = 0$ if the target grid is of uniform resolution. See Sec 2.1 again.

A.2.3 Composition of mappings

The matrix formulation in eqn 3 shows that first order conservative mapping from grid A to grid C via grid B is just given by the product of the underlying matrices: $w^1_{A\text{->}C}=w^1_{B\text{->}C}w^1_{A\text{->}B}$. Putting C=A, we see that if fluxes on grid A are “bounced” off grid B, back to A, they are transformed according to the product matrix $w^1_{B\text{->}A}w^1_{A\text{->}B}$. For instance, for the toy problem described in Sec 2.1, we have

$$\mathbf{w}^1{}_{A->A} = \mathbf{w}^1{}_{O->A}\mathbf{w}^1{}_{A->O} = \begin{pmatrix} 3/4 & 1/4 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/4 & 3/4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5/6 & 1/6 & 0 \\ 1/6 & 2/3 & 1/6 \\ 0 & 1/6 & 5/6 \end{pmatrix}$$

Note that $w^1_{A\text{->}A} \neq id_A$, the identity operator on A. This is to be expected if the gridcells of one grid do not lie wholly within those of the other, as flux from a source gridcell will be spread over a wider area before being mapped back.

A.2.4 Reverse mappings

Although $w^1_{G\text{->}g}$ may not be the inverse of $w^1_{g\text{->}G}$, there is still some relation between these two matrices. For, from the definition (eqn A.3a in Sec A.1),

$(w^1_{g\text{->}G})_{Ii} = \text{area}(A_I \cap a_i)/\text{area}(A_I)$ and $(w^1_{G\text{->}g})_{iI} = \text{area}(A_I \cap a_i)/\text{area}(a_i)$, and therefore

$(w^1{}_{g\text{->}G})_{Ii}$ = $(w^1{}_{G\text{->}g})_{iI}$ area(a_i)/area(A_I). If, further, grids *g* and *G* have uniform gridcell areas a^0 and A^0 respectively, then we see that each matrix is proportional to the transpose of the other:

$$\mathbf{w}^1{}_{g->G} = (a^0 / A^0)(\mathbf{w}^1{}_{G->g})^T$$

This can be seen in the examples in Sec A.2.3 above, for which *g=A*, *G=O* and a^0/A^0 = 4/3.

A.2.5 Coastal adjustment

This formalism allows us to express the coastal adjustment algorithm described in Sec 2.2.1 as

$$w^{2-3}_{IL^{-1}(M_O^{-1}(\{0\}))} \equiv 0$$

Here, M_O is the ocean model mask (1/0 for sea/land), and therefore $M^{-1}{}_O(\{0\})$ is the set of indices of land points on the ocean grid. *L* labels the links from *i* to *I*, and therefore L^{-1} of $M^{-1}{}_O(\{0\})$ is the set of indices of atmosphere gridcells that are linked to land points in the ocean model. By zeroing out $\mathbf{w^2}$ and $\mathbf{w^3}$ for *all* the links from these (coastal) cells, we revert to 1st order interpolation in precisely those cells where it is needed to retain conservative mapping (see Fig 4). The same condition can be written as

$$w^{2-3}_{I\alpha^{-1}([0,1))} \equiv 0$$

where α is the sea-fraction. This expression states that those links from atmosphere gridcells with a sea-fraction less than unity are nullified.

A.3 Masking and normalisation

A.3.1 General theory

We now consider the effects of masking the source and destination fields. The first is important in determining the flux incident upon the destination grid; the second is important when calculating flux budgets.

Remembering that we agreed that grids are masked and that fields are not, we need to explicitly account for the masks (=1 (0) where valid (invalid)) when defining conservative interpolation over some region *R*, thus:

$$\int_R MFdS = \int_R mfds$$

Taking *R* to be destination gridcell A_I leads to

$$F_I = A_I^{-1}\int_{A_I} mfds$$

unless M_I=0 and $\int mfds$ is not, in which case there's a an inconsistency. This happens whenever source flux falls on masked destination cells. See for example the top row of Fig 5. Short of *redistributing* the flux over valid target cells, all that can be done in this case is to account for the masking when calculating budgets.

Even if M_I is non-zero, the resulting flux as given above can be diluted by the absence of source flux over the whole of the destination cell. See for example the bottom row of Fig 5. This can be remedied by normalising by the *unmasked* source area, thus:

$$F_I = \int_{A_I} mfds \Big/ \int_{A_I} mds$$

This is just the FRACREA option in SCRIP.

Considerations like these lead to the following generalisations of the basic conservative interpolation theory, appropriate for masked grids. DESTAREA and FRACAREA normalisations need separate treatment.

