

THE AERODYNAMIC TERM IN THE PENMAN FORMULA

COMPARISON OF RESULTS OF EVALUATIONS USING VARIOUS SELECTIONS OF DATA

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1. INTRODUCTION

Using the form of the A-Term (SVP-VP) $(u + \frac{100}{21})$ evaluations were made with the following selections of data. (u = wind speed in m/s at 10m)

1. Instantaneous values of SVP, VP and u at 0300Z
2. " " " " " " " " 0900Z
3. " " " " " " " " 1500Z
4. " " " " " " " " 2100Z
5. Mean of instantaneous SVP, VP and u at 03Z and 15Z
6. " " " " " " " " 09Z and 21Z
7. " " " " " " " " 09Z and 15Z
8. " " " " " " " " 03Z, 09Z and 21Z
9. " " " " " " " " 03Z, 09Z and 15Z
10. " " " " " " " " 09Z, 15Z and 21Z
11. " " " " " " " " 03Z, 15Z and 21Z
12. " " " " " " " " 03, 09, 15 and 21Z
13. The approximation using (24-hour run of wind
(0900Z VP
(SVP from max and min temps.
14. The 24-hour mean of hourly evaluations of the A-Term. (u being mean wind for "past" hour).
15. The A-Term evaluated daily from means of hourly values of SVP, VP and u.

2. DATA AND COMPUTER TREATMENT

All the measurements were made at London Airport in 1965. The programme divided a 360 day "year" (Jan 1-Dec 26) into twelve 30-day "months" and printed out

- (a) Daily values of A-Terms 1 to 15
- (b) 30 day means of daily values of A-Terms 1 to 15
- (c) 30 day mean values of A-Terms 1 to 13 and 15 evaluated from 30 day mean values of the variables (SVP, VP and u).

Table I (available on request) contains the following data:

- (a) 30 day means of daily evaluations of A-Term
- (b) 30 day mean A-Term values from 30-day mean data
- (c) b as percentage of a

for each month and each selection of data.

3. ANALYSIS OF EVALUATIONS

Wright has already given reasons for considering that the whole Penman formula can not be evaluated for periods of less than 24 hours, on radiation considerations.

We shall assume, therefore, that the best value of the Aerodynamic Term (A-Term) is that obtained from the mean of continuous records of temperature, vapour pressure and wind speed. The selection of data nearest to continuous records is the mean of hourly values and 1-hour runs of wind and we shall, therefore, compare all other evaluations with those obtained from selection 15, which Wright has called "Product of the Means".

Wright has already shown, from the same data (Evap Memo No 4) the difference between the 24 hour mean of hourly evaluations of the A-Term ("Mean of the Products") 14 and the A-Term evaluated from 24 hour means of the variables (15).

Comparison of 30-day means of Daily Evaluations

The 30-day means of daily evaluations are plotted in Fig 1 as follows:

Product of Means (Type 15) in heavy black line.

Corresponding to each point on the Type 15 graph points are plotted at 4 units above and below. The "+4 band" is bordered by dashed lines and is shaded; no points falling within the shaded area are plotted for Types 1-14 to avoid needless complication of the figure.

Evaluations made with the other selections of data are plotted (where they fall outside the shaded area) in accordance with the column of symbols at the extreme right of the upper part of the figure. All graphs and "part-graphs" are labelled according to the data used.

The "calendar" at the top of the figure shows, for convenience, whether the evaluation lies above (+) or below (-) the shaded area or within the shaded area.

30-day means of daily evaluations of the A-Term (by "methods" 1 to 14) were divided into corresponding 30-day means of daily evaluations by "Product of Means" (method 15); the results, which are multiplying factors, as they stand, and would be percentages if multiplied by 100, are set out in Table II.

Totals and "yearly" (360-day) means are given for each "method" and also the difference between the greatest and least 30-day value of the factor for each method.

The "factors" in Table II are plotted "method by method" in Figs 2.1 to 2.14.

Each graph was inspected and either (i) rejected, as in the cases of 2.1, 2.4 and 2.8, as showing no reliable relationship between the "methods" in question and "method" 15 or (ii) thought flat enough to have the 360-day mean factor applied throughout or (iii) divided into parts of a "year" with appropriate "seasonal" mean factors calculated from 30-day values.

Graphs 2.2, 2.3, 2.5-7 and 2.9-14 can be summarised as follows:

- 2.2 (09Z data) Good for May-August. "Tolerable" for colder months.
- 2.3 (15Z data) Little variation throughout the year.
- 2.5 (03 and 15Z data) Good for October-April and quite good for warmer months.
- 2.6 (09 and 21Z data) Good for May-August. "Tolerable" for colder months.
- 2.7 (09 and 15Z data) Good for March-November but a slightly different factor needed in winter.
- 2.9 (03, 09 and 15Z data) Very good.
- 2.10 (09, 15 and 21Z data) Very good if "seasonal" factors are used.
- 2.11 (03, 15 and 21Z data) Very good if "seasonal" factors are used.

2.12 (03, 09, 15 and 21Z data) Very good.

2.13 ("Approximation") Very good except for December.

2.14 ("Mean of Products") Very good.

The December value in Fig 2.13 deserves comment. It appears to have been due to the occurrence of three days when the evaluation gave large (spurious) negative values. These are, of course, nonsense; they arose as a result of the arrival during the day, of mild, humid, windy weather causing a rapid rise of temperature. On days such as these the "approximation" (method 13) is clearly inappropriate.

The factors from Figs 2 are set out in Table III

METHOD	DATA	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	Nov	DEC
2	09	← 1.402 →		← 1.008 →			← 1.298 →						
3	15	← 0.485 →											
5	03 15	← 0.865 →			← 1.008 →				← 0.821 →				
6	09 21	← 1.355 →			← 1.141 →				← 1.381 →				
7	15 09	0.873	← 0.697 →					← 0.873 →					
9	03 15	← 1.013 →											
10	15 09 21	← 0.933 →		← 0.824 →				← 0.914 →					
11	03 15 21	← 0.985 →			← 1.096 →				← 0.986 →				
12	03 09 15 21	← 1.085 →											
13	APP	← 0.964 →						← 1.223 →					
14	M&P	← 0.933 →											

The factors in Table III were applied to the 30-day mean values of daily evaluations by each method (in first lines of each Section of Table I) to produce adjusted values of the A-Term; these adjusted values are set out in the first lines of each section of Table IV (available on request).

In the second line of each section is the "error", obtained by subtracting the corresponding 30-day mean of daily values by P of M (Col 15) from each "adjusted value" (first line).

In the third line of each section the "error" is expressed as a percentage of the P of M value of the Term. The percentages are given in Table IVA.

The percentage errors were added and meaned over the 360-day "year".

The 360-day means of percentage errors are listed in ascending order in Table V from which we see that 8 of the "methods" give annual average errors of less than 5% of the "monthly" values of the A-Term by the "best method"

TABLE V

<u>METHOD</u>	<u>DATA</u>	<u>MEAN OF 12 %AGE ERRORS</u>
11	03 15 21	1.4
14	M of P	1.9
12	03 09 15 21	2.5
9	03 09 15	3.1
13	Approx	3.4
10	09 15 21	3.5
5	03 15	3.6
7	09 15	4.5
6	09 21	5.7
2	09	7.4
3	15	14.2

This "ranking" of "methods" with respect to the P of M method is, of course, a very coarse approach and it must not be forgotten that it is based on 30-day means of daily evaluations from only one station-year of data.

Comparison of Daily Evaluations

Making an arbitrary selection of the 10th and 20th day in each 30-day period and leaving out "methods" 1, 4 and 8 the daily evaluations of the A-Term by the remaining "methods" were divided into the corresponding evaluations by "method" 15 "P of M". The "multiplying factors" so obtained are set out in Table VI (available on request).

The daily ratios of Table VI are plotted in Figs 3 and are there compared with the 30-day multiplying factors from Table III.

An X is shown in the graphs where a value of the A-Term was negative (and therefore nonsensical).

A few very large values are written in figures instead of being plotted as points on the graphs.

Although all days and several years of data from several stations would have to be analysed to produce firm conclusions, the following deductions, made from Figs 3 have limited validity in view of the arbitrary selection of days from the 360-day sample.

Relationship of daily ratios to 30-day ratios

- 3.2 (09Z data) Poor for January-April, moderate for May-August and quite good for September-December except for two values.
- 3.3 (15Z data) Poor for January but quite good for the rest of the year.
- 3.5 (03 and 15Z data) Quite good for colder months but only moderate in the warmer months.
- 3.6 (09 and 21Z data) Mostly moderate.
- 3.7 (09 and 15Z data) Moderate in January but quite good in the rest of the year.
- 3.9 (03, 09, 15Z data) Mostly moderate. All near or above the 30 day value.
- 3.10 (09, 15, 21Z data) Poor in January but good for rest of the year.

- 3.11 (03, 15, 21Z data) Very good for January-April and quite good for the rest of the year.
- 3.12 (03, 09, 15, 21Z data) Good except for October to December
- 3.13 (Approximation) Quite good apart from winter.
- 3.14 (Mean of Products) Very good.

From careful examination of the graphs (Figs 3) we see that, on the basis of daily comparisons, a similar "ranking" to that of Table V appears.

TABLE VII

<u>METHOD</u>	<u>DATA</u>	<u>RELATIONSHIP BETWEEN DAILY AND 30-DAY RATIOS</u>
14	M of P	Very Good
11	03, 15, 21Z	Mostly Good
12	03, 09, 15, 21Z	Mostly Good
10	09, 15, 21Z	Mostly Good
7	09, 15Z	Mostly Quite Good
3	15Z	Mostly Quite Good
9	03, 09, 15Z	Moderate
5	03, 15Z	Fair

Occurrence of Negative and very small values of the A-Term

All values of the A-Term (daily) for all "methods" were examined and cases were counted as follows:

- (i) Number of negative values
- (ii) Number of positive values < 2
- (iii) Number of positive values > 2 < 4

The distribution of the above classes of values by "methods" and 30-day periods is shown in Table VIII A, B and C.

If we now discard all "methods" giving a negative value of the A-Term and rank the remainder on the basis of numbers of evaluations less than 2 we arrive at the rough classification of Table IX.

TABLE IX

<u>METHOD</u>	<u>DATA</u>	<u>CLASS</u>
15	P of M	FIRST
14	M of P	
11	03, 15, 21Z	
10	09, 15, 21Z	
5	03, 15Z	SECOND
7	09, 15Z	
12	03, 09, 15, 21Z	THIRD
3	15Z	
9	03, 09, 15Z	FOURTH
4	21Z	FIFTH

It will be noticed from inspection of Table VIII A, B and C that methods which give some negative evaluations also give higher numbers of very small values of the A-Term (with the exception of method 13 with 7-ve values but only one +ve value < 2).

Relationship between 30-day means of the A-Term and values of the A-Term evaluated from 30-day mean data

Values of the A-Term evaluated both ways from each selection of data were plotted for comparison.

Values of Mean A-Term from 30-day mean data
30-day Mean of A-Term

were calculated for each case. These percentages, labelled x/o are written below each pair of graphs. Figs 4.1-14 (available on request) and are shown in Table X. To summarise the percentages we can say that all were $\leq 94\%$ during the period October-March (inclusive), with the exception of "1500Z method" in January and October, and that most were $\leq 89\%$. Most percentages were ≥ 95 and ≤ 105 during the period April-September (inclusive) with the notable exception of the "0300Z method".

TABLE X

MEAN A-TERM FROM 30-DAY MEAN DATA
30-DAY MEAN OF A-TERM.

METHOD	1	2	3	4	5	6	7	8	9	10	11	12	13	14
DATA	03	09	15	21	03 15	09 21	09 15	0309 21,15	0309 21,15	09 15	03 15	0309 15,21	APP	M&P
JAN	65	62	95	67	83	69	83	70	78	82	82	79	81	76
FEB	83	85	94	90	92	89	92	89	90	92	92	91	93	91
MAR	60	69	89	87	88	79	87	76	84	88	89	85	86	80
APR	75	84	101	100	97	95	96	91	94	99	98	96	96	92
MAY	91	103	97	101	100	103	101	102	102	101	101	102	100	97
JUN	84	98	104	98	101	99	104	97	101	102	100	100	97	94
JUL	93	95	99	95	97	97	98	97	97	98	98	98	98	94
AUG	92	95	99	101	100	99	99	99	99	100	101	100	98	95
SEP	69	84	101	93	99	93	99	90	96	100	100	98	96	91
OCT	63	69	95	86	91	81	90	79	87	90	91	88	92	82
Nov	60	60	85	69	79	68	78	67	74	77	78	75	77	73
DEC	64	65	92	73	86	78	85	76	82	87	87	85	88	79

BLACK BORDER ENCLOSSES PERCENTAGES 95-105

4. CONCLUSIONS

1. On the basis of radiation considerations it was assumed that the Perman Formula for calculating Potential Evaporation can not be evaluated, reliably, for periods of less than 24 hours. Hence we assumed that the best value of the Aerodynamic Term (A-Term) is to be obtained from continuous values of temperature, vapour pressure and wind speed. The nearest approximation to continuous values is the hourly measurement of temperature and vapour pressure and run of wind; i.e. "Method" 15 which Wright has called Product of the Means where the A-Term is evaluated daily from means of hourly values of the variables.

2. Comparison of 30 day means of daily values of the A-Term with Product of Mean 30-day values led to the rejection of methods using (1) 03Z data, (4) 21Z data and (8) 03, 09 and 21Z data and to the calculation of provisional yearly or seasonal multiplying factors (Table III) to convert A-Term values obtained in various ways to A-Term values by Product of Means.

3. The multiplying factors of Table III were used to adjust 30-day means (of daily evaluations of the A-Term by various methods) and final errors expressed as %age of the Product of Means A-Term mean values for 30-day periods. This process gave a provisional order of "excellence" Table V.

4. Daily evaluations of the A-Term were compared with those obtained by Product of Means, ratios were obtained and compared with the ratios (multiplying factors) of Table III. This process gave a provisional order of "excellence" Table VII

5. Methods giving any negative evaluations were rejected (for all-year use) and the remainder were "ranked" in inverse ratio to the number of values of the A-Term < 2 in 360 days; this process gave a provisional order of "excellence" Table IX.

6. The findings of (3), (4) and (5) "combined" suggest the following rough order

(Best method 15) P of M

Second: (14) M of P and (11) 03, 15, 21Z
Third: (12) 03, 09, 15, 21Z and (10) 09, 15, 21Z
Fourth: (7) 09, 15Z
Fifth: (5) 03, 15Z
Sixth: (9) 03, 09, 15Z (3) 15Z

7. The 30 day means of daily evaluations of the A-Term were compared with 30 day evaluations of the A-Term from 30-day means of the variables and it was found that whilst differences were small in the warmer months there were large differences in the colder months.

8. It must not be forgotten that the whole analysis has been based on one station-year of data.

NOTE This memo is circulated for discussion purposes only. Comments and suggestions will be welcomed by the authors.

Met O Sc
Bracknell
January 1970
LRN

TABLE II

30-DAY MEANS OF DAILY EVALUATIONS OF THE A-TERM DIVIDED BY THE CORRESPONDING 30-DAY MEANS OF DAILY EVALUATIONS BY "PRODUCT OF MEANS" (TYPE 15) METHOD.