DESTAREA

mdi	**36**	**100**

↓

mdi	**mdi**	**$57\frac{1}{3}$**	**100**

↓

mdi	**$28\frac{2}{3}$**	**$89\frac{1}{3}$**

Calculation:

$$\mathbf{F = \int mfds \,/ \int ds,}$$

which implies

$F_I = \sum_i m_i f_i A_{Ii} / \sum_i A_{Ii} = \sum_i m_i f_i A_{Ii} / A_I$

where $A_{Ii} = \int_{AI \cap ai} ds$ as before.

Flux budgets:

$$\mathbf{\int \varphi mfds = \int MFdS,}$$

where φ is the open fraction of grid G on grid g, ie

FRACAREA

mdi	**36**	**100**

↓

mdi	**mdi**	**$57\frac{1}{3}$**	**100**

↓

mdi	**$57\frac{1}{3}$**	**$89\frac{1}{3}$**

Calculation:

$$\mathbf{F = \int mfds \,/ \int mds,}$$

which implies

$F_I = \sum_i m_i f_i A_{Ii} / \sum_i m_i A_{Ii}$

where $A_{Ii} = \int_{AI \cap ai} ds$ as before.

Flux budgets:

$$\mathbf{\int \varphi mfds = \int \Phi MFdS,}$$

where φ is as before, and Φ is the open fraction of grid g on grid G,

$\varphi = \int MdS \,/\, \int dS$ which implies

$\varphi_i = \sum_I M_I A_{Ii} \,/\, \sum_I A_{Ii} = \sum_I M_I A_{Ii} \,/\, a_i.$

Proof:

$\int \varphi mfds = \sum_i \varphi_i m_i f_i a_i$

$= \sum_i m_i f_i \sum_I M_I A_{Ii}$ by definition of φ_i

$= \sum_I M_I \sum_i m_i f_i A_{Ii}$

$= \sum_I M_I F_I A_I$ by definition of F_I

$= \int MFdS.$ ▌

These can be checked using the numbers in the toy model problem above. The budget integrals equal 118/3 in each case.

The difference in the unmodified integrated atmospheric flux is 18/3, which is precisely that lost to the ocean grid masked cells; multiplying appropriately by the open fraction accounts for this.

ie $\Phi = \int mds \,/\, \int ds$ which implies

$\Phi_I = \sum_i m_i A_{Ii} \,/\, \sum_i A_{Ii} = \sum_i m_i A_{Ii} \,/\, A_I.$

Proof:

$\int \varphi mfds = \sum_i \varphi_i m_i f_i a_i$

$= \sum_i m_i f_i \sum_I M_I A_{Ii}$ by definition of φ_i

$= \sum_I M_I \sum_i m_i f_i A_{Ii}$

$= \sum_I M_I F_I \sum_i m_i A_{Ii}$ by definition of F_I

$= \sum_I M_I F_I \Phi_I A_I$ by definition of Φ_I

$= \int \Phi MFdS.$ ▌

Note the pleasing symmetry in the flux budget integrals: the flux on *each* grid needs to be multiplied by the open fraction of the other grid. As for DESTAREA, the budget integrals in the toy problem above are all 118/3. Note that in this case we appear to have gained some (unmodified) flux on the atmosphere grid; again, proper budgeting accounts for this.

The following special cases can be derived.

A.3.2 Target mask covers source mask

Here, masked source cells are fully overlain by masked target cells. The source mask is therefore not relevant to the remapping, since all valid target cells are wholly covered by valid source cells. *This is the situation when mapping from atmosphere to ocean in a GCM.* In this case, $\varphi=\alpha$, the sea-fraction defined in Sec 2.2, and $\Phi\equiv1$, because of the way ocean and atmosphere grids are defined in a GCM. The above theory and the top panels of Fig 5 make it clear that in this case:

- There's no difference between DESTAREA and FRACAREA in SCRIP (because the overlying source cells are unmasked, so $\Phi\equiv1$);
- One has to multiply the source flux by the sea-fraction (Sec 2.2) to balance the flux budgets: $\int \alpha m_A f_A ds_A = \int M_O F_O dS_O$. This is an unavoidable consequence of some of the source flux falling on masked target cells.

A.3.3 Source mask covers target mask

Here, unmasked target cells may be partially overlain by masked source cells. The source mask is therefore important, since it determines the amount of flux falling on the target cell. *This is the situation when mapping from ocean to* *atmosphere in a GCM.* In this case $\varphi\equiv1$ because of the way ocean and atmosphere grids are defined in a GCM, and $\Phi=\alpha$, the sea-fraction defined in Sec 2.2. The above theory and the bottom panels of Fig 5 make it clear that in this case:

- The DESTAREA option in SCRIP (in which the target fluxes are normalised wrt to the total target gridcell area) preserves integrated fluxes, at the expense of some dilution of the local flux as a result of partial masking of the overlying source cells: $\int m_O f_O ds_O = \int M_A F_A dS_A$;
- The FRACAREA option in SCRIP (in which the target fluxes are normalised wrt to the area of that part of the target gridcell which overlies unmasked source cells) preserves local fluxes, at the expense of some apparent non-conservation of integrated flux. One needs to multiply the target flux by the sea-fraction to balance the flux budgets: $\int m_O f_O ds_O = \int \alpha M_A F_A dS_A$.