	1/15	2/15	3/15	4/15	5/15	6/15	7/15	8/15	9/15	10/15	11/15	12/15	13/15	14/15
"MONTH"	03	09	15	21	03 15	09 21	09 15	03 09 21	03 09 15	09 15 21	03 15 21	03 09 15 21	APP	M of P
JAN	1.229	1.342	0.640	1.050	0.861	1.259	0.914	1.290	0.999	0.998	0.975	1.062	0.927	0.970
FEB	1.760	1.402	0.531	1.047	0.890	1.229	0.822	1.394	1.030	0.900	0.955	1.050	1.008	0.954
MAR	2.611	1.671	0.379	1.557	0.832	1.639	0.719	1.938	1.031	0.901	1.006	1.138	0.950	0.903
APR	2.869	1.192	0.415	1.325	0.875	1.293	0.633	1.632	0.971	0.825	1.002	1.055	0.986	0.932
MAY	3.196	1.016	0.467	1.269	1.027	1.143	0.678	1.550	1.031	0.822	1.105	1.089	0.998	0.923
JUN	3.515	1.037	0.546	1.117	1.118	1.085	0.748	1.495	1.103	0.844	1.118	1.106	1.000	0.945
JUL	2.969	0.976	0.471	1.415	0.953	1.193	0.661	1.549	0.968	0.837	1.095	1.071	0.925	0.937
AUG	4.028	1.004	0.440	1.191	1.008	1.144	0.657	1.624	1.035	0.792	1.069	1.076	1.028	0.913
SEP	2.828	1.230	0.436	1.439	0.933	1.411	0.710	1.779	1.035	0.894	1.091	1.145	0.951	0.907
OCT	3.521	1.463	0.380	1.375	0.776	1.480	0.677	1.926	1.019	0.841	1.000	1.107	0.917	0.912
NOV	1.439	1.124	0.529	1.299	0.839	1.285	0.765	1.367	0.936	0.922	0.985	1.038	0.918	0.950
DEC	1.167	1.376	0.584	1.067	0.849	1.348	0.883	1.351	1.002	0.999	0.972	1.079	1.223	0.946
TOTALS	31.132	14.833	5.818	15.151	10.961	15.509	8.896	18.895	12.160	10.575	12.373	13.015	11.831	11.192
YEAR	2.594	1.236	0.485	1.263	0.913	1.292	0.741	1.575	1.013	0.881	1.031	1.085	0.985	0.933
MAX MIN	2.861	0.695	0.261	0.510	0.342	0.559	0.257	0.587	0.135	0.207	0.163	0.100	0.306	0.067

TABLE IV A

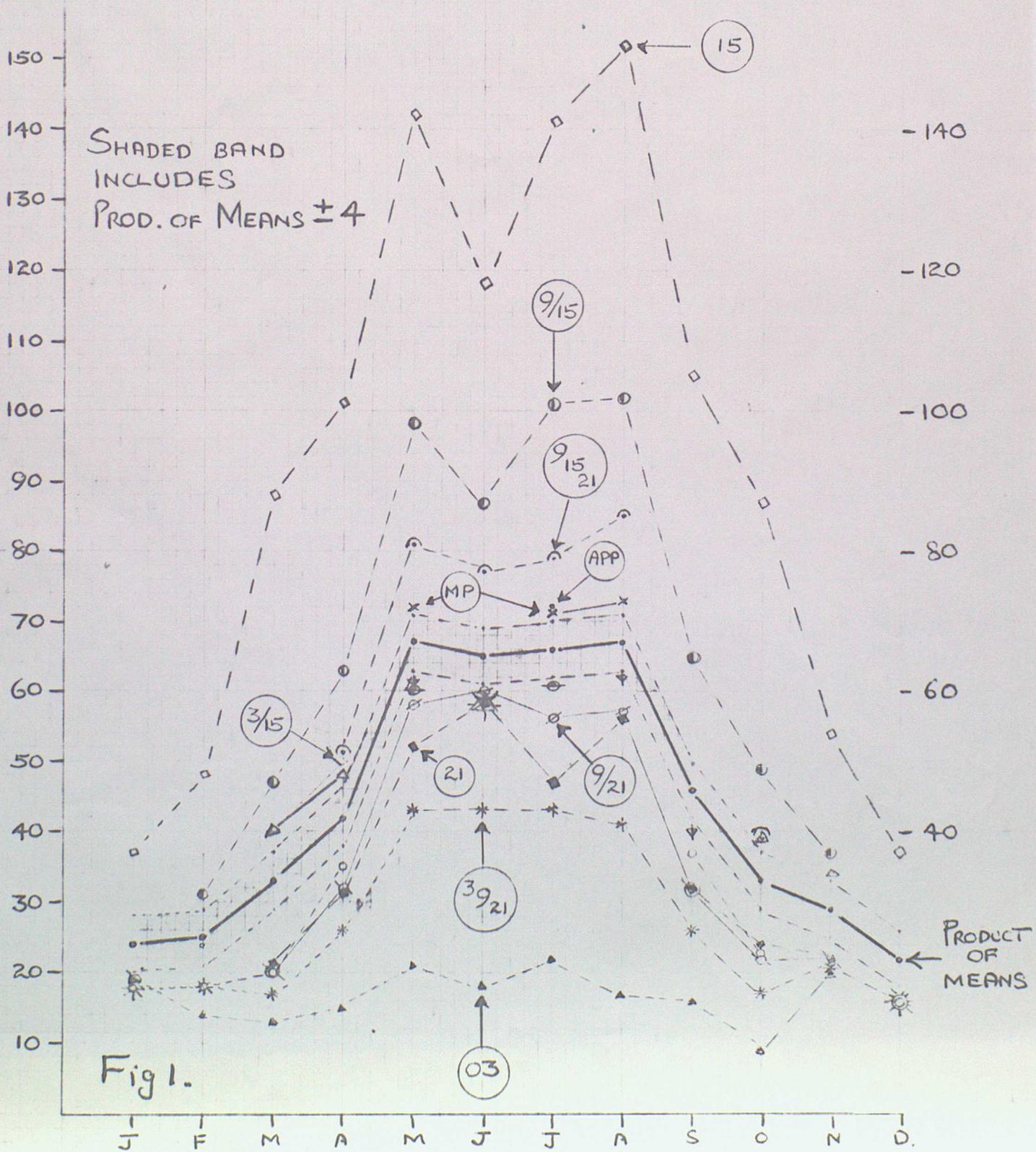
(Adjusted 30-day mean of daily A-Term) - (T of M 30 day mean of daily A-Term)
 (P of M 30 day mean of daily A-Term)

"METHOD"	2	3	5	6	7	9	10	11	12	13	14	
DATA	⁰⁹ 15	⁰³ 15	⁰⁹ 21	⁰⁹ 15	^{03 09} 15	⁰⁹ 15	^{03 09} 15 21	⁰⁹ 15	^{03 09} 15 21	^{03 09} 15 21	APP.	M of P
JAN	4.3	24.2	0.3	7.5	4.5	1.2	6.6	0.9	2.2	3.9	3.8	
FEB	0	8.6	2.9	10.3	6.1	1.8	3.6	3.0	3.3	4.4	2.2	
MAR	16.1	27.9	4.1	17.3	3.0	1.7	3.6	2.1	4.6	1.5	3.4	
APR	17.6	17.0	1.0	4.8	5.3	4.4	0.1	1.7	2.9	2.2	0	
MAY	0.6	3.7	1.9	0.2	2.6	1.8	0.1	0.9	0.4	3.4	1.1	
JUN	2.9	11.3	9.9	5.2	6.8	8.1	2.3	2.0	1.9	3.6	1.3	
JUL	3.3	3.0	5.7	4.4	5.5	4.6	1.7	0.1	1.3	4.2	0.4	
AUG	5.7	10.1	0.1	0.3	5.8	2.2	3.9	2.4	0.7	6.2	2.1	
SEP	5.5	11.2	8.1	2.1	1.8	2.1	2.3	0.5	5.2	1.4	2.8	
OCT	11.3	27.7	4.2	6.6	2.9	0.6	8.6	1.3	2.0	5.1	2.3	
NOV	15.5	8.4	2.1	7.4	9.0	8.2	0.8	0	4.5	5.0	1.7	
DEC	5.7	16.9	3.3	2.5	1.1	1.0	8.5	1.4	0.5	0	1.4	
TOTAL	88.5	170.0	43.6	68.6	54.4	37.7	42.1	16.3	29.5	40.9	22.5	
MEAN	7.4	14.2	3.6	5.7	4.5	3.1	3.5	1.4	2.5	3.4	1.9	

VALUES OF THE AERODYNAMIC TERM OBTAINED WITH VARIOUS DATA. (30-DAY MEANS OF DAILY EVALS.)

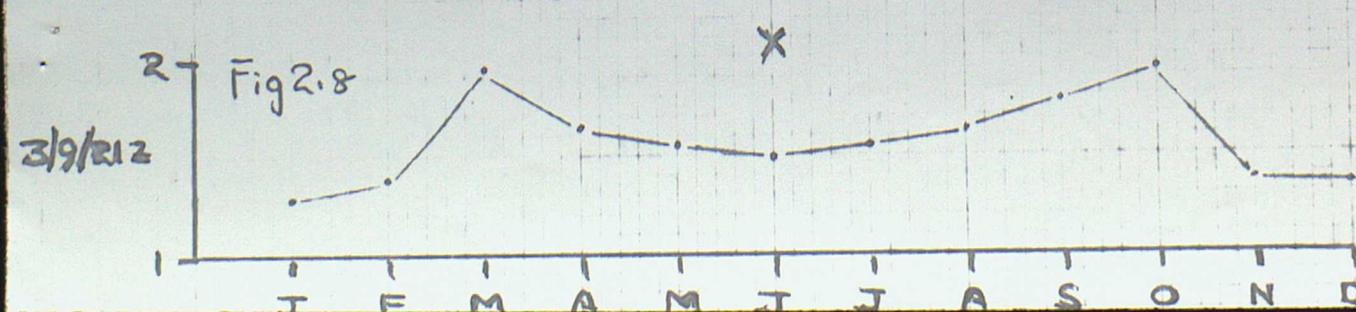
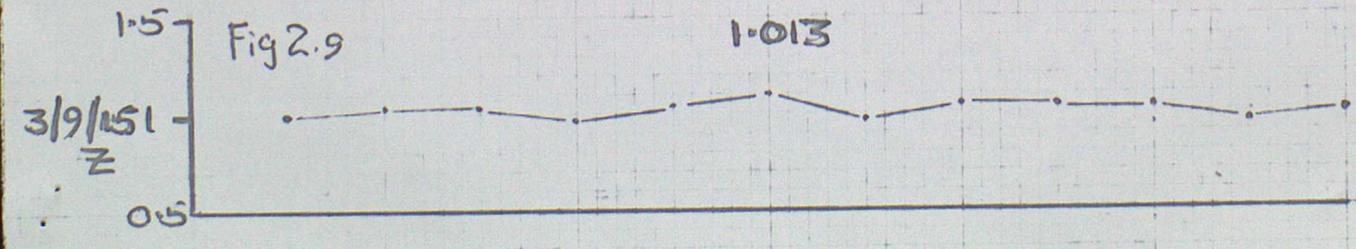
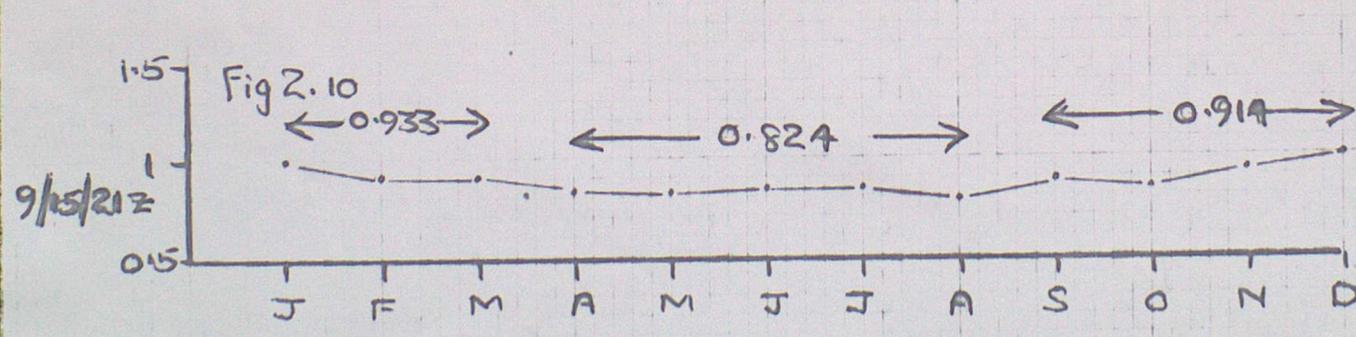
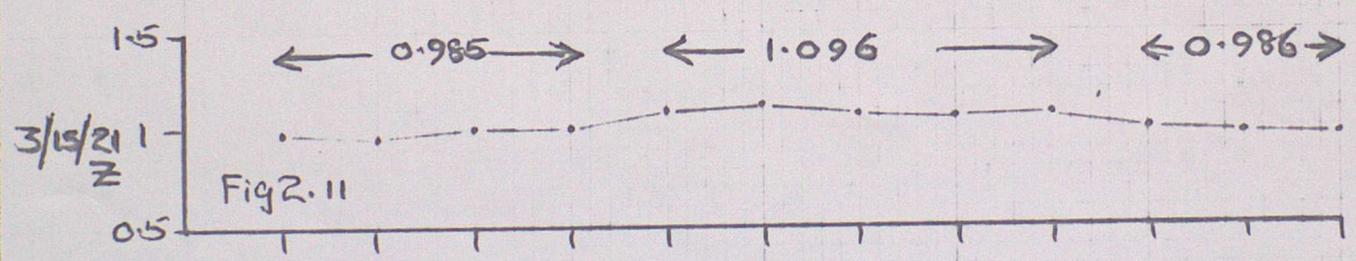
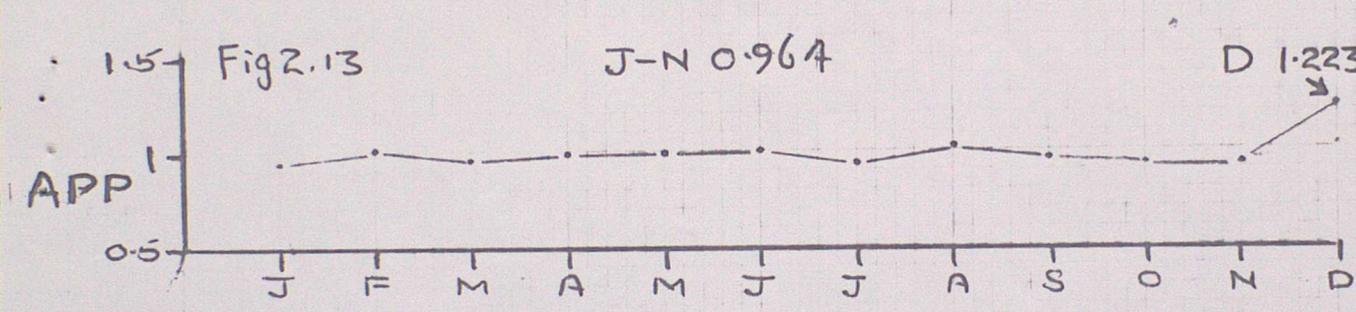
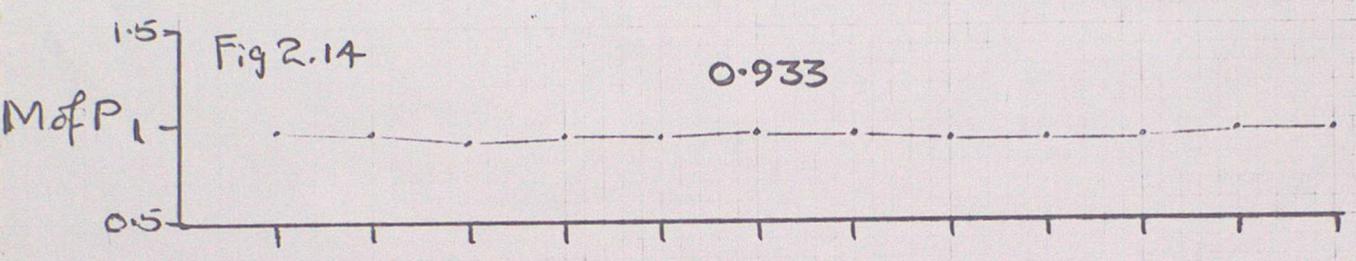
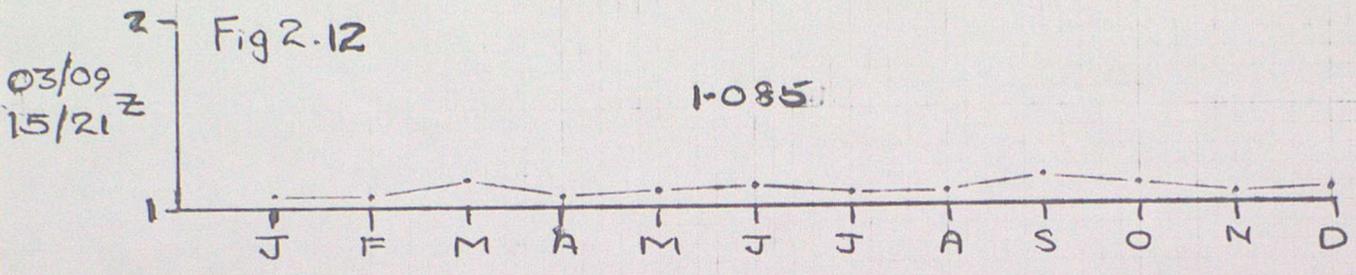
DIAGRAM SHOWS RELATIONSHIP TO "PRODUCT OF MEANS" $\left\{ \begin{array}{l} + = > 4 \text{ ABOVE} \\ \square = \text{WITHIN } \pm 4 \\ - = > 4 \text{ BELOW} \end{array} \right.$