All these results are encapsulated in Fig 5. The key thing to remember is that the calculated fluxes may need to be modified by an "open fraction" when calculating flux budgets.

The FRACAREA normalisation is analogous to that applied to the (1st order conservative) remapping used in "transo2a", the ocean-to-atmosphere coupler used in HadGEM1 and 2. To interpolate from the atmosphere to the ocean, these models use a scheme called "transa2o", a brief description of which now follows.

A.4 The atmosphere-to-ocean HadGEM1&2 remapping scheme: transa2o

For the record, we discuss the transa2o remapping schemes used to conservatively map fluxes from the atmosphere to the ocean in HadGEM1 and HadGEM2. As noted in Sec 2.1, this scheme uses a predictor-corrector method to adjust a first guess solution, derived by bilinear interpolation, to conserve integrated flux. Additive corrections are applied to fields of indeterminate sign; multiplicative corrections are applied to fields that are positive or negative everywhere. We examine each in turn.

A.4.1 *Additive correction*

Fig A.1 explains the method as applied to the 3x1 atmosphere to 4x1 ocean toy problem described in Sec 2.1.

The idea should be clear: by conservatively mapping the bilinear first guess back to the source grid, we derive a difference field which, when mapped conservatively to the target grid and added to the 1st guess, compensates for the lack of conservation in the original bilinear mapping, and thereby guarantees $g^1 dS_O = f^0 dS_A$.

Formally, we seek an additive correction to the prediction Bf^0 such that the integrated flux is conserved. Thus, write $g^1 = Bf^0+\varepsilon$, where $(Bf^0+\varepsilon)dS_O = f^0dS_A$. If we write $f^0dS_A = Cf^0dS_O$ (because C is conservative), this implies $\varepsilon\, dS_O = (C\text{-}B)f^0dS_O$, which (weakly) suggests that $\varepsilon=(C\text{-}B)f^0$. Hence, $g^1 = Bf^0+\varepsilon = Cf^0$, ie just the 1st order conservative interpolation (aka area-averaging). But we know from Sec 2.1 that this doesn't respect gradients in the input fields particularly well, and can be "blocky" when mapping from atmosphere to ocean. So we seek a better method.

Another possible approach is to map the bilinear approximation back to the source grid, where it can be differenced from the original field. We have $\varepsilon\, dS_O = f^0dS_A - Bf^0dS_O = f^0dS_A - cBf^0dS_A$ (because c is conservative) $= (Cf^0 - CcBf^0)dS_O$ (because C is conservative). This suggests $\varepsilon=(C\text{-}CcB)f^0$, and thus $g^1= Bf^0+\varepsilon = (B+C\text{-}CcB)f^0$. This is indeed the transa2o method, as Figure A.1 makes plain. Inclusion of the contribution from B ensures that gradients in the input field are respected more closely (see also Fig 3) than they would be by using C. Note that the fact that $Cc \neq id_O$ (Sec A.2.3) is crucial to making the two methods different, since $g^1= (C+(id_O\text{-}Cc)B)f^0$.

f^0					$f^0dS_A = 46.6667$
	4	36	100		

↓B

$g^0 = Bf^0$					$g^0dS_O = 48.0000$
	0	24	60	108	

↓c

$f^1 = cBf^0$ | 6 | 42 | 96 | $f^0 dS_A = 48.0000$

↓-

$\Delta f = f^0 - f^1 = (1 - cB)f^0$ | -2 | -6 | 4 | $\Delta f dS_A = -1.3333$

↓C

$\Delta g = C\Delta f = C(1-cB)\, f^0$ | -2 | -4.66 | -2.66 | 4 | $\Delta g dS_O = -1.3333$

↓+

$g^1 = g^0 + \Delta g = (B+C-CcB)\, f^0$ | -2 | 19.33 | 57.33 | 112 | $g^1 dS_O = 46.6667$

Fig A.1: Additive transa2o method applied to the 3X1 to 4X1 toy problem of Sec 2.1. B is the bilinear mapping from A to O, c is the1st order conservative remapping from O to A, and C is the 1st order conservative remapping from A to O.