DATA	J	F	M	A	M	J	J	A	S	O	N	D	TYPE
APPROX							+						13
03	Z	-	-	-	-	-	-	-	-	-	-	-	1
09	Z	-	-	-	-	-	-	-	-	-	-	-	2
15	Z	+	+	+	+	+	+	+	+	+	+	+	3
21	Z			-	-	-	-	-	-	-	-	-	4
03 15	Z			+	+	-	-	-	-	+	+		5
09 21	Z	-		-	-	-	-	-	-	-	-	-	6
09 15	Z		+	+	+	+	+	+	+	+	+		7
03 09 21	Z	-	-	-	-	-	-	-	-	-	-	-	8
03 09 15	Z									+			9
09 15 21	Z				+	+	+	+	+	+			10
03 15 21	Z				-	-	-	-	-	-			11
03 09 15 21	Z				-	-	-	-	-	-			12
MEAN OF PRODS.					+	+	+	+					14



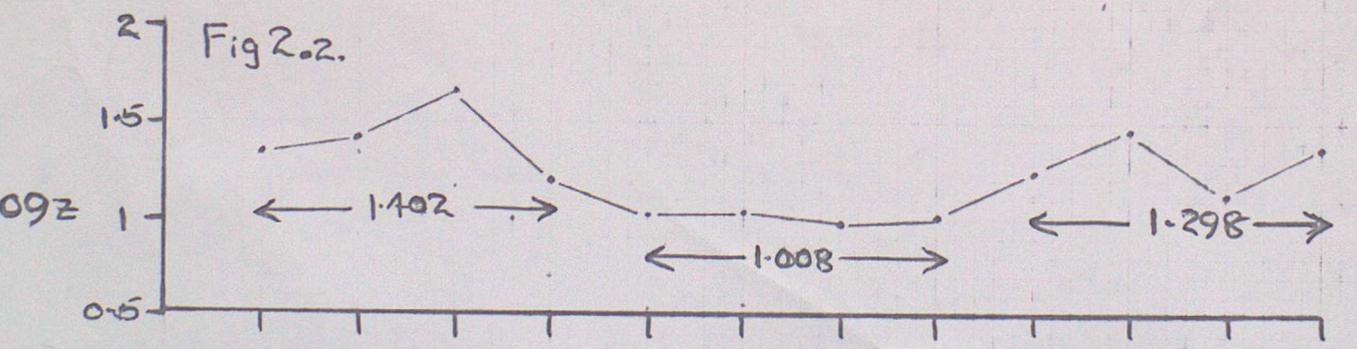
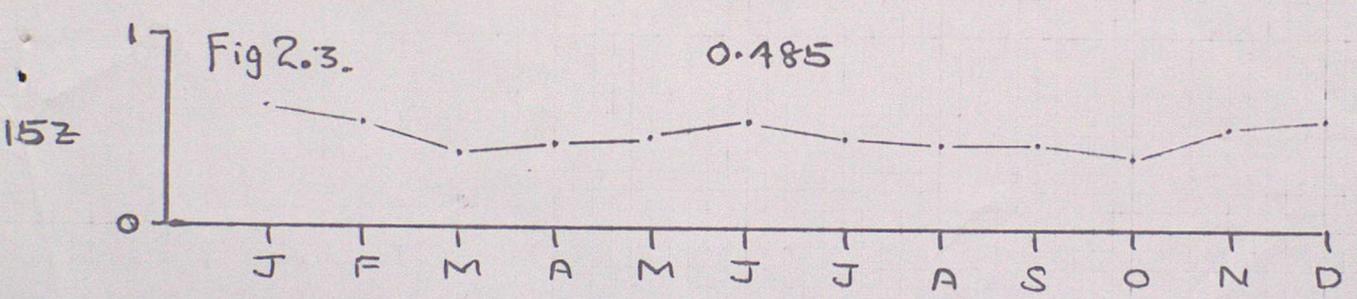
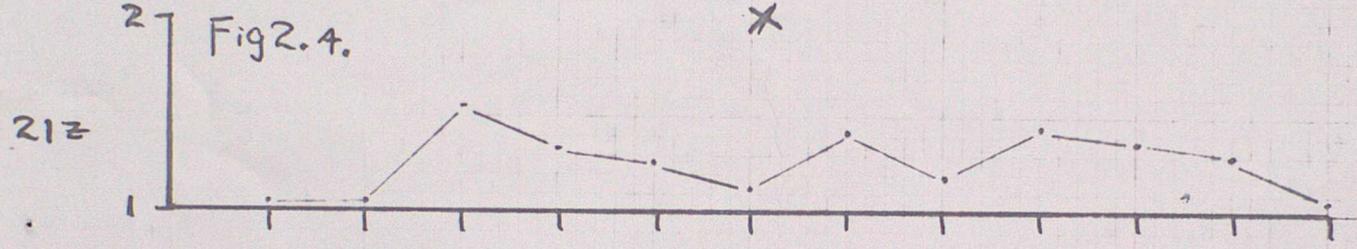
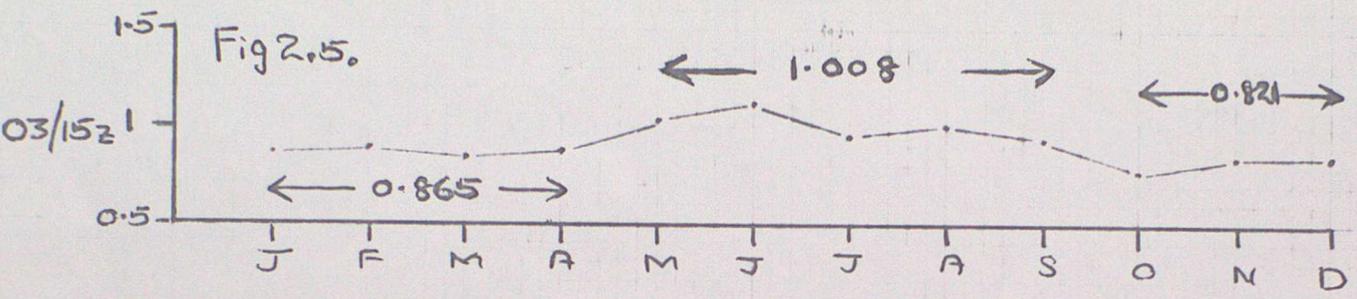
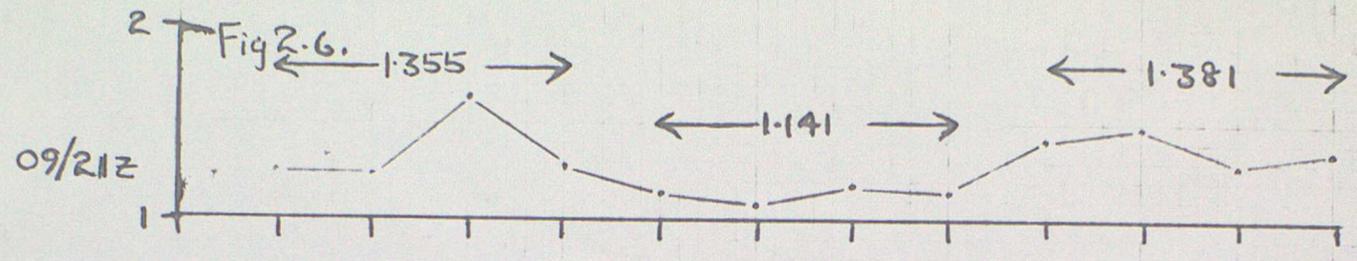
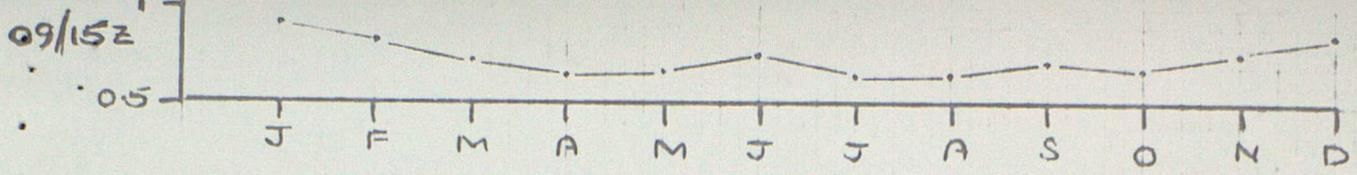
30-DAY MEAN OF DAILY A-TERM BY PRODUCT OF MEANS

30-DAY MEAN OF DAILY A-TERM BY VARIOUS METHODS



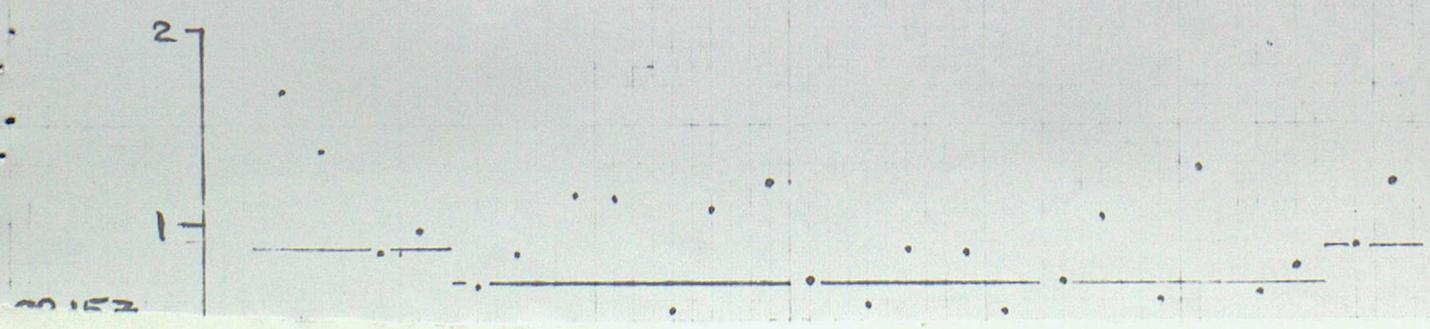
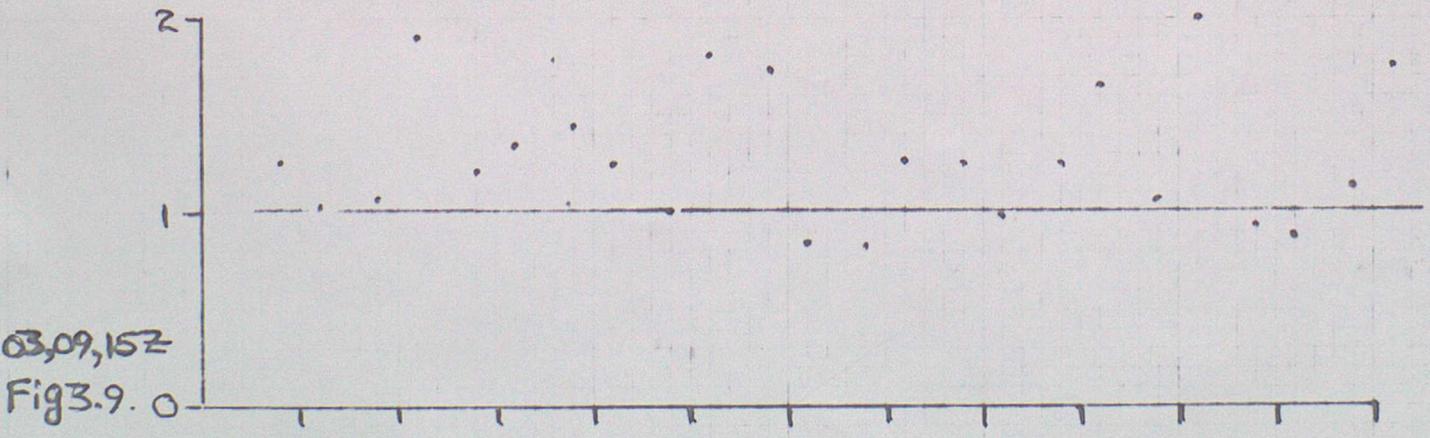
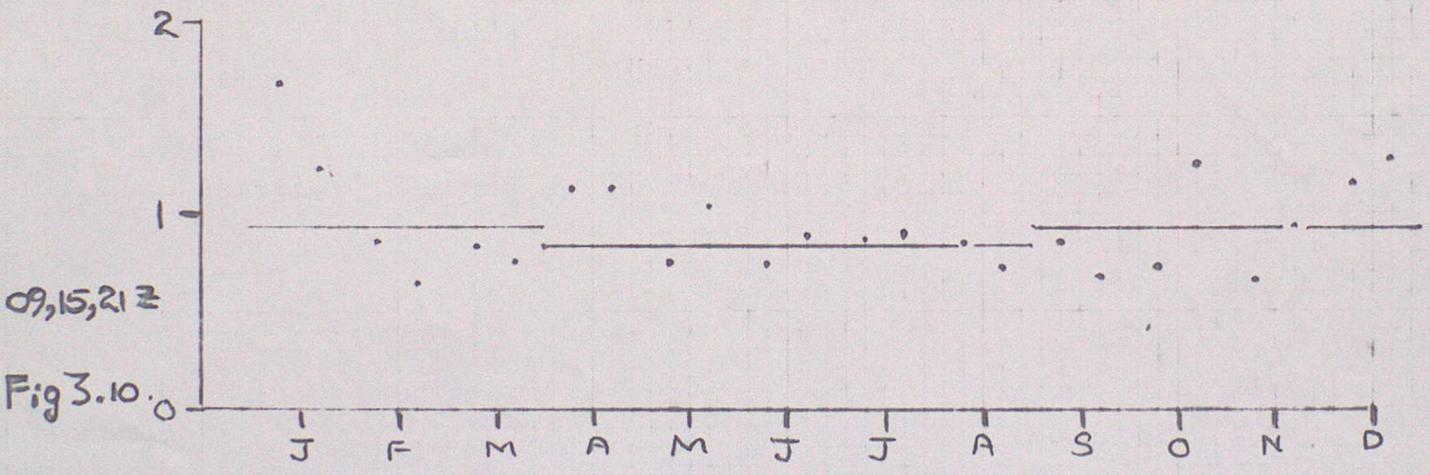
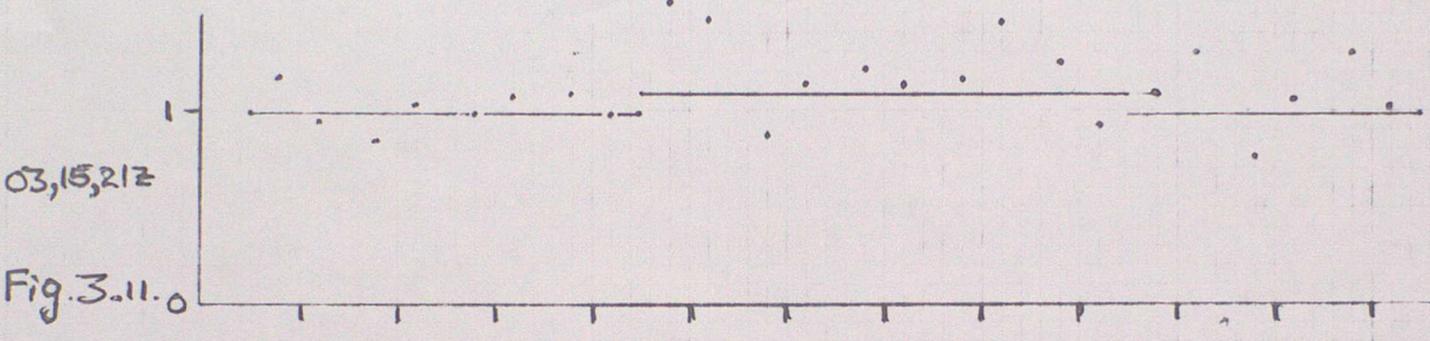
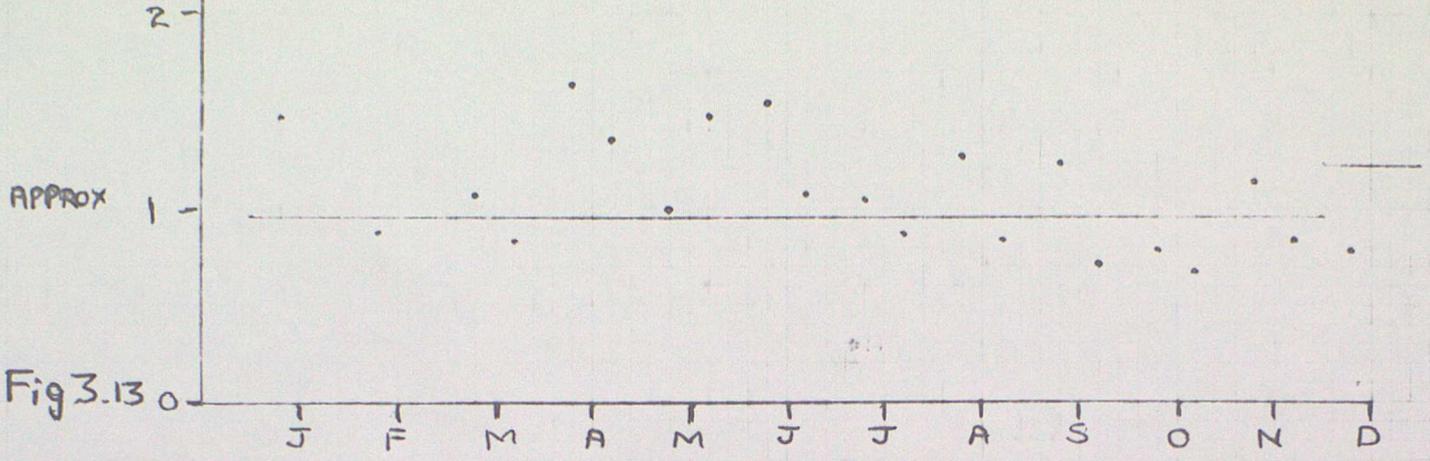
30-DAY MEAN OF DAILY A-TERM BY PROD. OF MEANS
 30-DAY MEAN OF DAILY A-TERM BY VARIOUS METHODS