For this toy problem, C (=$w^1_{A\text{->}O}$) and c (=$w^1_{O\text{->}A}$) are quoted in Sec A.2.3, and B is easily shown to be given by

$$\mathbf{B}_{A\text{->}O} = \begin{pmatrix} 9/8 & -1/8 & 0 \\ 3/8 & 5/8 & 0 \\ 0 & 5/8 & 3/8 \\ 0 & -1/8 & 9/8 \end{pmatrix}.$$

Therefore, the additive transa2o remapping matrix is given by

$$\mathbf{transa2o(add)} = \mathbf{B} + \mathbf{C} - \mathbf{CcB} = \begin{pmatrix} 114/96 & -18/96 & 0 \\ 26/96 & 82/96 & -12/96 \\ -12/96 & 82/96 & 26/96 \\ 0 & -18/96 & 114/96 \end{pmatrix},$$

as quoted in Sec 2.1. (Note that the regridding in the transa2o routine is not explicitly treated as a linear transformation in this way.)

A.4.2 *Multiplicative correction*

Fig A.2 explains the method as applied to the 3x1 atmosphere to 4x1 ocean toy problem described in Sec 2.1.

The idea should be clear: by conservatively mapping the bilinear first guess back to the source grid, we derive a *ratio* field which, when mapped conservatively to the target grid and *multiplied* by the 1st guess, compensates for the lack of conservation in the original bilinear mapping, and thereby guarantees $g^1 dS_O = f^0 dS_A$.

Because of the non-linear nature of the divisions and multiplications implicit in this method, it is not possible to express the algorithm as a linear mapping between source and grid[5]. However, the expression for the I^{th} element of g^1 (eg see Fig A.2),

$$g_I^1 = g_I^0 \sum_i C_{Ii} \left(\frac{f_i^0}{\sum_J c_{iJ} g_J^0} \right) \qquad \text{(A.6)},$$

(where $g^0 = Bf^0$) allows conservation to be shown directly, thus:

$$\int g^1 dS_O = \sum_I A_I g_I^1 = \sum_I A_I g_I^0 \sum_i C_{Ii} \left(\frac{f_i^0}{\sum_J c_{iJ} g_J^0} \right)$$

$$= \sum_I \sum_i A_I C_{Ii} g_I^0 \left(\frac{a_i f_i^0}{\sum_J A_J C_{Ji} g_J^0} \right) \quad \text{(by the result in Sec A.2.4 that } c_{iJ} = C_{Ji}(A_J/a_i)\text{)}$$

$$= \sum_i a_i f_i^0 \sum_I A_I C_{Ii} g_I^0 \left(\frac{1}{\sum_J A_J C_{Ji} g_J^0} \right)$$

$$= \sum_i a_i f_i^0 = \int f^0 dS_A \quad \blacksquare .$$

---

[5] Eqn A.6 shows that the mapping f^0 to g^1 is homogeneous of degree 1 (ie λf^0 is mapped to λg^1). However, eqn A.6 also shows that it is non-linear, because $f_A^0 + f_B^0$ is not mapped to $g_A^1 + g_B^1$. Here, g_X^k is the image of f_X^k under the transa2o remapping.

f^0	4	36	100		$f^0 dS_A = 46.6667$
	$\downarrow$B				
$g^0 = Bf^0$	0	24	60	108	$g^0 dS_O = 48.0000$
	$\downarrow$c				
$f^1 = cBf^0$	6	42	96		$f^0 dS_A = 48.0000$
	$\downarrow\div$				
$\mathcal{P}f = f^0/f^1$	0.667	0.857	1.042		$\mathcal{P}f dS_A = 0.85516$
	$\downarrow$C				
$\mathcal{P}g = C\mathcal{P}f$	0.667	0.793	0.918	1.042	$\mathcal{P}g dS_O = 0.85516$
	$\downarrow\times$				
$g^1 = g^0 * \mathcal{P}g$	0	19.05	55.12	112.50	$g^1 dS_O = 46.6667$

Fig A.2: Multiplicative transa2o method applied to the 3X1 to 4X1 toy problem of Sec 2.1. B is the bilinear mapping from A to O, c is the1st order conservative remapping from O to A, and C is the 1st order conservative remapping from A to O.

It may be possible to emulate the additive transa2o scheme in SCRIP/OASIS, by calculating the matrix *C+(1-CcB)* offline, as we have done in Sec A.4.1, but the non-linearities implicit in the multiplicative predictor-corrector scheme, which is needed for always positive or always negative fields, would appear to make this difficult for OASIS.

Met Office
FitzRoy Road, Exeter
Devon, EX1 3PB
UK

Tel: 0870 900 0100
Fax: 0870 900 5050
enquiries@metoffice.gov.uk
www.metoffice.gov.uk