Fig JFD M-N
 0.873 0.697
 2.7.



DAILY A-TERM BY PRODUCT OF MEANS
 DAILY A-TERM BY VARIOUS METHODS
 compared with "30-day factors" from Table III

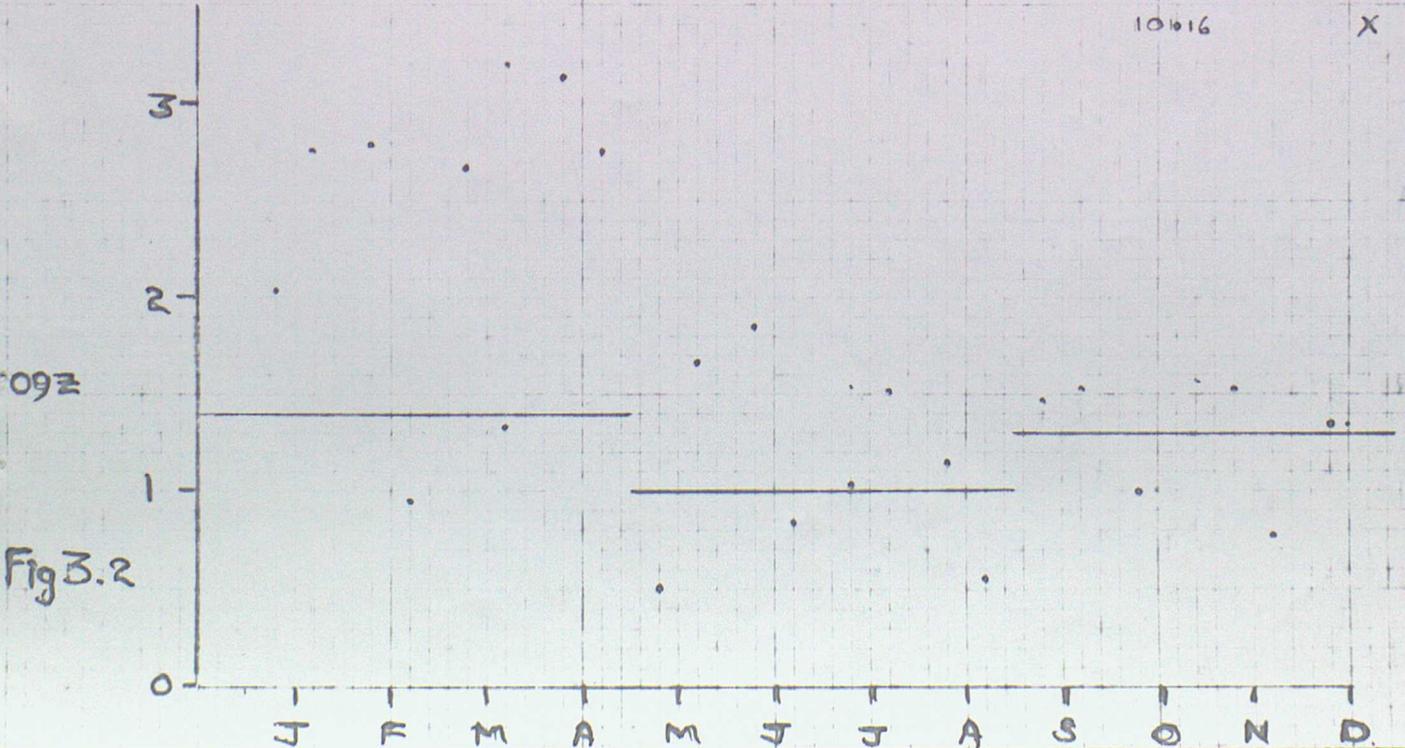
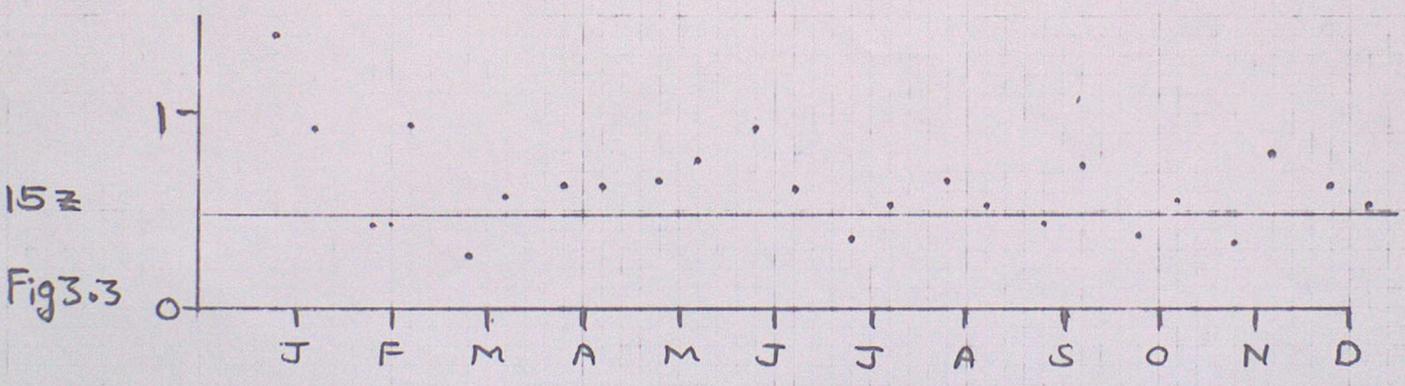
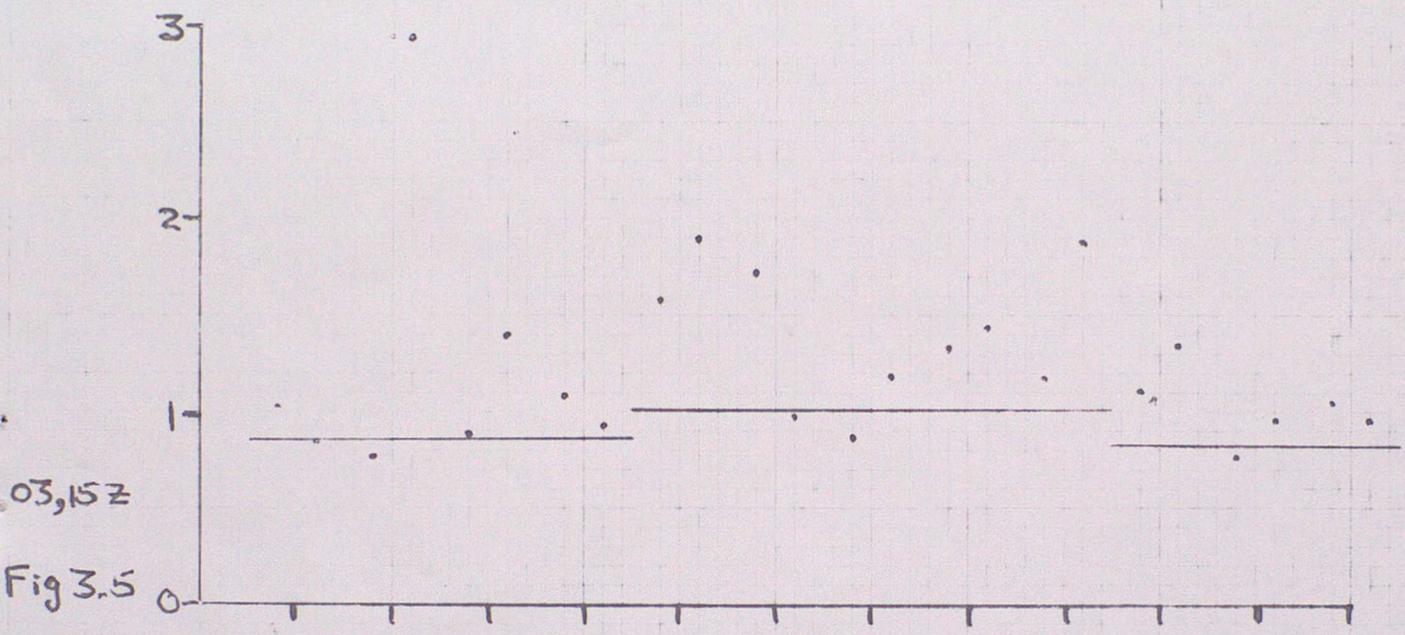
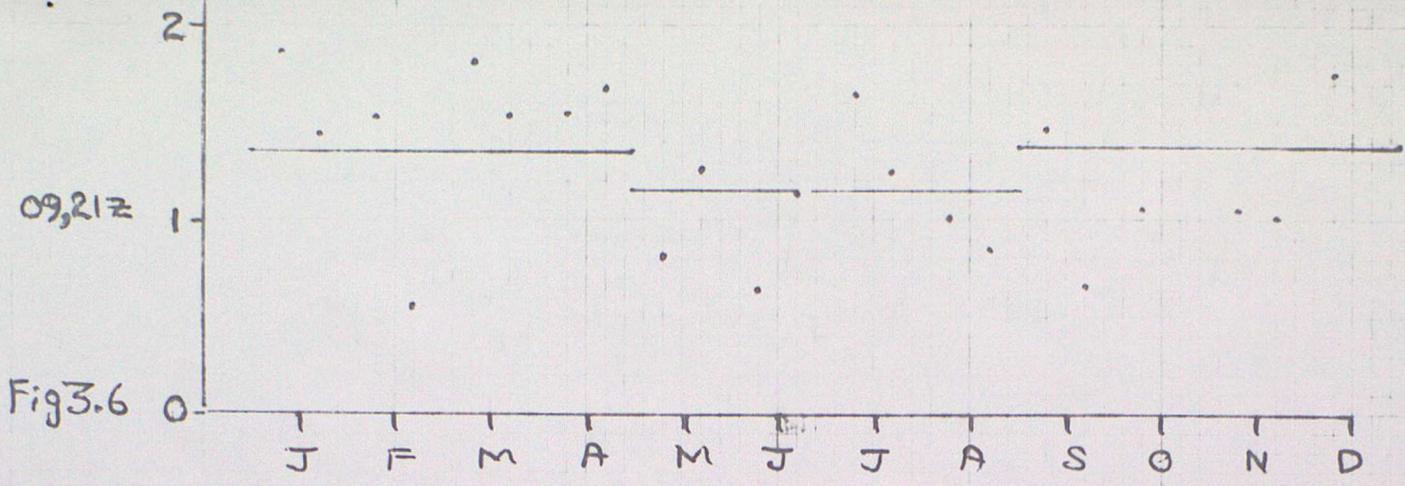
x x



DAILY A-TERM BY PRODUCT OF MEANS

2-198

DAILY A-TERM BY VARIOUS METHODS
compared with "30-day factors" from Table III



DAILY A-TERM BY PRODUCT OF MEANS
DAILY A-TERM BY VARIOUS METHODS
 compared with "30-day factors" from Table III

